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THE GILBERT ARITHMETICS



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THE GILBERT ARITHMETICS

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BOOK TWO

PART ONE

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PREFACE

THE authors have endeavored to present such new material and method, culled from the mass of recent contributions to the subject of arithmetic, as is of real value for children in school, and, at the same time, to retain or restore the admirable features found in the older arithmetics.

The older arithmetics were strongly scientific and treated arithmetic seriously, as a subject demanding systematic presentation, orderly statement, and clear scientific definitions and rules.

They were, however, authoritative in method, ignoring the order in which the mind naturally acquires its knowledge and secures its generalizations, and they gave little heed to the broader, cultural values of the subject.

The principal merits of the newer books are psychological, and consist in the use of the inductive method of presentation and in the employment of practical and vital problems. Their weakness is in the failure to present arithmetic as a science deserving careful and orderly development.

The method of presentation in these books is inductive, until *principles have been thoroughly established and illuminated*. Then the principles are stated "in good set terms." After such statement has been made it is utilized, and illustrations and problems are founded upon it.

The books recognize both the merits and the limitations of the so-called "spiral plan."

Up to a certain point this plan is psychologically sound. Repeated experiences, if attended with the interest incident to the addition of new features at each recurrence, unques-

tionably fix and intensify impressions. They furnish the natural way of learning of infants and of the unschooled.

But the schooled adult soon acquires from his experiences that great economizer, the generalization, which he thereafter uses constantly; hence the limitations of the spiral plan in an arithmetic. It should be used long enough to establish generalizations. When this has been done for any subject, it should be dropped. The period for induction has passed. Thereafter topics should be treated topically, except as frequent reviews are used to freshen and deepen impressions.

The problems are drawn, in so far as possible, from fields of activity within the experience or ready comprehension of children, and likely to appeal to their interest. But when, to illustrate a principle, there is need of problems that are not derived naturally from the common experiences of children, the authors have deemed it better to supply good problems that meet the mathematical need than to introduce inferior illustrations that stimulate interest. Vital problems are important, but *the mathematical principle is the thing*, after all.

This book is intended for the fifth and sixth years of the average graded school. The normal age of the pupils will range from ten to twelve years.

This is the psychological age for drill, when memory is at its best and reason is just beginning to show strength. Consequently, memory should be utilized and reason developed.

The arrangement of this book is increasingly topical, only a few of the more difficult subjects, as percentage, being treated recurrently, in preparation for their final discussion in Book Three.

As many pupils leave school after completing the sixth year, a few important business applications are given, such as interest and the simplest facts of banking.

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PART I

INTRODUCTION

REVIEW AND PREPARATION

NOTATION AND NUMERATION

EXERCISES

1. Write in figures and also in Roman characters :
All the numbers from 1 to 20.
All the tens from 20 to 100.
All the hundreds from 100 to 1000.
2. How many different figures did you use in writing ?
3. How many different Roman letters did you use ?
4. Read 165,276.
5. Name each figure from its position.
6. Into what does the comma divide the number ?
7. Write, divide into periods, and read the following :
5673218.
Sixteen thousand, five hundred one.
One million.
One thousand, one hundred one.
7230841.
25036.
8. Write in figures, and read :
CDVI. CC. XCIH. MD.
CXIII. XXXIX. XIX. LXXIV.

ADDITION AND SUBTRACTION

ORAL EXERCISES

Give the sums rapidly :

Add 1 to: 1, 2, 3, 4, 5, 6, 7, 8, 9.

Add 2 to: 2, 3, 4, 5, 6, 7, 8, 9.

Add 3 to: 3, 4, 5, 6, 7, 8, 9.

Add 4 to: 4, 5, 6, 7, 8, 9.

Add 5 to: 5, 6, 7, 8, 9.

Add 6 to: 6, 7, 8, 9.

Add 7 to: 7, 8, 9.

Add 8 to: 8, 9.

Add 9 to: 9.

WRITTEN EXERCISES

1. Write a problem, using the signs $+$, $-$, $=$.

2. Add: 267

376

Tell how you find each figure of the result.

What name is given to the result?

ORAL EXERCISES

3. Subtract rapidly :

1 from: 2, 3, 4, 5, 6, 7, 8, 9.

2 from: 3, 4, 5, 6, 7, 8, 9.

3 from: 4, 5, 6, 7, 8, 9.

4 from: 5, 6, 7, 8, 9.

5 from: 6, 7, 8, 9.

6 from: 7, 8, 9.

7 from: 8, 9.

8 from: 9.

4. Subtract: 453
164

Tell how you find each figure of the result.

What name is given to 453? To 164? To the result?

5. First add, then subtract:

$$\begin{array}{r} \$26.73 \\ \underline{18.94} \end{array}$$

$$\begin{array}{r} \$3.02 \\ \underline{1.83} \end{array}$$

$$\begin{array}{r} \$5.00 \\ \underline{2.65} \end{array}$$

$$\begin{array}{r} \$176.31 \\ \underline{102.05} \end{array}$$

MULTIPLICATION

ORAL EXERCISES

Multiply rapidly:

$$2 \times: 2, 3, 4, 5, 6, 7, 8, 9.$$

$$3 \times: 3, 4, 5, 6, 7, 8, 9.$$

$$4 \times: 4, 5, 6, 7, 8, 9.$$

$$5 \times: 5, 6, 7, 8, 9.$$

$$6 \times: 6, 7, 8, 9.$$

$$7 \times: 7, 8, 9.$$

$$8 \times: 8, 9.$$

$$9 \times: 9.$$

WRITTEN EXERCISES

1. Write an example using the signs $\times$ and $=$.

2. Multiply: 465
37

Tell how you find each figure of the result.

What name do you give to 465? To 37? To the result?

3. Multiply :

$$\begin{array}{r} 365 \\ \underline{47} \end{array}$$

$$\begin{array}{r} \$42.50 \\ \underline{25} \end{array}$$

$$\begin{array}{r} 6200 \\ \underline{12} \end{array}$$

$$\begin{array}{r} 4800 \\ \underline{2000} \end{array}$$

DIVISION

ORAL EXERCISES

Divide rapidly :

$$4 \div 2.$$

$$6 \div 2; 6 \div 3.$$

$$8 \div 2; 8 \div 4.$$

$$9 \div 3.$$

$$10 \div 2; 10 \div 5.$$

$$12 \div 2; 12 \div 3; 12 \div 4.$$

$$14 \div 2; 14 \div 7.$$

$$15 \div 3; 15 \div 5.$$

$$16 \div 2; 16 \div 4; 16 \div 8.$$

$$18 \div 2; 18 \div 3; 18 \div 6; 18 \div 9.$$

$$20 \div 2; 20 \div 4; 20 \div 5; 20 \div 10.$$

WRITTEN EXERCISES

1. Divide :

$$100 \text{ by } 2; \text{ by } 4; \text{ by } 5; \text{ by } 10.$$

$$99 \text{ by } 3; \text{ by } 9; \text{ by } 11.$$

$$98 \text{ by } 2; \text{ by } 7.$$

$$96 \text{ by } 2; \text{ by } 3; \text{ by } 4; \text{ by } 6; \text{ by } 8; \text{ by } 12.$$

$$95 \text{ by } 5; \text{ by } 19.$$

$$94 \text{ by } 2.$$

$$93 \text{ by } 3.$$

$$92 \text{ by } 2; \text{ by } 4.$$

$$90 \text{ by } 2; \text{ by } 3; \text{ by } 5; \text{ by } 6; \text{ by } 9; \text{ by } 10.$$

2. Write a problem using the signs $\div$ and $=$.

3. Divide: $21 \overline{)252}$.

Tell how you find each figure of the result.

What name do you give to 21? To 252? To the result?

4. Divide:

$$21 \overline{)105}$$

$$31 \overline{)310}$$

$$41 \overline{)615}$$

$$61 \overline{)920}$$

FRACTIONS

REDUCTION

ORAL EXERCISES

1. Read: $\frac{1}{2}$; $\frac{3}{4}$; $\frac{20}{25}$.
2. What are these expressions called?
3. What name is given to the figure above the line? To the figure below the line?
4. Reduce to eighths: $\frac{1}{2}$; $\frac{1}{4}$.
5. Reduce to ninths: $\frac{1}{3}$.
6. Reduce to twelfths: $\frac{1}{2}$; $\frac{1}{3}$; $\frac{1}{4}$; $\frac{1}{6}$.
7. Reduce to fifteenths: $\frac{1}{3}$; $\frac{1}{5}$.
8. Reduce to sixteenths: $\frac{1}{2}$; $\frac{1}{4}$; $\frac{1}{8}$.
9. Reduce to eighteenths: $\frac{1}{2}$; $\frac{1}{3}$; $\frac{1}{6}$; $\frac{1}{9}$.
10. Reduce to twentieths: $\frac{1}{2}$; $\frac{1}{5}$; $\frac{1}{10}$.
11. Reduce to twenty-seconds: $\frac{1}{2}$; $\frac{1}{11}$.
12. Reduce to twenty-fourths: $\frac{1}{2}$; $\frac{1}{3}$; $\frac{1}{4}$; $\frac{1}{6}$; $\frac{1}{8}$; $\frac{1}{12}$.

ADDITION AND SUBTRACTION

ORAL EXERCISES

1. Add: $\frac{1}{2} + \frac{1}{3}$. $\frac{1}{3} + \frac{1}{4}$. $\frac{3}{4} + \frac{1}{5}$.
 $\frac{2}{5} + \frac{1}{6}$. $\frac{1}{2} + \frac{1}{4}$. $\frac{1}{2} + \frac{1}{5}$.
 $\frac{1}{2} + \frac{1}{6}$. $\frac{2}{3} + \frac{1}{8}$. $\frac{5}{8} + \frac{1}{4}$.

2. Subtract:

$\frac{2}{3} - \frac{1}{3}.$

$\frac{2}{3} - \frac{1}{4}.$

$\frac{2}{3} - \frac{1}{5}.$

$\frac{1}{4} - \frac{1}{8}.$

$\frac{3}{4} - \frac{1}{2}.$

$\frac{3}{4} - \frac{1}{8}.$

$\frac{1}{5} - \frac{1}{10}.$

$\frac{1}{6} - \frac{1}{12}.$

$\frac{5}{8} - \frac{3}{8}.$

MULTIPLICATION

ORAL EXERCISES

1. Multiply: $\frac{2}{3} \times 3.$ $\frac{3}{4} \times 5.$ $\frac{15}{16} \times 2.$
2. How do you multiply a fraction by a whole number?
3. Multiply: $\frac{1}{5} \times \frac{2}{3}.$ $\frac{3}{4} \times \frac{1}{2}.$ $\frac{5}{6} \times \frac{9}{8}.$
4. What is $4\frac{1}{5}$ called?
5. How many fifths do $4\frac{1}{5}$ equal?
6. What name is given to $2\frac{1}{5}$?

WRITTEN EXERCISES

1. Multiply $6 \times 2\frac{1}{3}$:
 $6 \times 2 = ?$ $6 \times \frac{1}{3} = ?$ $6 \times 2\frac{1}{3} = ?$
2. Multiply:
 $\frac{1}{3} \times \frac{2}{3}.$ $\frac{1}{4} \times \frac{1}{2}.$ $\frac{1}{6} \times \frac{5}{7}.$
 $16 \times 1\frac{1}{2}.$ $10 \times 2\frac{1}{5}.$ $8 \times 3\frac{1}{4}.$

MEASURES

ORAL EXERCISES

Read, and supply the missing numbers:

1. 1 yd. = — ft. ; = — in.
2. 1 mi. = — rd. ; = — yd. ; = — ft.
3. 1 sq. rd. = — sq. yd.
4. 1 sq. yd. = — sq. ft. ; = — sq. in.
5. 1 cu. yd. = — cu. ft.
6. 1 cu. ft. = — cu. in.

7. 1 gal. = — qt.; = — pt.; = — gi.
 8. 1 bu. = — pk.; = — qt.
 9. 1 yr. = — mo.; = — wk.
 10. 1 da. = — hr.; = — min.
 11. 1 dollar = — dimes; = — cents; = —
 “nickels.”
 12. 1 lb. = — oz.

WRITTEN EXERCISES

Solve:

1. Mr. Henderson bought 150 bu. of pears at \$1.25 a bushel. He sold them for 50 cts. a peck. What was his profit?

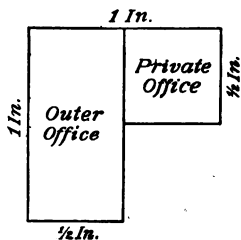
2. Mrs. Ames invited 50 children to a Christmas tree. Each child received a box containing 4 oz. of candy. What part of a pound did each receive? How many pounds had Mrs. Ames to buy for all? How much had the candy cost at 60 cts. a pound?

3. Each child received a book that cost 75 cts. How much had all the books cost?

4. This is a diagram of Dr. Williams's offices. It is drawn on a scale of 1 in. to 20 ft. What is the area of the floor of each office?

5. How many cubic feet of air are there in a room having a floor area of 20 ft. $\times$ 15 ft., and a height of 9 ft.?

6. Mr. Allen's garden is 16 rd. $\times$ 10 rd. How much land does it contain?



7. Draw a diagram of it on the scale of 1 in. to 2 rd.

8. John planted peanuts in his garden, in 12 rows of 16 hills each. How many hills did he have?

9. He allowed for each hill a $1\frac{1}{2}$ ft. square. How many feet long was each side of the bed?

10. He gathered $1\frac{1}{2}$ qt. of peanuts to a hill. How many pecks did he gather in all?

PERCENTAGE

1. How many fourths are there in 1? Write the answer as a fraction.

2. How many hundreds are there in 1? Write the answer as a fraction. Write it as a number per cent.

3. The whole of anything is what per cent. of it? One half is what per cent.?

ORAL EXERCISES

Solve:

1. What per cent. does each of the following fractions equal:

$\frac{1}{2}$?	$\frac{1}{4}$?	$\frac{3}{4}$?	$\frac{1}{8}$?	$\frac{3}{8}$?	$\frac{5}{8}$?	$\frac{7}{8}$?
$\frac{2}{3}$?	$\frac{1}{5}$?	$\frac{2}{5}$?	$\frac{3}{5}$?	$\frac{4}{5}$?	$\frac{1}{6}$?	$\frac{1}{12}$?
$\frac{1}{16}$?	$\frac{1}{10}$?	$\frac{3}{10}$?	$\frac{7}{10}$?	$\frac{9}{10}$?	$\frac{1}{3}$?	$\frac{1}{8}$?

2. Mr. Weston has 30 acres of land. How many acres are there in 10% of it? In 50%? In 25%? In 20%?

3. There are 160 rods in an acre. How many rods in .5 of an acre? In .05 of an acre? In .005 of an acre? In $12\frac{1}{2}$ % of an acre?

4. There are 250 sheep in a yard. 20% of them were sold. How many remained? $12\frac{1}{2}$ % of the remainder were put into a pasture. How many were put into the pasture? How many then remained in the yard?

5. John lost 40 marbles, which were 25% of all he had. How many had he?

SUGGESTION: 40 is 25% of what number?

6. James lost 30 marbles, which was $12\frac{1}{2}\%$ of all he had. How many had he?

SUGGESTION: 30 is $12\frac{1}{2}\%$ of what number?

7. 50 acres is 50% of Mr. Milton's farm. It is equal to 25% of Mr. Brown's farm, $33\frac{1}{3}\%$ of Mr. Elston's farm, $16\frac{2}{3}\%$ of Mr. Clinton's farm, $8\frac{1}{3}\%$ of Mr. Hudson's farm, 20% of Mr. Washington's farm, 40% of Mr. Raymond's farm, and $12\frac{1}{2}\%$ of Mr. Hilton's farm. How many acres has each?

$$8. \frac{100}{100} - \frac{25}{100} = 100\% - 25\% =$$

$$9. 100\% - 12\frac{1}{2}\% =$$

$$19. 100\% - 80\% =$$

$$10. 100\% - 87\frac{1}{2}\% =$$

$$20. 1 - 12\frac{1}{2}\% =$$

$$11. 100\% - 6\% =$$

$$21. 1 - 83\% =$$

$$12. 100\% - 90\% =$$

$$22. 1 - 85\% =$$

$$13. 100\% - 40\% =$$

$$23. 100\% - 62\frac{1}{2}\% =$$

$$14. 1 - 75\% =$$

$$24. 100\% - 2\% =$$

$$15. 1 - 95\% =$$

$$25. 100\% - 60\% =$$

$$16. 100\% - 37\frac{1}{2}\% =$$

$$26. 100\% - 8\% =$$

$$17. 100\% - 20\% =$$

$$27. 1 - 87\frac{1}{2}\% =$$

$$18. 100\% - 4\% =$$

$$28. 1 - .10 =$$

28. James Dayton sold .35 of his oats. How many hundredths had he left?

29. Carl Simons spent .82 of his income. What per cent. of his income was left? How many hundredths of his income were left?

30. Mary Dunn lost 45% of her money? How many hundredths had she left?

31. If she had \$50 at first, how much remained?

WRITTEN EXERCISES

Solve :

1. Find 6 % of : 450 ; 785 ; 4762 ; 18 ; 1052 ; 3001 ; 25.
2. Find 5 % of : \$ 35 ; \$ 4225 ; \$ 19,020 ; \$ 74,750.
3. Find $4\frac{1}{2}$ % of : \$ 200 ; \$ 1652 ; \$ 4350 ; \$ 1774 ; \$ 26,740 ; \$ 51,818.
4. A girl graduate of a trade school went to work in a millinery shop at \$3.50 a week. After a year she was receiving 40 % more per week than at the start. (1) How much was she then getting ?
The next year her wages were advanced 20 %. (2) How much did she then receive a week ?
Later she received a further increase of $33\frac{1}{3}$ %. (3) What was her final weekly wage ?
5. Mr. Allison bought 125 yd. of silk, at 75 cts. a yard. What was the amount of the bill ?
6. He was allowed to subtract from it 6 % for paying cash ? How much did he take off ? How much did he pay ?
7. Mr. Brown had a farm of 320 acres. 20 % of it was woodland. How many acres were woodland ?
8. 30 % was pasture. How many acres were pasture ?
The remainder was in wheat. What per cent was in wheat ? How many acres ?
9. He paid \$15 an acre for the woodland and pasture. How much did it cost him ?
10. For the wheat land he paid 50 % more. What did that cost him ?
11. What did the whole farm cost ?
12. He sold the farm for 15 % more than it cost him. How much did he receive for it ?

CHAPTER I

NUMBERS TO BILLIONS — THE FOUR OPERATIONS — THE SOLUTIONS — DENOMINATE NUMBERS AND MEASUREMENT

NOTATION AND NUMERATION

ARABIC NUMBERS

1. All numbers, however large, may be expressed by the use of one or more of ten characters called **figures**: 1, 2, 3, 4, 5, 6, 7, 8, 9, 0. Of these figures, 1, 2, 3, 4, 5, 6, 7, 8, 9, are called **digits**. 0 is called **cipher**, **zero**, or **naught**. It has no value when standing alone.

2. Writing numbers according to a regular plan, or system, is called **notation**.

Numbers expressed by the use of figures are said to be written in the **Arabic notation**. This is so called because Europeans learned it from the Arabs.

Reading numbers that have been written according to a system of notation is called **numeration**.

3. In 8,765, the 5 is called a **figure of the first order**; it stands for five units. The 6 is a **figure of the second order**; it stands for six tens or 60. For what does the 7 stand? The 8?

Orders, in the Arabic notation, are numbered from right to left. The first order at the right is called **unit's order**; the second, **ten's order**; and so on. Name the third, fourth, and fifth orders.

Write the value of each order in 8,765.

ORAL EXERCISES

4. Read and answer :

1. How many units make a ten? How many tens make a hundred? How many hundreds make a thousand?

2. In 111, what is the ratio of the 1 in ten's place to the 1 in unit's place? Of the 1 in hundred's place to that in ten's?

3. In what ratio does the value of a figure in a number increase when moved one order from right to left?

4. How does the value of a figure change when moved one order from left to right?

5. What is the value of 0? What does 0 stand for in 10? In 205?

6. Give the names of the places filled by the ciphers in the following numbers: 20; 200; 1,025; 20,106.

7. In 8,765, of what order is 8? What is its value? Why are three ciphers used in 8,000 but not in 8,765?

8. In 5,468,921, write the value of the figure of the second order. The sixth. The third. The seventh. The fifth.

5. In Arabic notation, numbers larger than 999 are written and read in groups, called **periods**. The periods are separated by commas. Each period contains three orders. The period at the left, however, may be incomplete. It may contain but one or two orders, as in 17,824 or 5,630,375.

Each period is named from its right-hand order.

Thus, in 4,212,256, the period 256 is called **unit's period**, from its right-hand order, units; the period 212 is called **thousand's period**, from its right-hand order, thousands. What is the name of the incomplete period?

A number is read from the left. Each period is read as if it were a separate number and, except unit's period, with the name of the period added.

For example, 60,400,900 contains :

THIRD PERIOD	SECOND PERIOD	FIRST PERIOD
Millions	Thousands	Units
60,	400,	900.

It is read, "Sixty million, 4 hundred thousand, 9 hundred."*

6. One billion is written 1,000,000,000.

					1,000,000,000
					256,000,000
					805,000
					370
					1,256,805,370
Billions	Hundred Millions Ten Millions Millions	Hundred Thousands Ten Thousands Thousands	Hundreds Tens Units		
1,	2 5 6,	8 0 5,	3 7 0.		

1. Read this number. How many periods are there in it?

2. Name: the first period at the right; the second period; the third period; the fourth period.

3. What is the ratio of the second order of a period to the first? To the third? To the second order of the preceding period?

4. How many figures are there in each period in the number above?

5. Which period contains less than three figures? Why may this period have less than three figures? If, in any other period, there is no digit of a certain order, what takes its place? What instances of this are there in the number above?

*The second order of each period is the ten of that period, and the third order the hundred. Thus, in 215,674, the 1 in thousand's period is 1 ten thousand; the 2 is 2 hundred thousand.

ORAL EXERCISES

7. Read :

- | | | |
|-------------|---------------------|------------------|
| 1. 20,306. | 7. 327,408,375,604. | 13. 10,200,500. |
| 2. 75,208. | 8. 515,036,002. | 14. 45,360,720. |
| 3. 99,364. | 9. 720,306,040. | 15. 54,207,804. |
| 4. 104,206. | 10. 4,446,284. | 16. 60,500,125. |
| 5. 315,724. | 11. 16,370,703. | 17. 100,200,300. |
| 6. 9,215. | 12. 98,098,098. | 18. 200,146,725. |

WRITTEN EXERCISES

8. Copy and separate into periods :

- | | | |
|---------------|---------------|----------------|
| 1. 3486502. | 5. 420415002. | 9. 34056784. |
| 2. 5213869. | 6. 90009. | 10. 705864903. |
| 3. 4013. | 7. 21300405. | 11. 40005. |
| 4. 600700070. | 8. 215375. | 12. 73050. |

9. Write in figures :

1. 24 million, 307 thousand, 6 hundred 12.
2. 6 million, 24 thousand, 9 hundred 25.
3. 40 thousand, 4 hundred.
4. 1 million, 1 thousand, 1.
5. The millions from twenty-five to thirty-five.
6. One hundred millions.
7. The hundred millions, from one hundred to nine hundred.
8. One hundred twenty billions.

10. Write in words :

- | | | |
|------------|-------------|-----------------|
| 1. 27,300. | 4. 526,700. | 7. 19,018,007. |
| 2. 85,040. | 5. 408,520. | 8. 156,240,300. |
| 3. 96,005. | 6. 925,006. | 9. 704,060,110. |

ROMAN NUMBERS

11. Another system of notation is called the **Roman system**, because it was used by the ancient Romans. It is not so good a system as the Arabic, especially for expressing large numbers. Its use now is confined almost wholly to the faces of clocks and watches, and to the numbering of chapters and certain pages in books.

The Roman characters and their Arabic equivalents are :

I = 1 V = 5 X = 10 L = 50 C = 100 D = 500
M = 1000

These characters may be so combined as to represent all numbers.

If a Roman numeral follows one of equal or greater value, its value is added.

If a Roman numeral is placed before one of greater value, its value is subtracted.

ORAL EXERCISES

12. State the values of the following :

XIV.	XL.	XC.	CCXII.	MCCIII.
XV.	LV.	XCIX.	CCLXIX.	MD.
XVI.	LVI.	CC.	CLXXIX.	MDCCCC.
XIX.	LX.	CD.	CM.	MCM.
XXX.	LXIV.	DCC.	MC.	MDCCCCVIII.

WRITTEN EXERCISES

13. Write the following in Roman numerals :

57	86	114	146	800
64	94	120	159	869
77	100	124	210	550
84	105	140	240	1500

ADDITION

14. How many hours are there in a week?

SOLUTION :	Sunday	=	24 hours
	Monday	=	24 hours
	Tuesday	=	24 hours
	Wednesday	=	24 hours
	Thursday	=	24 hours
	Friday	=	24 hours
	Saturday	=	24 hours
	A week	=	<u>168</u> hours

We combine the number of hours in the different days into a single number of hours, 168.

The numbers combined we call **addends**. The result we call the **sum**.

Finding the sum of two or more numbers is called **addition**.

Addition may be indicated by the sign $+$, read "plus" or "and." Read: $5 + 6 = 11$.

ORAL EXERCISES

1.	3, 5, 2, 8, 5, 7, 3, 4, 6, 9
2.	4, 4, 1, 3, 4, 5, 6, 7, 1, 5
3.	3, 6, 3, 7, 6, 1, 2, 3, 8, 3
4.	5, 5, 2, 3, 3, 6, 6, 5, 3, 9
5.	6, 7, 4, 5, 4, 5, 1, 7, 8, 4
6.	4, 9, 6, 1, 8, 4, 9, 6, 2, 8
7.	9, 5, 5, 4, 7, 8, 5, 1, 7, 7
8.	7, 2, 7, 9, 6, 9, 7, 8, 9, 1
9.	8, 8, 7, 6, 1, 1, 7, 8, 5, 6
10.	2, 3, 8, 2, 9, 2, 4, 2, 4, 2

15. Add the numbers in rows 10 and 9; as $2 + 8 = 10$, $3 + 8 = 11$, giving results only. Then, in like manner, the numbers in rows 9 and 8, 8 and 7.

Add the three numbers in rows 10, 9, and 8; as $2 + 8 + 7$, and in other adjacent rows.

Add the numbers in four rows.

Find the sums of the several columns. Also, find the sums of the several rows.

16. In the following table, find in the different columns, a, b, c, d , etc., also in the rows, A, B, C , etc., numbers whose sum is 10. For example, in column $a : 3, 7; 4, 6; 2, 4, 4$.

	a	b	c	d	e	f	g	h	i	j	k	l
A	8	3	7	3	4	5	1	9	2	1	7	2
B	3	7	3	8	5	4	9	1	2	7	1	8
C	7	8	3	4	1	9	6	7	5	1	4	3
D	4	4	8	9	2	4	4	6	3	2	4	6
E	6	7	1	2	9	1	5	4	3	7	3	8
F	4	2	5	3	2	9	3	5	2	4	7	2
G	5	1	4	5	8	6	4	2	9	0	1	3
H	2	4	7	6	8	3	9	1	3	5	4	4
I	4	2	6	5	3	7	8	1	1	4	2	2
J	4	6	1	4	8	1	5	9	8	1	7	6
K	8	2	9	3	5	7	1	2	3	5	6	4
L	5	7	3	9	1	2	5	3	4	6	8	3

17. Add rapidly, naming only the sums:

9	8	7	6	5	4	3	2	1	4	3
<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	<u>5</u>	<u>6</u>	<u>7</u>	<u>8</u>	<u>9</u>	<u>4</u>	<u>5</u>
8	7	6	5	4	3	2	1	7	6	5
1	2	3	4	5	6	7	8	1	2	3
<u>1</u>	<u>1</u>	<u>1</u>	<u>1</u>	<u>1</u>	<u>1</u>	<u>1</u>	<u>1</u>	<u>2</u>	<u>2</u>	<u>2</u>
1	6	5	4	3	2	1	5	4	3	2
7	1	2	3	4	5	6	1	2	3	4
<u>2</u>	<u>3</u>	<u>3</u>	<u>3</u>	<u>3</u>	<u>3</u>	<u>3</u>	<u>4</u>	<u>4</u>	<u>4</u>	<u>4</u>

ADDING 9 AND 8 TO OTHER NUMBERS

$$18. \quad 12 + 9 = 21. \qquad 23 + 9 = 32. \qquad 34 + 9 = 43.$$

In each of these examples, how does the unit's figure of the sum compare with the unit's figure of the number to which 9 is added? How do the ten's figures compare?

If the number ends in 0 or 1, this relation is not found.

$$10 + 9 = 19. \qquad 21 + 9 = 30.$$

To add 9 to a number whose unit's figure is neither 0 nor 1, add 1 to the ten's figure of the number and subtract 1 from the unit's figure.

$$19. \quad 13 + 8 = 21. \qquad 24 + 8 = 32. \qquad 33 + 8 = 41.$$

But $10 + 8 = 18$, $21 + 8 = 29$, $32 + 8 = 40$.

To add 8 to a number whose unit's figure is not 0, 1, nor 2, add 1 to the ten's figure of the number and subtract 2 from the unit's figure.

ORAL EXERCISES

20. Add:

1.	9	9	8	8	9	9	8	8
	<u>12</u>	<u>14</u>	<u>15</u>	<u>17</u>	<u>13</u>	<u>16</u>	<u>19</u>	<u>18</u>
2.	9	8	9	9	8	8	9	9
	<u>45</u>	<u>54</u>	<u>62</u>	<u>23</u>	<u>35</u>	<u>76</u>	<u>99</u>	<u>108</u>
3.	8	8	9	9	8	8	9	8
	<u>124</u>	<u>175</u>	<u>188</u>	<u>177</u>	<u>162</u>	<u>152</u>	<u>144</u>	<u>107</u>

REVIEW

21. Add:

1.	5 + 6.	2.	5 + 7.	3.	5 + 8.	4.	5 + 9.	5.	5 + 10.
	15 + 6.		15 + 7.		15 + 8.		35 + 9.		45 + 10.
	25 + 6.		25 + 7.		25 + 8.		75 + 9.		85 + 10.

22. Add in turn :

1. 6, 7, 8, 9, 10, to all numbers whose unit figure is 5, from 5 to 95.

2. 6, 7, 8, 9, 10, to all numbers whose unit figure is 6, from 6 to 96.

3. 7, 8, 9, 10, to all numbers whose unit figure is 7, from 7 to 97.

4. 8, 9, 10, to all numbers whose unit figure is 8, from 8 to 98.

5. 9 and 10, to all numbers whose unit figure is 9, from 9 to 99.

6. 15, 16, 17, 18, 19, to all numbers whose unit figure is 5, from 5 to 95.

SUGGESTION: Add 15 to 25. $15 = 10 + 5$. $25 + 10 = 35$; $35 + 5 = 40$.

7. 16, 17, 18, 19, to all numbers whose unit figure is 6, from 6 to 96.

8. 17, 18, 19, to all numbers whose unit figure is 7, from 7 to 97.

9. 18 and 19, to all numbers whose unit figure is 8, from 8 to 98.

10. 19, to all numbers whose unit figure is 9, from 9 to 99.

11. 20, 30, 40, 50, 60, 70, 80, 90, to all numbers whose unit figure is 2, 3, 4, 5, 6, 7, 8, or 9.

23. Add :

SUGGESTION: Add 35 to 27. $27 = 20 + 7$. $35 + 20 = 55$; $55 + 7 = 62$.

1. $25 + 23$.

5. $75 + 23$.

9. $29 + 37$.

2. $46 + 35$.

6. $65 + 33$.

10. $37 + 53$.

3. $81 + 25$.

7. $95 + 35$.

11. $59 + 26$.

4. $33 + 20$.

8. $36 + 42$.

12. $78 + 26$.

SUBTRACTION

24. How many more days has January than February?

SOLUTION: 31 days in January
 28 days in February
 3 days

There are 3 days more in January than in February, or 3 days less in February than in January.

$$31 - 3 = 28; \text{ or, } 28 + 3 = 31.$$

1. Which number is the **minuend**?
2. Which number is the **subtrahend**?

The **difference** is the number that remains if a smaller number is taken from a larger. It is also the number that, added to the smaller, will make a number equal to the larger.

Finding the difference of two numbers is called **subtraction**. To **test** subtraction, add the difference to the subtrahend.

3. What should be the sum of such addition? Why?

Subtraction may be represented by the sign **—**, read "minus," or "less."

4. Read: $25 - 9 = 16$.

ORAL EXERCISES

25. Read, supplying the missing numbers:

- | | | |
|---------------------------|----------------------------|------------------------------|
| 1. $13 + \text{—} = 22$. | 7. $18 + \text{—} = 26$. | 13. $82 + \text{—} = 90$. |
| 2. $12 + \text{—} = 21$. | 8. $25 + \text{—} = 34$. | 14. $124 + \text{—} = 132$. |
| 3. $17 + \text{—} = 25$. | 9. $35 + \text{—} = 43$. | 15. $132 + \text{—} = 141$. |
| 4. $19 + \text{—} = 27$. | 10. $47 + \text{—} = 56$. | 16. $146 + \text{—} = 154$. |
| 5. $15 + \text{—} = 23$. | 11. $56 + \text{—} = 64$. | 17. $154 + \text{—} = 162$. |
| 6. $14 + \text{—} = 23$. | 12. $64 + \text{—} = 73$. | 18. $162 + \text{—} = 171$. |

26. In making change it is customary to add to the amount of the purchase amounts represented by coins or bills until the amount of the bill or coin given for the purchase is equaled.



This is a picture of a “cash register,” such as is used in shops to show what sales have been made. Read the amount printed in the black panel.

To write this amount the clerk pressed three keys: the \$1 key; the 60 key; and the 5 key. \$1—60—5.

27. 1. What amounts of sales would be shown by pressing the following sets of keys:

\$8—70—4?	\$4	} —5?	\$7	} 40—9?
\$9—6?	\$9		\$8	
—90?	\$1—1—7?	—6—4?	—7?	

2. What keys would be pressed to show sales of:

\$1.23?	\$10.67?	\$15.06?	\$0.49?
\$3.78?	\$14.85?	\$ 0.06?	\$0.10?
\$9.99?	\$11.16?	\$ 0.17?	9 cents?

3. Count out the change that would be given in each case if payment is made as follows:

\$ 2.00.	\$20.00.	\$25.00.	1 half dollar.
\$ 5.00.	\$50.00.	\$10.15.	1 quarter dollar.
\$15.00.	\$12.00.	\$16.75.	1 dime.

REVIEW

28. Solve :

1. From 132 million, 756 thousand, 905 take 125 million, 36 thousand, 425 ; and prove the result.

2. During one year, the steamship lines named below made trips and carried passengers, as follows :

	PASSENGERS			
	NO. TRIPS	FIRST CABIN	SECOND CABIN	STEERAGE
White Star	99	11,978	12,949	49,453
Cunard	69	9,190	13,940	62,547
American	52	5,786	8,328	26,611
Hamburg American	104	13,494	16,018	128,811
North German Lloyd	98	12,796	23,992	101,996

- (1) How many trips were made by all the steamers ?
- (2) How many first cabin passengers did they carry ?
- (3) How many second cabin passengers did they carry ?
- (4) How many steerage passengers did they carry ?
- (5) How many more steerage passengers than first and second cabin combined were carried by the White Star Line ?
- (6) By the Cunard Line ?
- (7) By the American Line ?
- (8) By the Hamburg American Line ?
- (9) By the North German Lloyd Line ?

3. Hugh Keller gives his wife \$70 a month for household expenses. In January his wife paid bills as follows :

Rent	\$15.00
Groceries	15.72
Butcher's bill	9.05
Milk	3.18
Gas	2.45
Clothing	8.99

She laid aside \$5 for car fare, etc., and deposited the remainder in a savings bank. How much did she deposit?

4. In February Mrs. Keller's bills were as follows :

Rent	\$15.00
Groceries	16.02
Butcher's bill	8.73
Milk	4.24
Shoes	2.50
Schoolbooks	3.14
Doctor's bills	32.00

How much did she owe at the end of the month?

5. Mr. Warner had \$873.25 in the bank on Monday morning. He deposited \$362.17 and withdrew \$418.28. How much had he remaining at the close of business on Monday?

6. He deposited and drew out as follows :

	DEPOSITED	WITHDREW
Tuesday,	\$ 384.39	\$ 115.23
Wednesday,	541.62	610.18
Thursday,	748.15	329.34
Friday,	136.50	247.75
Saturday,	1263.89	921.94

(1) Including the balance of the day before, how much did he have remaining in the bank at the close of each day?

(2) How much did he deposit in all during the week?

(3) How much did he withdraw?

MULTIPLICATION

29. To find the number of hours in a week, we may add 7 twenty-fours together, as was done in the solution on page 16; or, the number of hours in each day being equal, we may repeat the number in one day as many times as there are days in a week; namely, 7.

$$\begin{array}{r} 24 \text{ hours} \\ 7 \\ \hline 168 \text{ hours} \end{array}$$

Equal numbers may be united into a new number equal to their sum, by "taking" one of them as many times as there are numbers to be repeated. This is called **multiplying**.

It may be represented thus: $7 \times 24 = 168$.

The sign $\times$ is read "times" or "multiplied by."

1. Can you find the number of days in the year from the number in each month by multiplying? Why?

Multiplication is a short process of adding *equal* numbers, by taking one of them a given number of times.

2. Define **multiplicand**, **multiplier**, **product**.

To **test** multiplication, divide the product by either the multiplier or the multiplicand.

3. What should the quotient equal?

WRITTEN EXERCISES

30. Solve:

1. First-class workers on ready-made women's garments receive the following weekly wages:

Fitters and cutters	\$25.00
Waist makers	20.00
Hand trimmers	15.00
Sleeve makers and trimmers	10.00
Plain sewers	5.00
Unskilled workers	2.00

Counting 50 working weeks in the year, how much more will a fitter and cutter earn in that time than each of the other workers?

2. In 1909, J. Anthony, a carpenter, received 45 cts. an hour. He worked 9 hr. per day, except Saturdays, when he worked 5 hr. On account of sickness, bad weather, strikes, and holidays, he lost 16 weeks' time during the year. His son, Robert, a helper, received 20 cts. an hour, and worked 30 wk.

The family's monthly expenses averaged as follows:

Rent	\$18.00
Groceries	17.25
Butcher's bill	10.40
Milk	3.80
Shoes	2.25
Clothing	15.00
Doctor's bill	2.50

- (1) What were Anthony's wages per week?
 - (2) What were Robert's wages per week?
 - (3) What were Anthony's wages for the year?
 - (4) What were Robert's wages for the year?
 - (5) What were the total earnings for both for the year?
 - (6) What was the amount for the year of Anthony's rent?
- Of his grocery bills? Of his butcher's bill? Of his milk bill? Of his bill for shoes? For clothes? Of his doctor's bill?
- How much was the sum total of his expenses?
- How much money did he have left?

31. When the multiplier contains more than one figure, where must the first figure of each **partial product** be placed? Why?

32. How do we multiply an integer by a fraction? An integer by a mixed number?

33. How do we multiply an integer by 10? By 100? By 1,000?

34. How do we multiply an amount of dollars and cents?

WRITTEN EXERCISES

35. Multiply:

1. $4,256 \times 27$.

5. $64 \times 2\frac{1}{4}$.

2. $1,830 \times 30$.

6. $48 \times 31\frac{2}{3}$.

3. $452,836 \times 114$.

7. $\$1,739.75 \times 5\frac{1}{2}$.

4. $\$8,590.50 \times 24$.

8. $\$420.50 \times 300$.

36. Solve:

1. A woman bought 8 pieces of lawn at 20 cts. a yard. Each piece contained 48 yd. She made it into skirts each requiring 4 yd. How many skirts did she make?

2. Her time was worth 10 cts. an hour, and she worked 4 hr. on each skirt. She sold the skirts at \$1.50 each. What was her profit?

3. Mr. Fisher has 15 acres of Maryland land planted with tomatoes. He gets 240 baskets per acre, which he sells at \$8 a ton.

(1) Counting 60 baskets to a ton, how much does he receive for his crop of tomatoes?

(2) How much does he receive per acre?

(3) How much per basket?

(4) The cost of labor was \$5 per acre. What was the cost for the whole acreage?

(5) It cost 60 cts. a ton to carry the tomatoes to the factory. How much did it cost for the entire crop?

(6) What did the crop of tomatoes cost?

(7) What was Mr. Fisher's profit?

(8) What was his profit per acre?

4. Mr. Fisher has another tomato field of 25 acres. John Jefferson works this field on shares. He furnishes all the labor and receives $\frac{1}{3}$ of the proceeds. This land produces 300 baskets per acre. The fruit brings the same price, \$8 a ton.

(1) What value does Mr. Fisher get from the field?

(2) Is this more or less than he would have received if he had worked the field himself? How much?

(3) Jefferson does all the work himself except carrying the tomatoes to the factory. After paying 60 cts. a ton for this, how much does he have for himself?

5. Mr. Jacobs has a large maple grove in Vermont. Last year he tapped 1,000 trees and collected the sap in 3-gallon buckets. The average flow of sap per tree was 10 buckets.

(1) How many gallons of sap were produced?

(2) 20 gal. of sap boiled down produced 1 gal. of syrup. How many gallons of syrup did Mr. Jacobs get?

(3) He sold 1,250 gal. of the syrup at 75 cts. a gallon. How much did he receive for it?

(4) The remainder of the syrup he boiled down to sugar, 1 gal. producing 8 lb. How many pounds of sugar were made?

(5) He sold it at 15 cts. a pound. How much did Mr. Jacobs receive for his sugar?

(6) How much did he receive for the entire product of his grove?

(7) How much more would he have received if he had boiled all the syrup down to sugar?

6. If an automobile goes at the rate of 15 miles an hour, how far will it go in $3\frac{1}{2}$ hour? In $5\frac{3}{4}$ hours?

7. In June Mr. Brown bought 12 tons of coal at \$5.75 per ton. The following February he bought 2 tons more at \$6.25 per ton. How much did he pay for coal?

DIVISION

37. There are 168 hours in a week. In each day there are 24 hours. How many days are there in a week?

We can find the result by subtracting 24 from 168, and then again from each remainder until nothing remains, and counting the number of subtractions, thus:

There are seven subtractions in all. That is, there are seven 24's in 168, or seven days in the week.

This is a slow process. We obtain the same result by dividing:

$$\begin{array}{r} 7 \\ 24 \overline{)168} \\ \underline{168} \\ 0 \end{array}$$

There are seven 24's in 168, or 24 is contained 7 times in 168.

The process may be represented also:

$$168 \div 24 = 7; \frac{168}{24} = 7; \text{ or, } \frac{1}{24} \text{ of } 168 = 7.$$

1. Which number is the **dividend**? The **divisor**? The **quotient**?

The sign $\div$ is read "divided by."

Division is the process of finding how many times one number is contained in another.

It is equivalent to the repeated subtraction of equal subtrahends, and is the opposite of multiplication; hence:

To **test** division, if there is no remainder, multiply the quotient by the divisor.

2. What should the product equal?

3. How many days do 172 hr. equal?

SOLUTION:

$$\begin{array}{r} 7 \\ 24 \overline{)172} \\ \underline{168} \\ 4 \end{array} \quad 172 \text{ hr.} = 7 \text{ da. } 4 \text{ hr.}$$

That is, if 24 hr. be taken 7 times from 172 hr., 4 hr. will still be left. 4 is the remainder.

4. Why is the remainder always less than the divisor?
To test the result, multiply 24 by 7, and add the remainder, 4.

5. Why do you add the remainder?

6. Define dividend, divisor, quotient, remainder.

ORAL EXERCISES

38. Divide rapidly, naming only the quotient:

1. By 3: 15, 27, 21, 12, 9, 33, 24, 18, 36.
2. By 6: 24, 48, 36, 18, 42, 30, 72, 12, 66.
3. By 9: 54, 27, 72, 45, 9, 108, 36, 63, 81.
4. By 4: 16, 28, 40, 20, 32, 12, 48, 36, 44.
5. By 8: 64, 16, 32, 56, 72, 24, 40, 96, 48.
6. By 5: 15, 45, 55, 30, 60, 35, 5, 20, 40.
7. By 7: 35, 56, 28, 63, 14, 77, 21, 84, 49.
8. By 12: 48, 24, 60, 36, 72, 96, 84, 12, 108.

WRITTEN EXERCISES

39. Copy, divide, and test:

- | | | | | |
|------------------------|------------------------|------------------------|-------------------------|-------------------------|
| 1. $7 \overline{)499}$ | 3. $6 \overline{)828}$ | 5. $8 \overline{)904}$ | 7. $11 \overline{)319}$ | 9. $7 \overline{)806}$ |
| 2. $5 \overline{)382}$ | 4. $9 \overline{)997}$ | 6. $4 \overline{)409}$ | 8. $12 \overline{)644}$ | 10. $9 \overline{)700}$ |

LONG DIVISION

40. $68,240 \div 216 = ?$ $6 \div 2$ gives the first quotient figure.

$75,296 \div 429 = ?$ $7 \div 4$ gives the first quotient figure.

These figures are called the **trial dividend** and the **trial divisor**.

$35,387 \div 475$. $35 \div 4$ gives the first quotient figure.

To find the first quotient figure, divide the first figure of the dividend by the first figure of the divisor.

When the first figure of the divisor is larger than the first figure of the dividend, divide the first two figures of the dividend by it.

ORAL EXERCISES

41. Give the first quotient figure in dividing each of the following :

1. $98,763 \div 435$.

5. $206,784 \div 5,278$.

2. $89,567 \div 376$.

6. $670,846 \div 8,725$.

3. $65,876 \div 483$.

7. $840,765 \div 5,174$.

4. $75,492 \div 294$.

8. $789,123 \div 3,649$.

42. Divide 427 by 23.

SOLUTION: 19

$$\begin{array}{r} 23 \overline{)437} \\ 23 \\ \hline 207 \\ 207 \\ \hline \end{array}$$

What is the trial divisor? What at first sight would you say the first quotient figure to be? Why is 2 not right?

$$\begin{array}{r} 207 \\ 207 \\ \hline \end{array}$$

In finding a quotient figure from the trial divisor and dividend, it is necessary to look far enough to make sure that the figure used is not too large.

Tell at sight what you think to be the first figure of the quotient for each of the following divisions:

1. $144 \overline{)1728}$

3. $48 \overline{)890}$

5. $44 \overline{)8437}$

2. $124 \overline{)2367}$

4. $361 \overline{)6739}$

6. $19 \overline{)370}$

7. Find by division whether your estimates were correct.

43. Divide 95,048 by 218.

SOLUTION: 6

30

400

218)95048

87200

7848

6540

1308

1308

Where do you place the first quotient figure?

218 is contained in the dividend 95,048, 400 times + a remainder.

218 is contained in the remainder 7,848, 30 times + a remainder.

218 is contained in the remainder 1,308, 6 times.

218 is contained in 95,048, 436 times.

In practice we omit the ciphers, as in the following:

44. Divide 95,096 by 218.

SOLUTION: $436\frac{48}{218}$

218)95096

872

789

654

1356

1308

48

We find how many times the divisor, 218, is contained in the first figures of the dividend that will contain it; that is, in 950. We write the quotient 4 over the last figure of the dividend used, the 0.

$$218 \times 4 = 872$$

$$950 - 872 = 78$$

To the remainder we annex the next figure of the dividend, 9, and divide as before. After all the figures of the dividend have been used, we write the last remainder, 48, as the numerator of a fraction of which the divisor is the denominator and add this fraction, $\frac{48}{218}$, to the quotient.

45. Divide 2,716 by 26.

SOLUTION: $104\frac{12}{26}$

$$26 \overline{) 2716}$$

$$\underline{26}$$

$$\underline{116}$$

$$\underline{104}$$

$$\underline{12}$$

The second dividend, 11, will not contain 26. We place 0 in the quotient and bring down the next figure.

If, after annexing a figure of the dividend to a remainder, the number thus formed is less than the divisor, a cipher must be placed in the quotient, and the next figure of the dividend annexed.

WRITTEN EXERCISES

46. Divide and prove:

1. $28 \overline{) 4968}$

5. $19 \overline{) 1124}$

9. $825 \overline{) 15055}$

2. $16 \overline{) 3799}$

6. $61 \overline{) 4835}$

10. $681 \overline{) 36976}$

3. $35 \overline{) 7034}$

7. $127 \overline{) 35094}$

11. $208 \overline{) 37402}$

4. $47 \overline{) 6283}$

8. $354 \overline{) 76283}$

12. $416 \overline{) 63006}$

47. What is the product of 0 and any number?

What is the quotient of any number by 0? Of 0 by any number?

1. Multiply in turn by 10, by 100, by 1,000: 1,500; 37; 425; 92; 840; 7; 206; 19; 100; 1,000; 2,000.

2. How is a number multiplied by 10? By 100? By 1,000?

3. Divide in turn by 10, by 100, by 1,000: 168,725; 49,060; 2,700; 8,463; 792; 185; 7,204; 10,000; 32,500; 87,250.

4. How is a number divided by 10? By 100? By 1,000?

5. Multiply 6,250 by 2,300.

6. How do you write the numbers when there are ciphers at the right of the multiplier, the multiplicand, or both?

48. Divide 1,728 by 30.

SOLUTION:
$$\begin{array}{r} 57\cancel{1}\cancel{8} \\ 30\overline{)1,72\cancel{8}} \end{array}$$

To divide when the divisor has ciphers at the right, cross off the ciphers of the divisor and an equal number of figures at the right of the dividend; divide the remaining figures of the dividend by those of the divisor; if there is no remainder from the division, the figures crossed from the dividend will be the remainder; if there is a remainder from dividing, write after it the figures crossed off from the dividend, for the whole remainder.

Tell why the result is the same as if the division were worked out in full.

WRITTEN EXERCISES

49. Solve:

1. Divide in turn by 20, by 30, by 200, by 400, by 50, by 60:

725; 864; 1,247; 6,824; 1,325.

2. Mr. Plum sold 840 quires of paper at 50 cts. a ream. What did he received for it?

3. Mr. Brant sent 1,200 roses to market in boxes, 60 roses in each box. He received \$1.50 per box for them. What did he receive for all?

4. A farmer sold 960 lb. of oats for 40 cts. a bushel. Allowing 30 lb. to a bushel, how much did he get for the oats?

50.

AVERAGE

1. Henry earned 25 cts. in one week and 31 cts. in the next. What was his average earning for the two weeks?

SOLUTION: $25 + 31 = 56$, the total for two weeks.
 $56 \div 2 = 28$, the average.

2. His father earned as follows, for the six days of a week : \$3.50, \$3, \$2.75, \$4, \$2.50, \$4.50. What were his average earnings ?

SOLUTION: $\frac{\$3.50 + \$3 + \$2.75 + \$4 + \$2.50 + \$4.50}{6} = \text{the average.}$

The **average** of several numbers is what each would be if the numbers were equal. It is found by dividing their sum by the number of addends.

WRITTEN EXERCISES

51. Solve :

NUMBER OF HOURS OF SUNSHINE IN NEW YORK IN ONE YEAR

1. January . . . 111	July . . . 307
February . . . 202	August . . . 226
March . . . 193	September . . . 251
April . . . 279	October . . . 153
May . . . 308	November . . . 175
June . . . 329	December . . . 103

What was the average number of hours of sunshine for each month of the year ?

2. The area of Manhattan is 22 sq. mi. Its population in 1905 was 2,112,697. What was the average population per square mile ? Per acre ?

3. Write the number of days in each month of the year. Find the average number of days in each month.

4. Find the average age of the members of your class.

5. Ask your teacher to give the necessary figures. Then find the average daily attendance of your school for one week, for one month.

6. The area of New York City is 326 sq. mi. Its population in 1905 was 4,014,304. What was the average population per square mile? Per acre?

52.

RATIOS

1. Add until 30 is reached : 3. Add until 66 is reached :

$$\begin{array}{r} 2\frac{1}{2} \\ 2\frac{1}{2} \\ \hline 5 \end{array}$$

$$\begin{array}{r} 2\frac{1}{2} \\ 7\frac{1}{2} \end{array}$$

$$\begin{array}{r} 5\frac{1}{2} \\ 5\frac{1}{2} \\ \hline 11 \end{array}$$

$$\begin{array}{r} 5\frac{1}{2} \\ 16\frac{1}{2} \end{array}$$

2. Add until 42 is reached : 4. Add until 54 is reached :

$$\begin{array}{r} 3\frac{1}{2} \\ 3\frac{1}{2} \\ \hline 7 \\ 3\frac{1}{2} \\ \hline 10\frac{1}{2} \end{array}$$

$$\begin{array}{r} 4\frac{1}{2} \\ 4\frac{1}{2} \\ \hline 9 \\ 4\frac{1}{2} \\ \hline 13\frac{1}{2} \end{array}$$

5. How many times is $2\frac{1}{2}$ contained in 5, $7\frac{1}{2}$, 10, and each multiple to 30?

6. How many times is $3\frac{1}{2}$ contained in 7, $10\frac{1}{2}$, 14, and each multiple to 42?

7. How many times is $5\frac{1}{2}$ contained in 11, $16\frac{1}{2}$, 22, and each multiple to 66?

8. How many times is $4\frac{1}{2}$ contained in 9, $13\frac{1}{2}$, 18, and each multiple to 54?

9. What is the ratio of 7 to $2\frac{1}{2}$?

ORAL EXERCISES

53. Find the following ratios :

1. 10 to $2\frac{1}{2}$.

2. 5 to $2\frac{1}{2}$.

3. 25 to $2\frac{1}{2}$.

- | | | |
|--|--|---|
| 4. 20 to $2\frac{1}{2}$. | 9. $12\frac{1}{2}$ to $2\frac{1}{2}$. | 14. 15 to $7\frac{1}{2}$. |
| 5. 30 to $2\frac{1}{2}$. | 10. $7\frac{1}{2}$ to $2\frac{1}{2}$. | 15. 27 to $4\frac{1}{2}$. |
| 6. 11 to $5\frac{1}{2}$. | 11. 22 to $5\frac{1}{2}$. | 16. $27\frac{1}{2}$ to $5\frac{1}{2}$. |
| 7. $49\frac{1}{2}$ to $5\frac{1}{2}$. | 12. 55 to $5\frac{1}{2}$. | 17. 33 to $5\frac{1}{2}$. |
| 8. 66 to $5\frac{1}{2}$. | 13. 44 to $5\frac{1}{2}$. | 18. $16\frac{1}{2}$ to $5\frac{1}{2}$. |

WRITTEN EXERCISES

54. Solve:

1. If 198 gal. of oil fill $5\frac{1}{2}$ bbl., how many gallons will fill 44 bbl. of the same size?

SUGGESTION: What is the ratio of 44 to $5\frac{1}{2}$?

2. Mr. Oliver's crop of potatoes from $2\frac{1}{2}$ acres is 625 bu. At that rate what is his crop from $17\frac{1}{2}$ acres worth at 75 cts. per bushel?

3. A merchant's profit on goods sold at a gain of $5\frac{1}{2}\%$ was \$2,480.75. What would have been his profits if he had gained 22%?

4. John said that the wheels of his bicycle turned around 1,980 times wheeling from Madison to Morristown, a distance of $4\frac{1}{2}$ miles. At that rate how many times would they turn around if he rode from Madison to Newark, a distance of 18 miles?

5. Mr. Salter saved \$2,786.85 in $3\frac{1}{2}$ years. At that rate how much would he save in $10\frac{1}{2}$ years?

6. Mr. Noe sold 3,780 roses in $4\frac{1}{2}$ days. At that rate how many roses would he sell in $22\frac{1}{2}$ days? What is the value of the roses at \$1.25 per dozen?

7. If Mr. DeMott's men can make 12,600 peach baskets in $4\frac{1}{2}$ weeks, how many baskets can they make in 54 weeks? What is their value at \$33.75 per thousand?

REVIEW

55. Solve:

1. A grocer bought eggs at the rate of 3 eggs for 5 cts. How many did he buy for 50 cts.? If he sold them at 2 cts. apiece, how much did he make?

SUGGESTION: The ratio of 50 cts. to 5 cts. = 10, therefore he bought 10×3 eggs = 30 eggs.

$\$.02 \times 30$ = selling price. Selling price - cost = gain.

2. Dick buys Journals at the rate of 8 Journals for 5 cts. and sells them for 2 cts. apiece. On Monday he bought 35 cts. worth; on Tuesday, 50 cts. worth; on Wednesday, 45 cts. worth; on Thursday, 60 cts. worth; on Friday, 75 cts. worth; on Saturday, \$1 worth. How much did he make each day? How much in 6 days?

3. Jack buys and sells oranges. On Monday morning he had 80 cts. With it he bought oranges at the rate of 3 oranges for 4 cts. and sold them 2 for 5 cts. How much did he make?

4. On Tuesday morning he spent \$1 for Florida oranges, at the rate of 2 oranges for 5 cts., and sold them for 4 cts. apiece. How much did he make on Tuesday?

5. On Wednesday he paid 90 cts. for oranges at the rate of 2 oranges for 3 cts., and sold them for 30 cts. a dozen. How much did he make on Wednesday? How much in the three days?

6. Joe's capital was 36 cts. With it he bought bananas at the rate of 5 bananas for 3 cts., and sold them at the rate of 3 bananas for 5 cts. How much did he make?

SUGGESTION: The ratio of 36 cts. to 3 cts. = 12, therefore he bought 12×5 = 60 bananas.

The ratio of 60 bananas to 3 bananas = 20, therefore he received 20×5 cts.

7. With 45 cts. of his new capital Joe bought and sold at the same rate. Find his gain.

8. Mr. Williams had \$10,000. He bought a farm for \$5,250; horses for \$610; harness, plows, and other farming implements for \$900; and with one half the remainder he bought cows at \$45 apiece. How many cows did he buy?

9. A farmer sold 65 cows at \$36.50 per head, and with the money purchased sheep at \$7.30 per head. How many sheep did he purchase?

10. A commission merchant bought 135 bbl. of cranberries at \$3.25 per barrel. 31 bbl. were spoiled. He sold the remainder at \$4.25 per barrel. Did he gain or lose, and how much?

11. Out of his year's earnings, Sam Morris paid \$35.50 per month for rent, \$9.75 for clothes, and \$45.68 for food. He has \$24.98 left. What did he earn in a year?

12. Mr. Ford bought berries at \$0.07 a quart. How many quarts could he pay for with \$14.35?

13. He sold the berries for \$0.09 a quart. How much did he make?

14. He bought 15 bbl. of flour for \$4.75 a barrel and sold it for \$5.25 per barrel. How much did he make?

15. He also bought a case of fruit containing 2 doz. cans, at \$0.16 per can, and sold it for \$0.20 a can. How much did he make?

16. What is the ratio of: A 2-inch square to a 4-inch square? A 3-inch square to a 6-inch square? A 6-inch square to a 12-inch square?

17. How many inches in the perimeter of a 12-inch square? How many yards?

18. A man's income is \$3,000 a year. His expenses are \$50 a week. How much can he save in a year?

19. Mr. Brandt has a peach orchard of 17 acres. There are 160 trees to an acre.

(1) How many trees are there in the orchard?

(2) If the trees bear 3 baskets of peaches each, how many baskets of peaches will he have for the season?

(3) If he pays 8 cts. a basket for picking, how much will it cost to pick them?

(4) How much would he receive by selling them at \$1.25 a basket?

(5) If he pays 5 cts. apiece for his baskets, 8 cts. per basket for picking, 12 cts. per basket to take them to market, and 6 cts. per basket commission, how much will he clear on a basket?

(6) How much will he clear on his entire crop?

20. (1) If a cotton mill manufactures 7,420 yd. of cotton cloth in one day, how many yards will it manufacture in 6 weeks, not including Sundays?

(2) If the cloth is sold for 8 cts. a yard, how much will be received for it?

(3) If it costs 5 cts. a yard to buy the cotton and to manufacture it, what will be the profit?

21. Mr. Platt bought a farm for \$13,325, at \$65 an acre. How many acres in the farm?

22. The distance from Newark to New York is 2,880 rd. There are 320 rd. in 1 mile. How many miles are the two cities apart?

23. A plot of land contains 34,560 square rods. There are 160 square rods in an acre. How many acres are there in the plot?

24. Mr. Lindsley sold 305 tons of hay for \$5,718.75. How much did he receive per ton?

25. He sold 415 bu. of corn for \$269.75. How much did he receive per bushel?

26. He also sold 86 bbl. of potatoes for \$184.90. How much did he receive per barrel?

27. How much did he receive for all of his crops? He has 215 acres. What was the average value of his crops per acre?

28. A North Carolina farmer planted 80 acres in cotton. His labor cost \$2,872; his fertilizers \$600. He raised 144 bales of cotton, of 500 lb. each, which he sold at 15 cts. a pound. What was his profit per acre?

29. On Saturday Mr. Brown sent to the bank for money to pay his men, using the following slip:

No.				
	Bills	each	\$ 100.00	Amount \$ 200
	Bills	each	10.00	Amount 500
	Bills	each	5.00	Amount 500
	Bills	each	2.00	Amount 160
	Bills	each	1.00	Amount 250
	Coins	each	\$20.00	Amount \$ 240
	Coins	each	1.00	Amount 100
	Coins	each	.50	Amount 35
	Coins	each	.05	Amount 10
	Coins	each	.01	Amount 1

(1) How many bills were called for?

(2) How many coins were called for?

(3) Write in the space at the left, the number of bills and coins of each denomination called for.

(4) Find the total amount.

30. Mr. Brown's cashier put the money for all but the office force into envelopes, as follows :

20 envelopes, \$ 5.00.

13 envelopes, 7.50.

52 envelopes, 9.00.

39 envelopes, 12.25.

5 envelopes, 20.00.

(1) How much was put into each group of envelopes ?

(2) How much was put into all the envelopes ?

(3) How much remained to pay the office force ?

56. Divide :

1. $24,194 \div 32$.

10. $50,560 \div 64$.

2. $46,275 \div 52$.

11. $66,940 \div 84$.

3. $60,156 \div 68$.

12. $109,950 \div 75$.

4. $51,875 \div 88$.

13. $108,820 \div 45$.

5. $405,558 \div 56$.

14. $861,437 \div 36$.

6. $429,436 \div 67$.

15. $513,822 \div 97$.

7. $151,436 \div 201$.

16. $143,276 \div 601$.

8. $545,812 \div 802$.

17. $416,259 \div 712$.

9. $345,571 \div 831$.

18. $463,471 \div 641$.

19. Measure your desk top in inches and find the number of square inches in it. How many dimensions must you measure ? Reduce your answer to square feet.

20. A certain car is 50 ft. long and has a seat along each side. Allowing 2 ft. 6 in. seating space for one person, how many persons will the car seat ?

21. How many books, averaging $1\frac{1}{4}$ in. in thickness, will a 5-ft. bookcase hold ?

22. How many cubic feet of ice are there in a block 3 ft. $\times$ 2 ft. $\times$ $1\frac{1}{2}$ ft ?

SOLVING A PROBLEM

57. To solve a problem readily, it is necessary to understand three things very clearly: first, **what facts are given**; second, **what is to be found**; third, **what steps are necessary**.

For example, in the problem:

"If a dozen lemons cost 60 cents, what will 3 lemons cost?"
What is given? What is to be found? What steps would you take to solve the problem?

Finding these three things about any problem is called **analysis**, which means "taking to pieces."

For the problem given, there are two sorts of analysis by which we may find the answer:

1. We may say, "If 1 dozen lemons cost 60 cents, 1 lemon will cost $\frac{1}{12}$ of 60 cents, or 5 cents; and 3 lemons will cost 3×5 cents or 15 cents."

This analysis is good, but rather long.

2. We may use ratio and say, "The ratio of 3 lemons to 12 lemons is $\frac{1}{4}$. Hence the cost of 3 lemons will be $\frac{1}{4}$ of the cost of 12 lemons, or $\frac{1}{4}$ of 60 cents, which is 15 cents."

But before using either analysis it is necessary first to **read the problem very carefully** and to think it all out. Then the simplest analysis should be used.

ORAL EXERCISES

58. Read the problems on page 40 of your Arithmetic and point out in each: first, what is told; second, what is to be found; and, third, what steps you would take to find the answer.

Solve problems **25** and **26** on page 40, following this order, and stating each step.

MEASUREMENT**WEIGHTS AND MEASURES****WRITTEN EXERCISES**

59. Write the table of :

- | | |
|----------------------|------------------------|
| 1. Dry measures. | 3. Measures of weight. |
| 2. Liquid measures. | 4. Linear measures. |
| 5. Measures of time. | |

ORAL EXERCISES

60. Read, supplying the missing numbers :

1. 5 bushels and 3 pecks = — pecks.
2. 6 bushels and 1 peck = — pecks.
3. 1 peck and 5 quarts = — quarts.
4. 3 pecks and 6 quarts = — quarts.
5. 10 gallons and 3 quarts = — quarts.
6. 12 gallons and 2 quarts = — quarts.
7. 1 pound and 4 ounces = — ounces.
8. 2 pounds and 6 ounces = — ounces.
9. 8 yards and 1 foot = — feet.
10. 7 yards and 2 feet = — feet.
11. 5 feet and 6 inches = — feet.
12. 8 feet and 4 inches = — feet.
13. 2 hours and 20 minutes = — minutes.
14. 12 years and 6 months = — months.
15. 8 weeks and 4 days = — days.

61. How much is :

1. 6 bushels minus 3 pecks ?
2. 3 pecks minus 6 quarts ?
3. 8 gallons minus 2 quarts ?
4. 10 yards minus 1 foot ?
5. 4 feet minus 6 inches ?
6. 1 pound minus 10 ounces ?
7. 2 pounds minus 8 ounces ?
8. 2 hours minus 15 minutes ?
9. 3 months minus 10 days ?
10. 4 weeks minus 5 days ?

62. Solve :

1. Wilson, Luke, and Minnie gathered chestnuts. Wilson gathered 3 pk. 6 qt., Luke gathered 1 bu. 2 qt., and Minnie gathered 1 bu.

- (1) How many quarts did each gather ?
- (2) How many quarts did they all gather ?
- (3) How many pecks did they all gather ?
- (4) How many bushels did they gather ?

2. They sold the nuts to Mr. Delno for 8 cts. a quart.

- (1) How much money did each receive ?
- (2) How much altogether did they receive ?

3. Mr. Delno sold the chestnuts for \$ 3.25 a bushel. How much did he make ?

4. Mr. Dustan had a barrel of sugar containing 300 lb. On Monday he sold 25 lb. 8 oz. ; on Tuesday, 30 lb. 4 oz. ; on Wednesday, 32 lb. 12 oz. ; on Thursday, 20 lb. 4 oz. ; on Friday, 26 lb. 8 oz. ; on Saturday, 50 lb. 12 oz.

How many pounds did he sell during the week ?

5. He paid $5\frac{1}{2}$ ¢ per pound for the sugar and sold it for 6¢ per pound. How much more money did he receive for the sugar than he paid for it? How many pounds had he left at the end of the week?

6. Mr. Dustan had a barrel of New Orleans molasses containing 100 gal., for which he paid 65 cts. per gallon, and from which he sold as follows: Monday, 4 gal. 1 qt.; Tuesday, 6 gal. 2 qt.; Wednesday, 10 gal. 3 qt.; Thursday, 8 gal.; Friday, 9 gal. 2 qt.; and Saturday, 14 gal. How many gallons did he sell during the week?

7. He sold the molasses for 80 cts. per gallon. How much did he make on his week's sale? How much is the remainder worth at the same price?

8. From a roll of ribbon, containing 30 yd., Hahne & Co. sold as follows: 3 yd. 2 ft., 5 yd. 1 ft., 6 yd., 8 yd. How many yards were sold?

9. The ribbon cost 15 cts. a yard and was sold for 18 cts. a yard. How much was made on the ribbon sold? What was the remainder worth at the selling price?

10. How many quarts equal 1 bushel?

11. At 6 cts. a quart what is the value of 1 peck of apples?

12. A grocer sold $\frac{1}{2}$ pk. of apples for $12\frac{1}{2}$ cts. What is the value of a bushel at that price?

13. A family picked 512 qt. of blackberries and sold them at \$2.75 per bushel. How much did they receive for the berries?

14. Mr. Babbitt has a field the perimeter of which is 7,029 ft. How many rods are there in the perimeter?

15. Four loads of coal weighed together 12,045 lb. The first weighed 3,429 lb.; the second 2,985 lb.; the third 3,197 lb. What was the weight of the fourth load?

63. Change:

- | | |
|------------------------------|-----------------------------|
| 1. 25 lb. 9 oz. to ounces. | 8. 96 bu. 3 pk. to pecks. |
| 2. 76 lb. 12 oz. to ounces. | 9. 78 bu. 2 pk. to pecks. |
| 3. 38 lb. 15 oz. to ounces. | 10. 28 da. 12 hr. to hours. |
| 4. 7 cwt. 50 lb. to pounds. | 11. 56 da. 15 hr. to hours. |
| 5. 19 cwt. 75 lb. to pounds. | 12. 12 yr. 6 mo. to months. |
| 6. 40 rd. 4 yd. to yards. | 13. 18 mo. 12 wk. to weeks. |
| 7. 24 rd. 5 yd. to yards. | 14. 15 yr. 7 mo. to months. |
-
- 15. 20 yr. 20 wk. to weeks.
 - 16. 400 in. to feet and inches.
 - 17. 92 ft. to yards and feet.
 - 18. 132 oz. to pounds and ounces.
 - 19. 825 lb. to hundredweights and pounds.
 - 20. 263 lb. to hundredweights and pounds.
 - 21. 148 hr. to days and hours.
 - 22. 296 hr. to days and hours.
 - 23. 1728 sq. in. to square feet and inches.
 - 24. 324 oz. to pounds and ounces.

SUMMARY OF CHAPTER I**NOTATION AND NUMERATION.**

Arabic numbers to billions.

Roman numbers to thousands.

FUNDAMENTAL PROCESSES.

Development of definitions and rules — of addition, subtraction, multiplication, division.

MEASUREMENT.

Denominate numbers.

CHAPTER II

FACTORS AND MULTIPLES—COMMON FRACTIONS— RATIO AND PROPORTION

FACTORS

64. What is the product of 2×3 ?

2 and 3 are factors of 6.

1. Of what number are 3 and 4 factors? 5 and 7?

2. Into how many 2's may 6 be divided? Into how many 3's?

2 and 3 are divisors of 6.

A whole number, as 2, 3, 4, is called an **integer**.

Any integer that will divide another integer without a remainder is a **divisor** of that integer.

The divisors that multiplied together make an integer are called the **factors** of that integer.

3. Find the factors or divisors of: 4; 6; 10; 14; 15; 21; 22, 25.

4. Read:

$$2 \times 2 \times 3 = 12.$$

$$2 \times 2 \times 2 \times 3 = 24.$$

A number may be the product of two or more factors.

5. Find all the factors of: 8; 18; 27; 16; 42; 44; 48.

6. Read:

$$\begin{array}{ll} 4 \times 6 = 24. & 24 \text{ is the product of each of three dif-} \\ 3 \times 8 = 24. & \text{ferent groups of factors:} \\ 2 \times 12 = 24. & 4 \times 6; 3 \times 8; 2 \times 12. \end{array}$$

7. Find the different groups of factors that each make: 28; 30; 32; 36; 40; 45; 48; 50.

65. $3 \times 1 = 3$; $5 \times 1 = 5$; $7 \times 1 = 7$.

1. Have 3, 5, and 7 any factors besides themselves and 1? 3, 5, and 7 are prime numbers.

A number that has no factors, except the number 1 and itself, is called a **prime number**.

A number that is made up, or composed, of factors other than itself and 1 is called a **composite number**. 12 and 24 are composite numbers.

2. Name five other composite numbers.
3. Name a composite number, the factors of which are: 3 and 5; 2 and 4; 6 and 3; 5 and 7; 2, 3, and 5; 2 and 6.

ORAL EXERCISES

66. Answer rapidly:

1. Of the following numbers, which are prime and which are composite: 5; 6; 7; 9; 13; 15; 17?

2. Name all the prime numbers from 1 to 100.

4 and 6 are factors of 24.

1. Are 4 and 6 prime or composite numbers?
2. What are the factors of 4? Of 6?
3. Are 2 and 3 prime or composite numbers?

$$2 \times 2 \times 2 \times 3 = 24.$$

2, 2, 2, 3, are the prime factors of 24.

The **prime factors** of a number are the prime numbers which multiplied together produce the number.

For example:

$$\left. \begin{array}{l} 6 \times 6 \\ 4 \times 9 \\ 3 \times 12 \\ 2 \times 18 \end{array} \right\} = 36 = \left\{ \begin{array}{l} 2 \times 3 \times 2 \times 3. \\ 2 \times 2 \times 3 \times 3. \\ 3 \times 3 \times 2 \times 2. \\ 2 \times 2 \times 3 \times 3. \end{array} \right.$$

Finding the factors of a number is called **factoring**.

WRITTEN EXERCISES

67. Factor:

- | | | | | |
|--------|--------|--------|---------|----------|
| 1. 54. | 3. 96. | 5. 66. | 7. 108. | 9. 100. |
| 2. 72. | 4. 65. | 6. 40. | 8. 120. | 10. 250. |

68. Recite in rotation, naming rapidly the prime and the composite factors of all the composite numbers from 1 to 100, thus: 1 is prime; 2 is prime; 3 is prime; 4 is composite, its factors are 2 and 2; and so on to 100.

69. What are the prime factors of 12? Of 15?

3. What number is a factor of both 12 and 15?

3 is a "common factor" or "common divisor" of 12 and 15. The word *common* means "belonging to more than one."

4. Name the factors of 3, 6, 12, 24, 36. Which factor is *common* to all the numbers?

A number that will divide each one of several numbers without a remainder is called a **common divisor** or **common factor** of the numbers.

70. 1. What is the largest number that is a common factor, or divisor, of 12, 24, 18, and 36?

6 is the greatest common divisor of 12, 24, 18, and 36.

The **greatest common divisor** of two or more numbers is the largest number that will divide each of them without a remainder. It may be expressed by the initials **g. c. d.**

ORAL EXERCISES

71. What is the greatest common divisor of:

- | | | |
|--------------------|----------------------|--------------------|
| 1. 2, 14, 16, 20? | 4. 14, 28, 56, 49? | 7. 9, 18, 27, 72? |
| 2. 10, 20, 30, 40? | 5. 6, 9, 15, 18, 27? | 8. 4, 62, 16, 8? |
| 3. 15, 45, 60? | 6. 8, 24, 36, 48? | 9. 21, 63, 49, 35? |

MULTIPLES

72. $6 \times 3 = 18$.

3 and 6 will exactly divide 18. The number 18, therefore, contains each of these numbers.

18 is called a **multiple** of 6, because it contains 6, or because 6 will divide it without a remainder. It is also a multiple of 3 because it contains 3, or because 3 will divide it without a remainder.

18 is also a multiple of 2 and 9.

Hence it is a common multiple of 2, 3, 6, and 9.

A **common multiple** of several numbers is a number that will contain each of them without a remainder.

ORAL EXERCISES

73. Name a common multiple of :

- | | | | |
|-------------|-------------|--------------|---------------|
| 1. 3 and 7. | 4. 2, 3, 4. | 7. 4, 7, 14. | 10. 5, 9, 3. |
| 2. 2 and 6. | 5. 6, 5, 2. | 8. 8, 12, 6. | 11. 10, 6, 5. |
| 3. 4 and 5. | 6. 3, 27. | 9. 5, 2, 10. | 12. 9, 5, 3. |

74. Name three common multiples of 2, 3, 4, and 6. Which is the smallest?

12 is the least common multiple of 2, 3, 4, and 6.

The **least common multiple** of several numbers is the smallest number that will contain each of them without a remainder. It may be expressed by the initials l. c. m.

ORAL EXERCISES

75. Name the least common multiple of :

- | | | | |
|-------------|-------------|--------------|---------------|
| 1. 2, 4, 5. | 4. 2, 7, 6. | 7. 18, 4, 3. | 10. 4, 5, 8. |
| 2. 2, 3, 5. | 5. 3, 7, 2. | 8. 7, 9, 3. | 11. 3, 6, 11. |
| 3. 4, 3, 8. | 6. 9, 2, 6. | 9. 5, 8, 10. | 12. 7, 4, 14. |

76. What is the least common multiple of 8, 6, and 3? You can readily see that it is 24. When the least common multiple is not easily found at sight, it may be found by factoring. The prime factors of all the numbers may then be multiplied together, and the result will be the least common multiple.

The prime factors of 8 are 2, 2, 2;
of 6 are 2, 3.

The prime factors of all the numbers are 2, 2, 2, 3.

The least common multiple $= 2 \times 2 \times 2 \times 3 = 24$.

24 is the least common multiple of 8, 6, and 3.

Find the l. c. m. of 10, 12, and 15.

SOLUTION: $10 = 5 \times 2$.
 $12 = 3 \times 2 \times 2$.
 $15 = 3 \times 5$.

The prime factors of the numbers are 5, 2, 3, and 2. Their product is 60.

Therefore 60 is the l. c. m. of 10, 12, and 15.

To find the least common multiple of two or more numbers, multiply together the prime factors of all the numbers, using each factor as many times as the largest number of times it occurs in any one of the numbers.

WRITTEN EXERCISES

77. Find, by factoring, the least common multiple of :

- | | | |
|----------------|------------------|-------------------|
| 1. 28, 24. | 7. 15, 36, 45. | 13. 27, 42, 12. |
| 2. 35, 60. | 8. 48, 66, 80. | 14. 16, 54, 15. |
| 3. 72, 96. | 9. 39, 6, 78. | 15. 81, 33, 21. |
| 4. 35, 75. | 10. 27, 42, 63. | 16. 36, 120, 72. |
| 5. 40, 80, 20. | 11. 15, 45, 75. | 17. 96, 160, 198. |
| 6. 36, 72, 48. | 12. 25, 75, 125. | 18. 88, 110, 330. |

COMMON FRACTIONS

78. How do you find one half of anything? One third? One fourth?

What is one half of 12 apples? One third of 12 apples? One fourth of 12 apples?

$\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ of 1 apple are **fractions** of the single apple, which we call the **unit**.

$\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ of 12 apples are fractions of the group, 12 apples, which in this case is the unit.

A **unit** is a single thing, or a group used as if it were a single thing.

A **fraction** is one or more of the equal parts of a unit.

A fraction shows three things:

(1) The unit is divided into a certain number of equal parts, named by the denominator.

(2) A certain number of these parts are used, counted by the numerator.

(3) The value of the fraction equals the numerator divided by the denominator.

In the fraction $\frac{3}{8}$, which figure shows into how many parts the unit is divided?

Which shows how many of these parts are used?

What is the figure below the line called?

What is the figure above the line called?

The word **denominator** means namer.

The word **numerator** means numberer.

The numerator and denominator are called the **terms** of the fraction.

REDUCTION

79. CHANGING THE FORM OF A FRACTION.

What kind of number is $6\frac{1}{2}$?

Can you express its value in another way?

What is a proper fraction? What is an improper fraction?

A mixed number?

In dealing with fractions, it is often necessary to change their form.

Mixed numbers may be changed to improper fractions.

$$\text{Thus: } 2\frac{2}{3} = \frac{8}{3}.$$

Improper fractions may be changed to whole or mixed numbers.

$$\text{Thus: } \frac{20}{3} = 6\frac{2}{3}.$$

A fraction may be changed to one of higher denomination or to one of lower denomination.

$$\text{Thus: } \frac{2}{4} = \frac{1}{2} = \frac{1}{2}.$$

The process of changing the form of a fraction without changing its value is called **reduction**.

The value of an improper fraction is always 1 or more than 1.

80. REDUCING AN IMPROPER FRACTION.

Change $\frac{18}{5}$ to a mixed number.

$$\text{SOLUTION: } \frac{3\frac{3}{5}}{5 \overline{)18}} \quad \frac{18}{5} = 3\frac{3}{5}.$$

Reduce $\frac{23}{8}$ to a mixed number.

To reduce an improper fraction to a whole or a mixed number, divide the numerator by the denominator. If there is no remainder, the result is an integer. If there is a remainder, place it over the denominator. The answer then will be a mixed number.

ORAL EXERCISES

81. Read and give the equivalent whole or mixed numbers :

1. $\frac{4}{2}, \frac{5}{2}, \frac{10}{2}, \frac{13}{2}, \frac{15}{2}, \frac{2}{2}, \frac{19}{2}, \frac{16}{2}$.
2. $\frac{8}{3}, \frac{10}{3}, \frac{15}{3}, \frac{21}{3}, \frac{22}{3}, \frac{30}{3}, \frac{18}{3}, \frac{37}{3}$.
3. $\frac{9}{4}, \frac{12}{4}, \frac{21}{4}, \frac{26}{4}, \frac{40}{4}, \frac{45}{4}, \frac{48}{4}, \frac{24}{4}$.
4. $\frac{12}{5}, \frac{15}{5}, \frac{27}{5}, \frac{40}{5}, \frac{44}{5}, \frac{65}{5}, \frac{60}{5}, \frac{32}{5}$.
5. $\frac{18}{6}, \frac{25}{6}, \frac{42}{6}, \frac{51}{6}, \frac{72}{6}, \frac{39}{6}, \frac{27}{6}, \frac{37}{6}$.
6. $\frac{15}{7}, \frac{21}{7}, \frac{38}{7}, \frac{42}{7}, \frac{84}{7}, \frac{50}{7}, \frac{60}{7}, \frac{75}{7}$.
7. $\frac{18}{8}, \frac{25}{8}, \frac{34}{8}, \frac{44}{8}, \frac{88}{8}, \frac{42}{8}, \frac{68}{8}, \frac{90}{8}$.
8. $\frac{27}{9}, \frac{37}{9}, \frac{48}{9}, \frac{88}{9}, \frac{65}{9}, \frac{108}{9}, \frac{75}{9}, \frac{72}{9}$.

WRITTEN EXERCISES

82. Reduce to whole or mixed numbers :

- | | | | |
|-----------------------|-----------------------|-----------------------|------------------------|
| 1. $\frac{138}{12}$. | 4. $\frac{230}{21}$. | 7. $\frac{412}{26}$. | 10. $\frac{525}{25}$. |
| 2. $\frac{184}{16}$. | 5. $\frac{312}{22}$. | 8. $\frac{425}{31}$. | 11. $\frac{400}{40}$. |
| 3. $\frac{244}{20}$. | 6. $\frac{198}{19}$. | 9. $\frac{225}{15}$. | 12. $\frac{520}{41}$. |

83. Express 4 as a fraction.

$$\frac{4}{1} = 4 \div 1 = 4. \quad 4 = 4 \times 1 = 4 \times \frac{1}{1} = \frac{4}{1}.$$

Any whole number may be written as a fraction having 1 as its denominator.

84. REDUCING A WHOLE NUMBER TO A FRACTION.

1. Reduce 4 to fourths.

SOLUTION: $4 \times \frac{1}{4} = \frac{4}{4}.$

2. Reduce 4 to eighths.

SOLUTION: $4 \times \frac{2}{8} = \frac{8}{8}.$

To reduce a whole number to an improper fraction having any denominator desired, multiply the whole number by 1 expressed as a fraction of the desired denomination.

ORAL EXERCISES

85. Change :

1. 3 to 12ths; to 18ths; to 20ths.
2. 10 to 15ths; to 100ths; to 1000ths.
3. 9 to 10ths; to 9ths; to 100ths.
4. 7 to 7ths; to 21sts; to 1000ths.

86. REDUCING A MIXED NUMBER TO A FRACTION.

Reduce $12\frac{3}{5}$ to an improper fraction.

SOLUTION :

$$12 \times \frac{5}{5} = \frac{60}{5}.$$

$$\frac{60}{5} + \frac{3}{5} = \frac{63}{5}.$$

Since there are 5 fifths in 1, in 12 there are 60 fifths.

60 fifths + 3 fifths = 63 fifths.

To reduce a mixed number to an improper fraction, multiply the integer by the denominator of the fraction ; to this product add the numerator, and write the sum over the denominator.

ORAL EXERCISES

87. Reduce to improper fractions :

1. $2\frac{1}{2}$, 6, $7\frac{1}{2}$, 8, $12\frac{1}{2}$.
2. $3\frac{1}{3}$, 4, $6\frac{2}{3}$, 7, $8\frac{2}{3}$.
3. $2\frac{1}{4}$, 10, $5\frac{3}{4}$, 9, $7\frac{1}{4}$.
4. 5, $6\frac{1}{5}$, $7\frac{2}{5}$, 8, $9\frac{4}{5}$.
5. $3\frac{1}{8}$, 5, $4\frac{5}{8}$, 7, $9\frac{1}{8}$.
6. $1\frac{2}{7}$, 7, $6\frac{3}{7}$, $8\frac{4}{7}$, 12.
7. 8, $4\frac{2}{8}$, $6\frac{5}{8}$, $7\frac{7}{8}$, 11.
8. $4\frac{1}{2}$, $4\frac{2}{3}$, $6\frac{3}{4}$, $7\frac{4}{7}$, $8\frac{5}{8}$.
9. $9\frac{6}{7}$, $10\frac{7}{10}$, $11\frac{8}{11}$, $12\frac{9}{12}$, $21\frac{1}{3}$.
10. $6\frac{4}{5}$, $7\frac{1}{2}$, $10\frac{1}{9}$, $9\frac{8}{9}$.

WRITTEN EXERCISES

88. Reduce to improper fractions :

1. $14\frac{5}{8}$.
2. $18\frac{8}{20}$.
3. $25\frac{11}{24}$.
4. $36\frac{15}{27}$.
5. $28\frac{9}{28}$.
6. $41\frac{16}{21}$.
7. $45\frac{15}{19}$.
8. $47\frac{16}{7}$.
9. $50\frac{12}{5}$.
10. $66\frac{21}{7}$.
11. $65\frac{11}{11}$.
12. $72\frac{11}{18}$.

COMPARISON, AND REDUCTION OF TERMS

89. Draw a line eight inches long.

Divide it into halves. Into fourths. Into eighths.

$\frac{1}{2}$ = how many fourths? How many eighths?

90. Fractions that have the same value can be written with different terms ; for example, $\frac{1}{2}$, $\frac{2}{4}$, $\frac{4}{8}$.

ORAL EXERCISES

91. Express :

1. $\frac{1}{4}$, in eighths ; in twelfths ; in sixteenths.
2. $\frac{1}{2}$, in sixths ; in twelfths ; in eighteenths.
3. $\frac{2}{3}$, in twelfths ; in sixths ; in ninths ; in eighteenths.
4. $\frac{1}{6}$, in tenths ; in twentieths ; in fortieths.
5. $\frac{2}{3}$, in tenths ; in twentieths ; in fortieths.
6. $\frac{3}{8}$, in tenths ; in twentieths ; in fortieths.
7. $\frac{1}{6}$, in twelfths ; in eighteenths ; in twenty-fourths ; in thirty-sixths.
8. $\frac{5}{8}$, in twelfths ; in eighteenths ; in twenty-fourths ; in thirty-sixths.

92. CHANGING TO LOWER OR HIGHER TERMS.

1. How do you change $\frac{4}{8}$ to the form $\frac{1}{2}$?
2. Multiply both numerator and denominator of $\frac{1}{4}$ by 3.
3. Is there any difference in value between $\frac{2}{12}$ and $\frac{1}{4}$?
4. Divide both numerator and denominator of $\frac{8}{16}$ by 8.
5. Does $\frac{8}{16}$ differ in value from $\frac{1}{2}$?

Multiplying or dividing both numerator and denominator of a fraction by the same number does not change the value of the fraction.

93. What are the "terms" of a fraction?

$\frac{30}{80} = \frac{30}{40} = \frac{10}{10} = \frac{5}{5}$. These fractions are equal, but are expressed in different terms.

The fraction $\frac{30}{80}$ is expressed in larger or higher terms than the others.

How can the fraction $\frac{30}{80}$ be changed to lower terms?

Can it be changed to higher terms?

94. Fractions may be reduced to lower terms by factoring both numerator and denominator, or canceling equal factors in both.

Thus: Reduce $\frac{10}{15}$ to lowest terms.

Factoring, we have: $\frac{2 \times 5}{3 \times 5} = \frac{2 \times \cancel{5}}{3 \times \cancel{5}} = \frac{2}{3}$.

We cross out or *cancel* 5 in each term, leaving 2 and 3 as the lowest terms of $\frac{10}{15}$.

Reduce $\frac{28}{42}$ to lowest terms.

Factoring, we have: $\frac{2 \times 2 \times 7}{2 \times 3 \times 7} = \frac{\cancel{2} \times 2 \times \cancel{7}}{\cancel{2} \times 3 \times \cancel{7}} = \frac{2}{3}$.

It is not necessary to write out the factors for cancellation. We may merely think the division by the common factor, and cancel the terms as they stand, and write the fraction as reduced.

By what number can both numerator and denominator of $\frac{30}{80}$ be divided to reduce it to the terms $\frac{5}{5}$?

What are the terms of: $\frac{9}{17}$, $\frac{18}{20}$, $\frac{25}{80}$, $\frac{44}{100}$?

By what number can both terms of $\frac{20}{24}$ be divided? Of $\frac{10}{12}$? Of $\frac{5}{6}$?

$\frac{5}{6}$ is in the lowest possible terms. Why?

A fraction is expressed in its **lowest terms** when no number greater than 1 will divide both numerator and denominator without a remainder.

ORAL EXERCISES

95. Reduce to lowest terms the following fractions: $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$, $\frac{6}{7}$, $\frac{7}{8}$, $\frac{8}{9}$, $\frac{9}{10}$.

Change to higher terms: $\frac{2}{3}$, $\frac{6}{10}$, $\frac{4}{15}$, $\frac{7}{18}$, $\frac{9}{21}$, $\frac{10}{24}$, $\frac{12}{25}$, $\frac{8}{27}$.

96. Fractions having like denominators are said to have a common denominator. For example, $\frac{1}{8}$, $\frac{2}{8}$, $\frac{5}{8}$, $\frac{7}{8}$, have the common denominator 8.

What does the word "common" mean?

97. Find a common denominator for $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$.

1. What is the least common multiple of 2, 3, 6, 4?
2. How many twelfths does $\frac{1}{2}$ equal? $\frac{1}{3}$? $\frac{1}{4}$? $\frac{1}{6}$?
3. How may both the numerator and the denominator of a fraction be changed without altering its value?
4. Name other multiples of 2, 3, 4, and 6.
5. Reduce $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$, to 24ths; to 36ths; to 48ths.
6. Reduce $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{10}$ to 20ths, to 40ths, to 60ths.

Any common multiple of the denominators of several fractions may be used as a common denominator of the fractions, but the *least common multiple* is generally used.

The least common multiple of the denominators is called the *least common denominator*. It may be expressed by the initials l. c. d.

ORAL EXERCISES

98. In each of the following exercises, find by inspection the least common denominator and reduce the fractions to fractions having this denominator:

- | | | |
|---|---|---|
| 1. $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$. | 5. $\frac{1}{2}$, $\frac{1}{7}$, $\frac{1}{14}$. | 9. $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{7}$, $\frac{5}{8}$. |
| 2. $\frac{1}{3}$, $\frac{1}{2}$, $\frac{1}{5}$. | 6. $\frac{1}{2}$, $\frac{5}{7}$, $\frac{3}{4}$, $\frac{1}{28}$. | 10. $\frac{3}{4}$, $\frac{5}{8}$, $\frac{7}{8}$, $\frac{11}{12}$. |
| 3. $\frac{2}{3}$, $\frac{5}{8}$, $\frac{1}{3}$, $\frac{5}{12}$. | 7. $\frac{2}{3}$, $\frac{3}{4}$, $\frac{7}{8}$, $\frac{5}{12}$. | 11. $\frac{2}{3}$, $\frac{5}{8}$, $\frac{3}{4}$, $\frac{1}{9}$. |
| 4. $\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{4}$, $\frac{1}{9}$. | 8. $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{8}$, $\frac{7}{16}$. | 12. $\frac{1}{2}$, $\frac{3}{7}$, $\frac{5}{14}$, $\frac{8}{28}$. |

ADDITION

99. Add:

1. 2 boys 2. 23 cows 3. 4 fifths 4. 16 twelfths
 3 boys 17 cows 3 fifths 14 twelfths
5. 6 girls + 4 girls = 10 what?
 6. $\frac{2}{8} + \frac{2}{8} = 5$ what? 7. $\frac{1}{8} + \frac{5}{8} = 6$ what?

$$\frac{2}{8} + \frac{5}{8} + \frac{1}{8} = \frac{14}{8} = 1\frac{6}{8} = 1\frac{3}{4}.$$

To add fractions having like denominators, add the numerators and place the sum over the common denominator. If the sum is an improper fraction, reduce it to a whole number, or to a mixed number having its fraction in the lowest terms.

ORAL EXERCISES

100. Add:

1. $\frac{5}{6}$ 2. $\frac{1}{12}$ 3. $\frac{5}{12}$ 4. $\frac{4}{8}$ 5. $\frac{8}{16}$ 6. $\frac{4}{16}$
 $\frac{4}{6}$ $\frac{8}{12}$ $\frac{7}{12}$ $\frac{8}{8}$ $\frac{5}{16}$ $\frac{7}{16}$
 $\frac{1}{6}$ $\frac{7}{12}$ $\frac{11}{12}$ $\frac{1}{8}$ $\frac{9}{16}$ $\frac{11}{16}$

101. Add 1 gallon, 2 quarts, 3 pints.

1. What is necessary before you can add these quantities?
 To what denomination do you change them?

Add 1 half, 2 thirds, 3 fourths.

2. What is necessary before you can add these fractions?
 To what common denominator will you change them?

You can take any common denominator as 12, 24, 36; but it is best to take the smallest, which is 12.

3. How many twelfths = $\frac{1}{2}$? $\frac{2}{3}$? $\frac{3}{4}$?

$$\frac{1}{2} = \frac{6}{12}$$

$$\frac{2}{3} = \frac{8}{12}$$

$$\frac{3}{4} = \frac{9}{12}$$

$$\frac{6}{12} + \frac{8}{12} + \frac{9}{12} = \frac{23}{12} = 1\frac{11}{12}.$$

$$1\frac{11}{12}$$

To add fractions having unlike denominators, change them to fractions having a common denominator, add the numerators, and treat the sum as in the addition of like fractions.

ORAL EXERCISES

102. Add:

- | | | | |
|----------------------------------|----------------------------------|------------------------------------|------------------------------------|
| 1. $\frac{2}{8} + \frac{3}{4}$. | 5. $\frac{2}{6} + \frac{5}{8}$. | 9. $\frac{2}{8} + \frac{4}{11}$. | 13. $\frac{2}{8} + \frac{5}{9}$. |
| 2. $\frac{5}{6} + \frac{3}{8}$. | 6. $\frac{3}{4} + \frac{5}{7}$. | 10. $\frac{3}{4} + \frac{5}{11}$. | 14. $\frac{1}{2} + \frac{2}{15}$. |
| 3. $\frac{3}{6} + \frac{2}{7}$. | 7. $\frac{5}{8} + \frac{2}{3}$. | 11. $\frac{3}{5} + \frac{2}{3}$. | 15. $\frac{2}{3} + \frac{5}{12}$. |
| 4. $\frac{5}{7} + \frac{3}{8}$. | 8. $\frac{7}{8} + \frac{3}{7}$. | 12. $\frac{3}{7} + \frac{2}{3}$. | 16. $\frac{4}{5} + \frac{1}{2}$. |

WRITTEN EXERCISES

103. Add:

- | | |
|---|---|
| 1. $\frac{1}{2} + \frac{2}{3} + \frac{3}{4}$. | 8. $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{4}{5}$. |
| 2. $\frac{3}{4} + \frac{5}{6} + \frac{4}{5}$. | 9. $\frac{1}{3} + \frac{2}{5} + \frac{1}{2} + \frac{3}{7}$. |
| 3. $\frac{2}{5} + \frac{5}{8} + \frac{2}{3}$. | 10. $\frac{3}{5} + \frac{2}{7} + \frac{1}{2} + \frac{2}{4}$. |
| 4. $\frac{3}{16} + \frac{5}{8} + \frac{2}{3} + \frac{1}{4}$. | 11. $\frac{2}{9} + \frac{5}{18} + \frac{7}{36} + \frac{5}{6}$. |
| 5. $\frac{2}{9} + \frac{5}{18} + \frac{7}{12} + \frac{7}{24}$. | 12. $\frac{5}{12} + \frac{7}{8} + \frac{2}{3} + \frac{7}{20}$. |
| 6. $\frac{3}{6} + \frac{3}{4} + \frac{5}{6} + \frac{3}{10}$. | 13. $\frac{4}{5} + \frac{2}{20} + \frac{6}{25} + \frac{7}{10}$. |
| 7. $\frac{3}{4} + \frac{2}{6} + \frac{5}{22} + \frac{7}{33}$. | 14. $\frac{5}{12} + \frac{7}{16} + \frac{3}{8} + \frac{11}{24} + \frac{3}{4}$. |

104. Solve:

1. $\frac{3}{7}$ of Mr. Ashland's wheat crop is at the depot, $\frac{2}{9}$ of it is at the mill, and $\frac{5}{21}$ of it is in a bin. What part of his wheat has been accounted for?

2. Mr. Jackson sold $\frac{1}{16}$ of his corn crop to the miller, $\frac{7}{32}$ of it to a dealer, $\frac{1}{4}$ of it to his neighbors, and $\frac{3}{8}$ of it he exchanged for wheat. What part of his crop did he dispose of?

3. Mr. Truesdell sold $\frac{1}{5}$ of his peaches in Morristown, $\frac{3}{10}$ of them in Madison, $\frac{2}{14}$ of them in Newark and $\frac{2}{5}$ of them in New York. What part of his crop did he sell?

4. Mr. Ray sells eggs to several customers: to Mrs. Lynch, $\frac{1}{8}$ of the eggs; to Mrs. Benson, $\frac{3}{16}$; to Mrs. Brown, $\frac{1}{4}$; to Mrs. Dunlap, $\frac{7}{40}$; and to Mrs. Holmes, $\frac{2}{20}$. What part of the eggs did he sell?

5. Mr. Trent went on a long journey; he traveled $\frac{2}{40}$ of the distance by rail, $\frac{11}{60}$ by boat, $\frac{7}{30}$ by stage and $\frac{29}{120}$ by automobile. What part of his journey did he travel in all of these ways?

105. Add:

1. 10 and $2\frac{2}{3}$.

$$\begin{array}{r} \text{SOLUTION: } 10 \quad 10 + 2 = 12. \\ \quad \quad \quad 2\frac{2}{3} \quad 10 + 2\frac{2}{3} = 12\frac{2}{3}. \\ \quad \quad \quad \hline 12\frac{2}{3} \end{array}$$

3. $3\frac{1}{2}$ and $2\frac{1}{3}$.

$$\begin{array}{r} \text{SOLUTION: } 3\frac{1}{2} = 3\frac{2}{3} \\ \quad \quad \quad 2\frac{1}{3} = 2\frac{2}{3} \\ \quad \quad \quad \hline 5\frac{4}{3} \end{array}$$

2. $2\frac{1}{3}$ and $3\frac{1}{3}$.

$$\begin{array}{r} \text{SOLUTION: } 2\frac{1}{3} \quad \frac{1}{3} + \frac{1}{3} = \frac{2}{3}. \\ \quad \quad \quad 3\frac{1}{3} \quad 2 + 3 = 5. \\ \quad \quad \quad \hline 5\frac{2}{3} \end{array}$$

4. $4\frac{2}{3}$ and $3\frac{3}{4}$.

$$\begin{array}{r} \text{SOLUTION: } \\ 4\frac{2}{3} = 4\frac{8}{12} \quad \frac{2}{3} = \frac{8}{12} \\ 3\frac{3}{4} = 3\frac{9}{12} \quad \frac{3}{4} = \frac{9}{12} \\ \quad \quad \quad \hline 8\frac{17}{12} \quad \frac{17}{12} = 1\frac{5}{12} \\ \quad \quad \quad 4 + 3 + 1\frac{5}{12} = 8\frac{5}{12}. \end{array}$$

In adding mixed numbers, add the fractions and the whole numbers separately, and then add the results.

WRITTEN EXERCISES

106. Add:

$$\begin{array}{r} 1. \quad 146\frac{2}{3} \\ \quad 145\frac{1}{3} \\ \hline \end{array}$$

$$\begin{array}{r} 4. \quad 408 \\ \quad 318\frac{6}{7} \\ \hline \end{array}$$

$$\begin{array}{r} 7. \quad 567\frac{7}{16} \\ \quad 438\frac{1}{4} \\ \hline \end{array}$$

$$\begin{array}{r} 10. \quad 390\frac{1}{4} \\ \quad 389\frac{4}{16} \\ \hline \end{array}$$

$$\begin{array}{r} 2. \quad 510\frac{1}{8} \\ \quad 500\frac{1}{16} \\ \hline \end{array}$$

$$\begin{array}{r} 5. \quad 4736\frac{1}{8} \\ \quad 3500\frac{1}{9} \\ \hline \end{array}$$

$$\begin{array}{r} 8. \quad 1860\frac{1}{6} \\ \quad 1760\frac{1}{10} \\ \hline \end{array}$$

$$\begin{array}{r} 11. \quad 230\frac{5}{12} \\ \quad 167\frac{1}{3} \\ \hline \end{array}$$

$$\begin{array}{r} 3. \quad 203\frac{1}{7} \\ \quad 421\frac{2}{4} \\ \hline \end{array}$$

$$\begin{array}{r} 6. \quad 489\frac{1}{11} \\ \quad 347\frac{8}{33} \\ \hline \end{array}$$

$$\begin{array}{r} 9. \quad 148\frac{5}{8} \\ \quad 217\frac{2}{3} \\ \hline \end{array}$$

$$\begin{array}{r} 12. \quad 250\frac{11}{12} \\ \quad 415\frac{5}{6} \\ \hline \end{array}$$

$$13. \quad 3\frac{1}{12} + 5\frac{2}{3} + 6\frac{3}{8}.$$

$$17. \quad 12\frac{5}{12} + 18\frac{5}{8} + 14\frac{1}{4}.$$

$$14. \quad 9\frac{2}{5} + 7\frac{2}{3} + 3\frac{5}{6}.$$

$$18. \quad 24\frac{7}{15} + 13\frac{2}{30} + 9\frac{2}{5}.$$

$$15. \quad 5\frac{5}{8} + 12\frac{3}{4} + 8\frac{5}{8}.$$

$$19. \quad 15\frac{3}{10} + 7\frac{2}{3} + 9\frac{2}{4}.$$

$$16. \quad 7\frac{7}{10} + 20\frac{5}{8} + 16\frac{2}{20}.$$

$$20. \quad 13\frac{3}{4} + 15\frac{3}{4} + 41\frac{3}{8}.$$

107. Solve:

1. Mr. Penrose sold $135\frac{1}{8}$ acres to Mr. Henry, $38\frac{3}{8}$ acres to Mr. Hanson, and $75\frac{3}{4}$ acres to Mr. Sexton. How many acres did he sell?

2. Mr. Wilbur, a grain dealer, sold $250\frac{3}{4}$ bu. of wheat, $340\frac{5}{16}$ bu. of corn, $125\frac{3}{8}$ bu. of oats, and $75\frac{1}{2}$ bu. of rye. How many bushels of grain did he sell?

3. On Monday Mr. Luce sold $45\frac{3}{4}$ lb. of butter; on Tuesday, 56 lb. 8 oz.; on Wednesday, $65\frac{3}{16}$ lb.; on Thursday, 35 lb. 4 oz. How many pounds did he sell?

SUGGESTION: 1 ounce = what fraction of a pound?

4. Mr. Cross, a coal dealer, sold 75 tons 500 lb. of pea coal for $\$287\frac{7}{8}$; 82 tons 1500 lb. of nut coal for $\$369\frac{5}{16}$; and 116 tons 1000 lb. of stove coal for $\$552\frac{3}{4}$. How many tons of coal did he sell? How much did he receive for it?

5. Mr. Knepp, a farmer, sold 4 loads of apples containing respectively $36\frac{5}{8}$, $42\frac{7}{32}$, $28\frac{3}{4}$, and $31\frac{9}{16}$ bushels. How many bushels did he sell?

6. A railroad train ran $35\frac{5}{10}$ miles the first hour, $38\frac{7}{10}$ miles the second hour, 40.25 miles the third hour, and 45.75 miles the fourth hour. How many miles did it run in the four hours?

7. Mr. Fisher sold on Monday, $56\frac{3}{8}$ lb. of butter; on Tuesday, 45 lb. 12 oz.; on Wednesday, $60\frac{3}{4}$ lb.; on Thursday, $38\frac{3}{4}$ lb.; on Friday, 47 lb. 8 oz.; on Saturday, 85 lb. 4 oz. How many pounds did he sell in all?

8. One side of a square is $8\frac{3}{4}$ in. What is the perimeter of the square?

9. The length of a rectangle is $25\frac{7}{8}$ in., the width is $12\frac{3}{4}$ in. What is its perimeter?

10. The sides of an equilateral triangle are $15\frac{5}{8}$ in. What is its perimeter?

SUBTRACTION

ORAL EXERCISES

108. Subtract:

1. $\frac{2}{4} - \frac{1}{4}$.

6. $\frac{3}{8} - \frac{1}{8}$.

11. $\frac{1}{18} - \frac{1}{18}$.

2. $\frac{4}{8} - \frac{3}{8}$.

7. $\frac{5}{16} - \frac{2}{16}$.

12. $\frac{7}{16} - \frac{4}{16}$.

3. $\frac{9}{12} - \frac{1}{12}$.

8. $\frac{7}{12} - \frac{5}{12}$.

13. $\frac{9}{10} - \frac{6}{10}$.

4. $\frac{15}{33} - \frac{11}{33}$.

9. $\frac{13}{72} - \frac{11}{72}$.

14. $\frac{13}{40} - \frac{5}{40}$.

5. $\frac{6}{27} - \frac{2}{27}$.

10. $\frac{25}{80} - \frac{16}{80}$.

15. $\frac{22}{120} - \frac{12}{120}$.

109. How do you subtract the fractions in the exercises above?

From $\frac{7}{8}$ take $\frac{5}{8}$.

SOLUTION: $\frac{7}{8} = \frac{21}{24}$
 $\frac{5}{8} = \frac{15}{24}$
 $\frac{21}{24} - \frac{15}{24} = \frac{6}{24}$

What is necessary before the subtraction can be performed?

What is the l. c. d. of $\frac{7}{8}$ and $\frac{5}{8}$?

To subtract fractions having like denominators, subtract their numerators, and place the result over the common denominator.

To subtract fractions having unlike denominators, reduce them to fractions having a common denominator, and subtract.

ORAL EXERCISES

110. Subtract:

1. $\frac{7}{8} - \frac{2}{8}$.

9. $\frac{3}{4} - \frac{2}{4}$.

17. $\frac{7}{9} - \frac{1}{9}$.

25. $\frac{5}{6} - \frac{3}{6}$.

2. $\frac{9}{10} - \frac{4}{10}$.

10. $\frac{5}{8} - \frac{5}{16}$.

18. $\frac{15}{16} - \frac{3}{4}$.

26. $\frac{7}{11} - \frac{1}{2}$.

3. $\frac{13}{12} - \frac{5}{6}$.

11. $\frac{7}{8} - \frac{5}{8}$.

19. $\frac{5}{6} - \frac{3}{6}$.

27. $\frac{4}{5} - \frac{3}{5}$.

4. $\frac{15}{16} - \frac{3}{8}$.

12. $\frac{4}{5} - \frac{3}{4}$.

20. $\frac{13}{16} - \frac{3}{8}$.

28. $\frac{21}{22} - \frac{11}{22}$.

5. $\frac{4}{8} - \frac{3}{8}$.

13. $\frac{3}{4} - \frac{2}{8}$.

21. $\frac{15}{16} - \frac{9}{16}$.

29. $\frac{11}{12} - \frac{3}{4}$.

6. $\frac{7}{8} - \frac{5}{8}$.

14. $\frac{13}{14} - \frac{2}{7}$.

22. $\frac{5}{6} - \frac{3}{6}$.

30. $\frac{4}{5} - \frac{3}{5}$.

7. $\frac{3}{8} - \frac{1}{4}$.

15. $\frac{3}{4} - \frac{5}{8}$.

23. $\frac{47}{50} - \frac{20}{50}$.

31. $\frac{17}{18} - \frac{13}{18}$.

8. $\frac{15}{17} - \frac{4}{17}$.

16. $\frac{7}{8} - \frac{3}{8}$.

24. $\frac{9}{10} - \frac{5}{10}$.

32. $\frac{19}{20} - \frac{6}{20}$.

WRITTEN EXERCISES

111. Copy and subtract :

- | | | | |
|--------------------------------------|-------------------------------------|--|--------------------------------------|
| 1. $\frac{7}{8} - \frac{5}{12}$. | 4. $\frac{11}{20} - \frac{9}{25}$. | 7. $\frac{13}{18} - \frac{5}{24}$. | 10. $\frac{8}{15} - \frac{9}{20}$. |
| 2. $\frac{11}{16} - \frac{13}{24}$. | 5. $\frac{7}{24} - \frac{7}{36}$. | 8. $\frac{109}{800} - \frac{3}{80}$. | 11. $\frac{17}{25} - \frac{3}{10}$. |
| 3. $\frac{9}{14} - \frac{11}{21}$. | 6. $\frac{27}{28} - \frac{9}{40}$. | 9. $\frac{121}{200} - \frac{12}{20}$. | 12. $\frac{22}{40} - \frac{7}{15}$. |

112. Solve :

1. Mr. Drake has a truck farm ; $\frac{3}{8}$ of it is planted with corn, $\frac{5}{16}$ of it with tomatoes, and $\frac{9}{32}$ with cucumbers.

(1) How much more is planted with corn than with cucumbers ?

(2) How much more is planted with corn and tomatoes together than with cucumbers ?

(3) How much more is planted with tomatoes and cucumbers together than with corn ?

2. Mr. Penrose sent his sons Leo, Richard, and Frank to Putnam, each with a load of wood. Leo had $5\frac{3}{4}$ of a cord, Richard had $1\frac{5}{6}$, and Frank had $\frac{3}{4}$ of a cord.

(1) How much more had Leo and Richard than Frank ?

(2) How much more had Richard and Frank than Leo ?

(3) How much did they all have ?

3. Mr. Zeliff has a large orchard of apple, peach, and pear trees. $\frac{5}{9}$ of the orchard is set out with apple trees, $\frac{5}{18}$ of it with peach trees, and $\frac{1}{6}$ with pear trees.

(1) How much more of the orchard is planted with apple trees than with pear trees ? With peach trees than with pear trees ?

(2) How much more of the orchard is planted with apple trees than with peach and pear trees together ?

4. If $\frac{1}{3}$ of a man's fruit crop was strawberries, $\frac{2}{5}$ blackberries, and the remainder currants, what part of his crop was currants ?

116. Subtract:

- 1.
- $10\frac{3}{4}$
- from
- $20\frac{3}{4}$
- .

$$\begin{array}{r} \text{SOLUTION: } 20\frac{3}{4} = 20\frac{9}{12} \\ 10\frac{3}{4} = 10\frac{9}{12} \\ \underline{10\frac{9}{12}} \end{array}$$

Why are the fractions changed in form?
Which is the larger?

- 2.
- $8\frac{1}{6}$
- from
- $15\frac{1}{3}$
- .

$$\begin{array}{r} \text{SOLUTION: } 15\frac{1}{3} = 15\frac{2}{6} = 14\frac{8}{6} \\ 8\frac{1}{6} = 8\frac{1}{6} = \frac{49}{6} \\ \underline{6\frac{1}{6}} \end{array}$$

When the fractions are reduced to a common denominator, which is the larger?

How is the fraction in the minuend increased?

To subtract a mixed number from a mixed number, subtract the fraction first and then the integer. If the fraction in the subtrahend is larger than that in the minuend, increase the fraction in the minuend by the value of 1 unit taken from the integer.

ORAL EXERCISES

117. Subtract:

$$\begin{array}{r} 1. \quad 21\frac{1}{6} \\ \underline{19\frac{5}{6}} \end{array}$$

$$\begin{array}{r} 3. \quad 25\frac{1}{3} \\ \underline{20\frac{1}{6}} \end{array}$$

$$\begin{array}{r} 5. \quad 84\frac{1}{4} \\ \underline{74\frac{1}{8}} \end{array}$$

$$\begin{array}{r} 7. \quad 40\frac{1}{8} \\ \underline{10\frac{1}{16}} \end{array}$$

$$\begin{array}{r} 9. \quad 91\frac{1}{2} \\ \underline{7\frac{5}{8}} \end{array}$$

$$\begin{array}{r} 2. \quad 7\frac{3}{8} \\ \underline{4\frac{5}{8}} \end{array}$$

$$\begin{array}{r} 4. \quad 13\frac{1}{3} \\ \underline{11\frac{2}{3}} \end{array}$$

$$\begin{array}{r} 6. \quad 7\frac{1}{3} \\ \underline{5\frac{1}{4}} \end{array}$$

$$\begin{array}{r} 8. \quad 6\frac{3}{12} \\ \underline{2\frac{1}{2}} \end{array}$$

$$\begin{array}{r} 10. \quad 9\frac{1}{9} \\ \underline{7\frac{1}{3}} \end{array}$$

WRITTEN EXERCISES

118. Copy and subtract:

$$\begin{array}{r} 1. \quad 25\frac{1}{2} \\ \underline{8\frac{1}{4}} \end{array}$$

$$\begin{array}{r} 4. \quad 30\frac{7}{8} \\ \underline{14\frac{1}{4}} \end{array}$$

$$\begin{array}{r} 7. \quad 13\frac{5}{18} \\ \underline{6\frac{7}{18}} \end{array}$$

$$\begin{array}{r} 10. \quad 18\frac{5}{8} \\ \underline{9\frac{7}{12}} \end{array}$$

$$\begin{array}{r} 2. \quad 62\frac{1}{2} \\ \underline{35\frac{5}{8}} \end{array}$$

$$\begin{array}{r} 5. \quad 16\frac{4}{9} \\ \underline{7\frac{2}{3}} \end{array}$$

$$\begin{array}{r} 8. \quad 43 \\ \underline{15\frac{5}{7}} \end{array}$$

$$\begin{array}{r} 11. \quad 50 \\ \underline{11\frac{7}{10}} \end{array}$$

$$\begin{array}{r} 3. \quad 28\frac{3}{8} \\ \underline{13\frac{7}{10}} \end{array}$$

$$\begin{array}{r} 6. \quad 35\frac{15}{16} \\ \underline{20\frac{9}{16}} \end{array}$$

$$\begin{array}{r} 9. \quad 120\frac{4}{19} \\ \underline{19\frac{5}{19}} \end{array}$$

$$\begin{array}{r} 12. \quad 45\frac{5}{7} \\ \underline{24\frac{2}{3}} \end{array}$$

- | | | |
|---|---|---------------------------------------|
| 13. $20 - 12\frac{1}{2}$. | 19. $45 - 24\frac{7}{12}$. | 25. $31 - 18\frac{5}{8}$. |
| 14. $40 - 25\frac{7}{16}$. | 20. $98 - 32\frac{4}{9}$. | 26. $30 - 20\frac{5}{8}$. |
| 15. $25\frac{3}{4} - 14\frac{1}{2}$. | 21. $84\frac{5}{8} - 21\frac{1}{4}$. | 27. $35\frac{5}{8} - 27\frac{3}{8}$. |
| 16. $40\frac{9}{16} - 25\frac{3}{8}$. | 22. $42\frac{11}{12} - 28\frac{5}{6}$. | 28. $56\frac{7}{9} - 19\frac{2}{3}$. |
| 17. $62\frac{1}{8} - 24\frac{1}{4}$. | 23. $76\frac{3}{4} - 45\frac{11}{12}$. | 29. $89\frac{1}{8} - 37\frac{3}{4}$. |
| 18. $100\frac{1}{4} - 84\frac{5}{16}$. | 24. $94\frac{4}{9} - 72\frac{5}{6}$. | 30. $47\frac{1}{2} - 14\frac{7}{8}$. |

119. Solve:

1. From a piece of silk containing 50 yd., $12\frac{5}{16}$ yd. were sold. How many yards were left?

2. Mr. Lane is $50\frac{3}{4}$ years old. Mr. Durand is $36\frac{3}{4}$ years old. How much older is Mr. Lane than Mr. Durand?

3. Newark is $9\frac{7}{8}$ mi. from New York. Morristown is $25\frac{5}{16}$ mi. from New York. How much farther from New York is Morristown than Newark?

4. Mr. Denman had 120 acres of land. He sold $45\frac{7}{8}$ acres to Mr. Baldwin and $22\frac{3}{4}$ acres to Mr. Rice. How many acres had he left?

5. Mr. Luther sold $28\frac{3}{4}$ lb. of butter Monday, $36\frac{7}{16}$ lb. Tuesday, and $40\frac{5}{8}$ lb. Wednesday. How much less than 125 lb. did he sell?

6. The perimeter of a parallelogram is $161\frac{9}{16}$ in. The sum of three of its sides is $113\frac{1}{16}$ in. What is the length of the other side?

7. Mr. Nelson starts to walk from Morristown to Jersey City, a distance of 30 mi. The first hour he walks $4\frac{3}{8}$ mi., the second hour $3\frac{5}{8}$ mi., the third hour $4\frac{5}{12}$ mi. How far from Jersey City is he at the end of the third hour?

8. How much less than 2 hours is the sum of $30\frac{3}{8}$ minutes, $45\frac{5}{8}$ minutes, and $25\frac{3}{4}$ minutes?

9. How much less than an acre is the sum of $25\frac{5}{16}$ square rods, $35\frac{5}{12}$ square rods, and 50 square rods?

MULTIPLICATION

120. 8 times 3 books = how many books?

8 times 3 fourths = how many fourths?

SOLUTION: $8 \times \frac{3}{4} = 2\frac{1}{4}$, or 6.

To multiply a fraction by a whole number, multiply the numerator by the whole number and place the product over the denominator.

The same result is obtained by taking $\frac{3}{4}$ of 8.

$$\frac{1}{4} \text{ of } 8 = 2;$$

$$\frac{3}{4} \text{ of } 8 = 6.$$

ORAL EXERCISES

121. Multiply:

1. $\frac{1}{2} \times 12.$

4. $16 \times \frac{1}{4}.$

7. $\frac{3}{4} \times 16.$

10. $\frac{1}{8} \times 20.$

2. $\frac{3}{8} \times 12.$

5. $24 \times \frac{5}{8}.$

8. $16 \times \frac{3}{4}.$

11. $\frac{3}{8} \times 20.$

3. $\frac{1}{4} \times 16.$

6. $40 \times \frac{7}{8}.$

9. $\frac{1}{6} \times 36.$

12. $18 \times \frac{5}{8}.$

122. What is $\frac{1}{2}$ of $\frac{1}{2}$? What is $\frac{1}{2}$ of $\frac{1}{4}$? What is $\frac{1}{2}$ of $\frac{1}{8}$?

Measure off a line 16 inches long on the blackboard and divide it into halves, fourths, eighths, and sixteenths.

You find by measurement that $\frac{1}{2}$ of $\frac{1}{4} = \frac{1}{8}$.

The same result is obtained by multiplying the numerators, 1 and 1, together; and the denominators, 2 and 4, together.

What is $\frac{1}{2}$ of $\frac{1}{8}$? $\frac{1}{2}$ of $\frac{1}{8} = \frac{1}{2} \times \frac{1}{8} = \frac{1}{16}$.

To find a fraction of a fraction, or to multiply a fraction by a fraction, we multiply the numerators together for a new numerator and the denominators together for a new denominator.

ORAL EXERCISES

123. Find:

1. $\frac{1}{2}$ of $\frac{3}{4}.$

4. $\frac{3}{8} \times \frac{1}{2}.$

7. $\frac{1}{2}$ of $\frac{5}{8}.$

10. $\frac{5}{8} \times \frac{1}{3}.$

2. $\frac{3}{4} \times \frac{1}{2}.$

5. $\frac{1}{2}$ of $\frac{3}{8}.$

8. $\frac{7}{8} \times \frac{1}{2}.$

11. $\frac{1}{4}$ of $\frac{1}{8}.$

3. $\frac{1}{2}$ of $\frac{1}{8}.$

6. $\frac{2}{3} \times \frac{1}{2}.$

9. $\frac{1}{8}$ of $\frac{1}{6}.$

12. $\frac{3}{4} \times \frac{3}{4}.$

WRITTEN EXERCISES

124. Solve:

1. $\frac{4}{5} \times \frac{5}{6}$. 3. $\frac{2}{3}$ of $\frac{5}{6}$. 5. $\frac{2}{5}$ of $\frac{3}{10}$. 7. $\frac{7}{10} \times \frac{3}{5}$.
 2. $\frac{2}{3}$ of $\frac{5}{6}$. 4. $\frac{7}{8} \times \frac{2}{3}$. 6. $\frac{3}{4}$ of $\frac{5}{6}$. 8. $\frac{7}{8} \times \frac{3}{4}$.

125. Solve:

1. At \$ $\frac{1}{2}$ a yard, what will $\frac{3}{4}$ of a yard of cloth cost?
 $\frac{3}{8}$ of a yard? $\frac{7}{8}$ of a yard? $\frac{5}{8}$ of a yard?
 2. At \$ $\frac{3}{4}$ a yard what is the value of $\frac{3}{4}$ of a yard of lace?
 $\frac{5}{8}$ of a yard? $\frac{3}{8}$ of a yard?

126. CANCELLATION IN MULTIPLICATION OF FRACTIONS.

$$\frac{2}{3} \times \frac{3}{5} = \frac{2 \times 3}{3 \times 5} = \frac{2}{5}.$$

In the example above, 3 is a factor in both numerator and denominator. Taking it out (**canceled** it) is dividing by it.

The division may be written $\frac{2}{\cancel{3}} \times \frac{\cancel{3}}{5} = \frac{2}{5}$, leaving only 2 in the numerator and 5 in the denominator.

Does dividing both numerator and denominator by the same number affect the value of the fraction? .

We draw a line through a figure to show that it is canceled.

Cancel factors that are common to numerator and denominator in the following and see if the quotient is changed:

$$\frac{6 \times 3}{3 \times 3}; \quad \frac{7 \times 2}{7 \times 1}; \quad \frac{10 \times 2}{2 \times 5}; \quad \frac{6 \times 8}{8 \times 6}.$$

Cancel the common factors of the numerators and denominators in each of the following before multiplying:

$$\frac{3}{44} \times \frac{12}{11}; \quad \frac{4}{6} \times \frac{15}{8}; \quad \frac{20}{7} \times \frac{35}{60}; \quad \frac{14}{15} \times \frac{15}{14}.$$

In multiplication of fractions the striking out of the same factor from both numerator and denominator is called **cancellation**. It is reducing to lowest terms by taking out common factors before multiplying.

1. Multiply: $\frac{2}{5} \times \frac{3}{7} \times \frac{3}{4} \times \frac{1}{2} \times \frac{7}{10}$.

SOLUTION: $\frac{2}{5} \times \frac{3}{7} \times \frac{3}{4} \times \frac{1}{2} \times \frac{7}{10} = \frac{9}{200}$.

After cancellation there are left 3, 3, 1, numerators; and 5, 4, 10, denominators. Multiplying, we have $\frac{9}{200}$.

2. Multiply: $\frac{7}{15}$ by $\frac{20}{21}$.

SOLUTION: $\frac{7}{15} \times \frac{20}{21} = \frac{4}{3 \times 3} = \frac{4}{9}$.

In this problem no numerators and denominators exactly cancel each other; but we cancel equal factors in numerators and denominators; that is, 5 in both 20 and 15, leaving the remaining factors 4 and 3; 7 in both 7 and 21, leaving the factors 1 and 3.

ORAL EXERCISES

127. Explain the cancellation in the following problems:

1. Multiply: $\frac{2}{8} \times \frac{2}{15} \times \frac{20}{21} \times \frac{15}{16} = \frac{5}{7 \times 16} = \frac{5}{112}$.

2. Multiply: $\frac{3}{8} \times \frac{4}{5} \times \frac{6}{7} \times \frac{14}{15}$.

3. Multiply: $\frac{5}{8} \times \frac{18}{21} \times \frac{7}{20}$.

4. Multiply: $\frac{12}{18} \times \frac{25}{45} \times \frac{33}{5}$.

128. Before multiplying mixed numbers it is usually simpler to reduce them to improper fractions, canceling the common factors.

$$3\frac{1}{2} \times \frac{7}{16} \times \frac{10}{11} \times \frac{2}{3} = \frac{16}{5} \times \frac{7}{16} \times \frac{10}{11} \times \frac{2}{3} = \frac{28}{33}$$

WRITTEN EXERCISES

129. Multiply, simplifying the work by cancellation :

- | | |
|---|--|
| 1. $\frac{3}{4} \times \frac{5}{8} \times 2\frac{1}{5} \times \frac{2}{3}$. | 6. $\frac{1}{2}\frac{1}{3} \times \frac{6}{7} \times \frac{5}{9} \times \frac{5}{11}$. |
| 2. $2\frac{1}{3} \times \frac{9}{14} \times \frac{7}{10} \times \frac{5}{16}$. | 7. $3\frac{1}{3} \times \frac{7}{15} \times \frac{5}{12} \times \frac{6}{7}$. |
| 3. $\frac{3}{8} \times \frac{5}{6} \times 4\frac{2}{3} \times \frac{4}{23}$. | 8. $\frac{2}{3}\frac{6}{7} \times \frac{2}{13} \times 3\frac{1}{4} \times \frac{7}{48}$. |
| 4. $6\frac{1}{4} \times \frac{3}{11} \times \frac{5}{8} \times \frac{7}{15}$. | 9. $4\frac{4}{10} \times \frac{1}{2}\frac{5}{2} \times \frac{3}{16} \times \frac{3}{40}$. |
| 5. $5\frac{1}{4} \times \frac{6}{7} \times 2\frac{1}{3} \times \frac{1}{20}$. | 10. $4\frac{2}{4} \times \frac{8}{15} \times \frac{7}{8} \times \frac{1}{21}$. |

130. Multiply 18 by $6\frac{1}{2}$.

SOLUTION: 18

$$\begin{array}{r} 6\frac{1}{2} \\ 108 \\ 9 \\ \hline 117 \end{array}$$

We find 6×18 ; to the product we add $\frac{1}{2}$ of 18. The answer is 117.

To multiply a whole number by a mixed number, multiply by the fraction and the integer separately and add the results.

WRITTEN EXERCISES

131. Multiply:

- | | | | | |
|----------------------------|----------------------------|----------------------------|----------------------------|----------------------------|
| 1. 20 | 2. 36 | 3. 72 | 4. 45 | 5. 28 |
| $\underline{5\frac{1}{4}}$ | $\underline{8\frac{1}{6}}$ | $\underline{8\frac{1}{9}}$ | $\underline{7\frac{1}{6}}$ | $\underline{7\frac{1}{7}}$ |

132. Multiply $3\frac{3}{4}$ by $8\frac{1}{2}$.

SOLUTION: $3\frac{3}{4} = \frac{15}{4}$, $8\frac{1}{2} = \frac{17}{2}$. $\frac{15}{4} \times \frac{17}{2} = \frac{255}{8} = 31\frac{7}{8}$.

To multiply a mixed number by a mixed number, reduce the mixed numbers to the form of improper fractions, and multiply.

WRITTEN EXERCISES

133. Find the cost of:

- $6\frac{1}{2}$ yd. of ribbon at $12\frac{1}{2}$ ¢ a yard.
- $8\frac{1}{2}$ lb. of sugar at $6\frac{1}{4}$ ¢ a pound.
- $7\frac{1}{4}$ yd. of dress goods at $24\frac{1}{2}$ ¢ a yard.
- $4\frac{1}{4}$ tons of coal at $\$5\frac{1}{2}$ a ton.

134. Multiply:

- | | | |
|---|--|--|
| 1. $\frac{2}{3} \times 2\frac{1}{2}$. | 11. $\frac{3}{4} \times \frac{4}{5}$. | 21. $3\frac{1}{4} \times \frac{5}{18} \times 8$. |
| 2. $\frac{3}{8} \times 2\frac{1}{2}$. | 12. $\frac{4}{5} \times \frac{5}{6}$. | 22. $\frac{1}{18} \times 5\frac{1}{2} \times \frac{5}{6}$. |
| 3. $\frac{4}{5} \times 3\frac{1}{8}$. | 13. $\frac{6}{7} \times \frac{1}{16}$. | 23. $\frac{8}{35} \times 16\frac{1}{2} \times \frac{5}{8}$. |
| 4. $2\frac{1}{2} \times \frac{7}{10}$. | 14. $\frac{5}{6} \times \frac{2}{5}$. | 24. $\frac{8}{11} \times 6 \times 5\frac{1}{4}$. |
| 5. $5\frac{1}{2} \times 3\frac{3}{8}$. | 15. $\frac{8}{11} \times \frac{3}{4}$. | 25. $\frac{7}{10} \times 10\frac{1}{2}$. |
| 6. $4\frac{1}{3} \times 5$. | 16. $\frac{2}{3} \times \frac{3}{4} \times \frac{4}{5}$. | 26. $33\frac{1}{8} \times \frac{9}{10}$. |
| 7. $6 \times 3\frac{1}{2}$. | 17. $\frac{3}{8} \times \frac{5}{7} \times 2\frac{1}{2}$. | 27. $\frac{1}{2} \times 6\frac{1}{4}$. |
| 8. $9 \times 3\frac{3}{8}$. | 18. $2\frac{1}{2} \times \frac{7}{10}$. | 28. $3\frac{1}{2} \times \frac{6}{7} \times 9$. |
| 9. $15 \times 2\frac{1}{3}$. | 19. $5\frac{1}{2} \times \frac{1}{2}$. | 29. $\frac{7}{4} \times 9 \times 3\frac{1}{8}$. |
| 10. $1\frac{3}{4} \times \frac{5}{7}$. | 20. $\frac{5}{9} \times \frac{8}{5} \times 19$. | 30. $\frac{7}{9} \times 8 \times 4\frac{1}{2}$. |

135. Solve:

- At \$3 $\frac{3}{4}$ a yard what will 18 $\frac{1}{2}$ yd. of silk cost?
- What are 5 $\frac{3}{8}$ bu. of apples worth, at 62 $\frac{1}{2}$ cts. a bushel?
- At 65 cts. a bushel, what are 25 bu. 3 pk. of corn worth?
- How many square feet in 8 boards, each 12 $\frac{1}{2}$ ft. long and 1 $\frac{1}{4}$ ft. wide?
- A stone block is 6 ft. long, 4 $\frac{3}{8}$ ft. wide, and 2 $\frac{1}{4}$ ft. thick. How many cubic feet are there in the block?
- Mr. Randall raised 28 $\frac{3}{4}$ bu. of wheat an acre. He sowed 4 $\frac{1}{2}$ acres. How many bushels did he raise?
- If a boy hoes 2 $\frac{1}{2}$ rows of potatoes in an hour, how many rows will he hoe in 4 $\frac{3}{4}$ hours?
- How far will a ship sail in 7 hours if it goes 15 $\frac{3}{4}$ knots an hour?
- At 18 $\frac{1}{2}$ cts. a pound, how much will 11 $\frac{1}{8}$ lb. of steak cost? 13 $\frac{1}{4}$ lb.?
- If a boy rows 3 $\frac{1}{2}$ mi. in an hour, how far will he row in 2 $\frac{3}{4}$ hr.?

DIVISION

136. Draw a line $\frac{1}{2}$ inch long. How many fourths of an inch are there in it?

1. How many fourths are there in 1 half?

$$2 \times \frac{1}{4} = \frac{1}{2}. \quad \frac{1}{2} \div \frac{1}{4} = 2.$$

The ratio of $\frac{1}{2}$ to $\frac{1}{4} = 2$.

The ratio of two numbers is the quotient that results from dividing one by the other.

This is true whether the numbers are whole numbers, fractions, or mixed numbers.

Measure a line 2 inches long. How many half-inches are there in it? There are four halves in two wholes.

2. Divide 2 by $\frac{1}{2}$. $\frac{1}{2}$ is contained in 2 four times.

$$\text{The ratio of 2 to } \frac{1}{2} = 4. \quad 2 \div \frac{1}{2} = 4.$$

3. Divide $2\frac{1}{2}$ by $\frac{1}{2}$ ($2\frac{1}{2} = \frac{5}{2}$).

$$\text{The ratio of } 2\frac{1}{2} \text{ to } \frac{1}{2} = 5. \quad 2\frac{1}{2} \div \frac{1}{2} = 5.$$

4. Divide $1\frac{1}{4}$ by $\frac{1}{4}$.

$$\text{The ratio of } 1\frac{1}{4} \text{ to } \frac{1}{4} = 5. \quad 1\frac{1}{4} \div \frac{1}{4} = 5.$$

ORAL EXERCISES

137. Find the ratio of each of the following to $\frac{1}{2}$:

- | | | | | |
|---------------------|--------------------|----------------------|----------------------|----------------------|
| 1. $6\frac{1}{2}$. | 5. $\frac{3}{4}$. | 9. $4\frac{1}{4}$. | 13. 6. | 17. $2\frac{3}{4}$. |
| 2. $3\frac{1}{2}$. | 6. 5. | 10. $1\frac{1}{4}$. | 14. $3\frac{1}{2}$. | 18. $4\frac{1}{2}$. |
| 3. $1\frac{1}{2}$. | 7. $\frac{1}{4}$. | 11. $5\frac{1}{2}$. | 15. $2\frac{1}{4}$. | 19. $7\frac{1}{2}$. |
| 4. $5\frac{1}{4}$. | 8. 1. | 12. $2\frac{1}{2}$. | 16. 3. | 20. $4\frac{3}{4}$. |

138. Solve:

If $\frac{1}{2}$ lb. of candy costs 20 cts., what will $3\frac{1}{2}$ lb. cost?

The ratio of $3\frac{1}{2}$ to $\frac{1}{2} = 7$.

Therefore, $3\frac{1}{2}$ lb. will cost 7×20 cts. = ?

WRITTEN EXERCISES

139. Find the ratio of the fractions in each of the following :

1. $\frac{1}{2} \div \frac{1}{8}$.

7. $\frac{1}{4} \div \frac{1}{8}$.

13. $\frac{6}{8} \div \frac{3}{8}$.

2. $\frac{1}{2} \div \frac{3}{8}$.

8. $\frac{3}{4} \div \frac{1}{8}$.

14. $\frac{3}{8} \div \frac{3}{8}$.

3. $\frac{1}{4} \div \frac{1}{2}$.

9. $\frac{5}{8} \div \frac{5}{8}$.

15. $\frac{3}{4} \div \frac{1}{2}$.

4. $3 \div \frac{1}{2}$.

10. $1 \div \frac{1}{4}$.

16. $6 \div \frac{1}{4}$.

5. $3\frac{1}{2} \div \frac{1}{2}$.

11. $2 \div \frac{1}{4}$.

17. $5 \div \frac{1}{2}$.

6. $2\frac{1}{2} \div \frac{1}{4}$.

12. $6\frac{1}{2} \div \frac{1}{4}$.

18. $3\frac{1}{2} \div \frac{1}{4}$.

140. Solve:

1. At Christmas Uncle John gave $5\frac{1}{2}$ bu. of sweet potatoes to several families. He gave $\frac{1}{2}$ bu. to each family. To how many families did he give? At 75 cts. a bushel, what did the potatoes cost him?

2. $2\frac{3}{4}$ yd. of ribbon were used in making bows. $\frac{1}{4}$ yd. was used for each bow. How many bows were made? What were they worth at 24 cts. apiece?

3. A mason built $5\frac{1}{2}$ rd. of stone wall, at the rate of $\frac{1}{2}$ rd. a day. How long did it take him? How much did the wall cost at \$4 $\frac{1}{2}$ a day? At the rate of \$7 $\frac{1}{2}$ a rod?

4. If it takes $\frac{1}{2}$ yd. of cloth for a waistcoat, how many waistcoats can be made from 6 yd.? What will they cost at \$3.25 apiece?

141. Read $\frac{1}{8}$, $\frac{2}{8}$, $\frac{4}{8}$.

1. What does the denominator of a fraction show?

2. What does the numerator show?

3. How many times is $\frac{2}{8}$ contained in $\frac{4}{8}$?
There are two 2 eighths in 4 eighths.

4. What is the ratio of $\frac{1}{8}$ to $\frac{2}{8}$?

5. What do you divide to find this?

142. Divide $\frac{3}{4}$ by $\frac{2}{4}$.

Dividing the numerator, 3, of the dividend by the numerator, 2, of the divisor we get $\frac{3}{2}$.

This may be stated thus:

SOLUTION: $\frac{3}{4} \div \frac{2}{4} = \frac{3}{2} = 1\frac{1}{2}$.

That is, there are one and one half 2 fourths in 3 fourths; so that the ratio of $\frac{3}{4}$ to $\frac{2}{4}$ is $1\frac{1}{2}$, which is $\frac{3}{2}$, or the ratio of 3 to 2, the ratio of the numerators.

When the denominators of two fractions are alike, the quotient or ratio of the fractions is found by dividing the numerator of the dividend by the numerator of the divisor.

ORAL EXERCISES

143. Find the quotient of each of the following:

- | | | | |
|-------------------------------------|--|---|--|
| 1. $\frac{5}{6} \div \frac{3}{6}$. | 4. $\frac{9}{10} \div \frac{4}{10}$. | 7. $\frac{30}{35} \div \frac{8}{35}$. | 10. $\frac{40}{51} \div \frac{8}{51}$. |
| 2. $\frac{7}{8} \div \frac{3}{8}$. | 5. $\frac{15}{16} \div \frac{5}{16}$. | 8. $\frac{12}{20} \div \frac{4}{20}$. | 11. $\frac{42}{48} \div \frac{10}{48}$. |
| 3. $\frac{8}{9} \div \frac{4}{9}$. | 6. $\frac{21}{24} \div \frac{7}{24}$. | 9. $\frac{26}{45} \div \frac{18}{45}$. | 12. $\frac{17}{18} \div \frac{3}{18}$. |

144. What is the ratio of $\frac{1}{2}$ to $\frac{1}{3}$?

SOLUTION: $\frac{1}{2} = \frac{3}{6}$. $\frac{1}{3} = \frac{2}{6}$. $\frac{3}{6} \div \frac{2}{6} = \frac{3}{2}$.

The same result may be obtained by *inverting* (turning upside down) the divisor, and multiplying as in multiplication of fractions, thus:

$$\frac{1}{2} \div \frac{1}{3} = \frac{1}{2} \times \frac{3}{1} = \frac{3}{2}.$$

What is the ratio of $\frac{1}{4}$ to $\frac{2}{3}$?

To divide a fraction by a fraction having a different denominator, invert the divisor and multiply the fractions.

NOTE TO TEACHERS. The full explanation of this convenient rule is given in Book III. What is here given is deemed sufficient at this stage.

WRITTEN EXERCISES

145. Find the quotient of:

1. $\frac{2}{3} \div \frac{3}{4}$.

4. $\frac{4}{5} \div \frac{3}{8}$.

7. $\frac{5}{7} \div \frac{2}{3}$.

10. $\frac{3}{16} \div \frac{5}{9}$.

2. $\frac{4}{15} \div \frac{2}{3}$.

5. $\frac{6}{7} \div \frac{4}{5}$.

8. $\frac{5}{12} \div \frac{4}{11}$.

11. $\frac{7}{12} \div \frac{3}{8}$.

3. $\frac{4}{18} \div \frac{3}{8}$.

6. $\frac{7}{20} \div \frac{3}{10}$.

9. $\frac{5}{16} \div \frac{4}{11}$.

12. $\frac{6}{25} \div \frac{4}{5}$.

What is the simplest form in which an integer may be expressed as a fraction?

$$6 = \frac{6}{1}.$$

146. Divide:

1. 6 by $\frac{2}{3}$.

SOLUTION: $\frac{6}{1} \div \frac{2}{3} = \frac{6}{1} \times \frac{3}{2} = \frac{1 \cdot 3}{1 \cdot 2} = 9.$

2. $\frac{2}{3} \div 6$.

SOLUTION: $\frac{2}{3} \div 6 = \frac{2}{3} \times \frac{1}{6} = \frac{2}{1 \cdot 3 \cdot 2} = \frac{1}{3}.$

To divide fractions when either dividend or divisor is a whole number, write the whole number as a fraction, with 1 for the denominator, and divide.

147. Divide $2\frac{1}{2}$ by $3\frac{1}{3}$.

SOLUTION: $2\frac{1}{2} \div 3\frac{1}{3} = \frac{5}{2} \div \frac{10}{3} = \frac{5}{2} \times \frac{3}{10} = \frac{1 \cdot 3}{2 \cdot 2} = \frac{3}{4}.$

When either the dividend or the divisor is a mixed number, change the mixed number to an improper fraction and divide.

WRITTEN EXERCISES

148. Divide:

1. 6 by $\frac{5}{8}$.

5. $3\frac{3}{4}$ by $\frac{5}{8}$.

9. $4\frac{2}{3}$ by $\frac{4}{5}$.

13. $\frac{3}{4}$ by 8.

2. 9 by $\frac{7}{10}$.

6. 12 by $\frac{5}{6}$.

10. $\frac{5}{6}$ by 12.

14. $\frac{4}{5}$ by $4\frac{3}{4}$.

3. $\frac{3}{4}$ by $2\frac{1}{2}$.

7. $\frac{5}{8}$ by $2\frac{1}{4}$.

11. $\frac{2}{3}$ by $4\frac{1}{2}$.

15. $3\frac{1}{5}$ by $\frac{5}{7}$.

4. $\frac{5}{6}$ by $3\frac{1}{5}$.

8. $2\frac{1}{4}$ by 3.

12. $3\frac{1}{8}$ by $\frac{5}{6}$.

16. $3\frac{1}{7}$ by $\frac{4}{5}$.

149. Solve:

1. David raised 22 bu. of potatoes, and put $2\frac{3}{4}$ bu. in a barrel. He sold them for $\$3\frac{3}{4}$ per barrel. How many barrels of potatoes did he have? How much did he receive for them?

2. Mr. Rhodes bought 60 bu. of cranberries, which he put into crates, $1\frac{1}{4}$ bu. in each crate. He sold them for $\$4\frac{1}{4}$ per crate. How many crates of cranberries did he have? How much did he receive for the cranberries?

3. A tailor had a piece of cloth containing $12\frac{3}{4}$ yd. Allowing $1\frac{1}{2}$ yd. for a waistcoat, how many waistcoats could he make, and how much cloth would he have left?

4. Brown bought 86 acres of land and divided it into fields averaging $5\frac{3}{8}$ acres each. How many fields had he?

5. A milliner made up $37\frac{1}{2}$ yards of ribbon into bows, each bow containing $4\frac{3}{8}$ yards. How many bows did she make?

6. For licensing automobiles, the Secretary of State received in one day $\$52.50$. As each license cost $\$2.50$, how many licenses did he give?

7. A mason received $\$23\frac{3}{8}$ for doing a certain piece of work. His wages were $\$2\frac{3}{4}$ per day. How many days did he work?

8. A grocer bought $7\frac{1}{2}$ bushels of potatoes, which he put up in bags containing $\frac{3}{4}$ bushel each. How many bags were there?

9. He bought the potatoes for $\$.50$ a bushel and sold them for $\$.60$ a bag. How much more did he get for them than he paid?

10. A freight train moving at the rate of $14\frac{3}{4}$ miles an hour went $51\frac{1}{2}$ miles. How many hours did it take to go the entire distance?

REVIEW

ORAL EXERCISES

150. Change the following fractions to their lowest terms:

- | | | | | | |
|---------------------|----------------------|----------------------|----------------------|-----------------------|-----------------------|
| 1. $\frac{6}{10}$. | 3. $\frac{10}{16}$. | 5. $\frac{15}{20}$. | 7. $\frac{48}{72}$. | 9. $\frac{6}{24}$. | 11. $\frac{15}{30}$. |
| 2. $\frac{8}{12}$. | 4. $\frac{12}{18}$. | 6. $\frac{18}{36}$. | 8. $\frac{35}{80}$. | 10. $\frac{14}{21}$. | 12. $\frac{16}{20}$. |

WRITTEN EXERCISES

151. Find the l.c.d. of the fractions in each of the following:

- | | | |
|--|--|-------------------------------------|
| 1. $\frac{15}{45}, \frac{18}{42}, \frac{21}{35}$. | 3. $\frac{35}{80}, \frac{15}{80}, \frac{15}{40}$. | 5. $\frac{24}{36}, \frac{26}{72}$. |
| 2. $\frac{18}{45}, \frac{25}{36}, \frac{24}{40}$. | 4. $\frac{27}{56}, \frac{18}{48}, \frac{13}{28}$. | 6. $\frac{36}{48}, \frac{24}{60}$. |

ORAL OR WRITTEN EXERCISES

152. Change the following improper fractions to whole or mixed numbers:

- | | | | | |
|----------------------|-----------------------|----------------------|-----------------------|-----------------------|
| 1. $\frac{45}{15}$. | 4. $\frac{60}{25}$. | 7. $\frac{40}{36}$. | 10. $\frac{17}{17}$. | 13. $\frac{80}{60}$. |
| 2. $\frac{75}{50}$. | 5. $\frac{100}{80}$. | 8. $\frac{90}{45}$. | 11. $\frac{48}{12}$. | 14. $\frac{35}{20}$. |
| 3. $\frac{38}{18}$. | 6. $\frac{170}{35}$. | 9. $\frac{45}{20}$. | 12. $\frac{26}{8}$. | 15. $\frac{48}{24}$. |

WRITTEN EXERCISES

153. Reduce to integers or to mixed numbers with fractions in lowest terms:

- | | | | | |
|-----------------------|-----------------------|------------------------|------------------------|--------------------------|
| 1. $\frac{125}{25}$. | 5. $\frac{316}{20}$. | 9. $\frac{148}{21}$. | 13. $\frac{524}{36}$. | 17. $\frac{2285}{125}$. |
| 2. $\frac{240}{35}$. | 6. $\frac{180}{15}$. | 10. $\frac{240}{75}$. | 14. $\frac{480}{24}$. | 18. $\frac{1050}{75}$. |
| 3. $\frac{144}{18}$. | 7. $\frac{344}{16}$. | 11. $\frac{460}{22}$. | 15. $\frac{640}{28}$. | 19. $\frac{1000}{25}$. |
| 4. $\frac{175}{12}$. | 8. $\frac{260}{30}$. | 12. $\frac{640}{80}$. | 16. $\frac{812}{30}$. | 20. $\frac{1210}{120}$. |

ORAL EXERCISES

154. Change the following mixed numbers to improper fractions:

- | | | | | |
|----------------------|----------------------|----------------------|---------------------|-----------------------|
| 1. $8\frac{1}{2}$. | 3. $15\frac{1}{2}$. | 5. $8\frac{7}{10}$. | 7. $9\frac{7}{8}$. | 9. $10\frac{3}{4}$. |
| 2. $9\frac{3}{10}$. | 4. $12\frac{3}{4}$. | 6. $10\frac{3}{4}$. | 8. $6\frac{4}{5}$. | 10. $13\frac{1}{2}$. |

WRITTEN EXERCISES

155. Add:

- | | |
|---|--|
| 1. $12\frac{1}{2} + 16\frac{3}{4} + 25\frac{3}{8}$. | 4. $21\frac{3}{4} + 19\frac{5}{8} + 80\frac{5}{16} + 14\frac{1}{2}$. |
| 2. $15\frac{3}{8} + 20\frac{5}{16} + 35\frac{7}{32}$. | 5. $60\frac{5}{8} + 28\frac{3}{8} + 42\frac{4}{15} + 19\frac{2}{10}$. |
| 3. $40\frac{1}{2} + 32\frac{7}{10} + 45\frac{11}{10}$. | 6. $20\frac{5}{8} + 16\frac{3}{4} + 15\frac{3}{20} + 27\frac{1}{2}$. |

156. Subtract:

- | | | |
|-------------------------------------|---------------------------------------|----------------------------|
| 1. $17\frac{5}{10} - 12\frac{3}{8}$ | 3. $30\frac{5}{8} - 20\frac{5}{8}$. | 5. $75 - 18\frac{7}{16}$. |
| 2. $100 - 75\frac{9}{24}$. | 4. $45\frac{7}{8} - 32\frac{9}{12}$. | 6. $42 - 41\frac{1}{2}$. |

ORAL EXERCISES

157. Multiply:

- | | | | |
|---------------------------------------|--|---|--|
| 1. $\frac{1}{2} \times \frac{1}{2}$. | 5. $\frac{1}{2} \times \frac{3}{8}$. | 9. $\frac{3}{8} \times \frac{3}{4}$. | 13. $\frac{5}{8} \times \frac{3}{8}$. |
| 2. $\frac{3}{8} \times \frac{5}{8}$. | 6. $\frac{4}{7} \times \frac{5}{11}$. | 10. $\frac{1}{2} \times \frac{1}{42}$. | 14. $\frac{4}{5} \times \frac{2}{3}$. |
| 3. $16 \times \frac{3}{4}$. | 7. $30 \times \frac{4}{6}$. | 11. $40 \times \frac{7}{20}$. | 15. $18 \times \frac{5}{6}$. |
| 4. $42 \times \frac{9}{7}$. | 8. $24 \times \frac{7}{8}$. | 12. $33 \times \frac{8}{11}$. | 16. $45 \times \frac{6}{9}$. |

WRITTEN EXERCISES

158. Multiply, using cancellation where possible:

- | | | |
|---|--|---|
| 1. $48 \times 12\frac{3}{8}$. | 5. $\$2.88 \times 25\frac{3}{12}$. | 9. $8.19 \times 36\frac{5}{9}$. |
| 2. $\$1.25 \times 18\frac{3}{5}$. | 6. $144 \times 36\frac{5}{8}$. | 10. $240 \times 20\frac{7}{12}$. |
| 3. $80 \times 20\frac{5}{10}$. | 7. $\$3.60 \times 20\frac{3}{8}$. | 11. $\$1.60 \times 15\frac{7}{20}$. |
| 4. $6\frac{3}{8} \times 7\frac{3}{8}$. | 8. $10\frac{1}{8} \times 4\frac{3}{4}$. | 12. $18\frac{3}{4} \times 3\frac{3}{8}$. |

159. Divide:

- | | | | |
|-------------------------------------|--|---------------------------------------|---|
| 1. $\frac{1}{6} \div \frac{1}{6}$. | 7. $\frac{1}{14} \div \frac{1}{6}$. | 13. $2\frac{1}{2} \div \frac{4}{6}$. | 19. $3\frac{1}{2} \div 4\frac{1}{2}$. |
| 2. $\frac{4}{5} \div \frac{5}{8}$. | 8. $\frac{3}{27} \div \frac{5}{8}$. | 14. $5\frac{1}{4} \div \frac{5}{8}$. | 20. $4\frac{1}{2} \div 3\frac{1}{2}$. |
| 3. $\frac{1}{6} \div \frac{1}{7}$. | 9. $\frac{7}{18} \div \frac{5}{9}$. | 15. $36 \div 1\frac{2}{9}$. | 21. $\frac{3}{8} \div 4\frac{1}{4}$. |
| 4. $\frac{5}{8} \div \frac{3}{8}$. | 10. $\frac{15}{18} \div \frac{7}{8}$. | 16. $\frac{12}{18} \div 36$. | 22. $16\frac{3}{8} \div 1\frac{1}{8}$. |
| 5. $\frac{1}{6} \div \frac{1}{6}$. | 11. $\frac{11}{12} \div \frac{1}{2}$. | 17. $75 \div \frac{25}{8}$. | 23. $9\frac{5}{9} \div \frac{25}{9}$. |
| 6. $\frac{5}{6} \div \frac{4}{6}$. | 12. $21 \div \frac{3}{4}$. | 18. $24 \div \frac{8}{15}$. | 24. $10\frac{1}{2} \div \frac{8}{11}$. |

160. Solve:

1. There are $35\frac{7}{16}$ A. in Mr. Adams's farm, $88\frac{5}{8}$ A. in Mr. Brown's farm, and $160\frac{3}{4}$ A. in Mr. Carroll's farm.

(1) How many acres are there in all the farms?

(2) How many more acres are there in Mr. Brown's farm than in Mr. Adams's?

(3) How many more acres are there in Mr. Carroll's farm than in Mr. Adams's and Mr. Brown's farms together?

2. Mr. James had a barrel of sugar containing 300 lb. On Monday he sold $25\frac{1}{2}$ lb.; on Tuesday, $50\frac{1}{4}$ lb.; on Wednesday, $100\frac{5}{8}$ lb.; and on Thursday, $112\frac{5}{16}$ lb.

(1) How many pounds did he sell during the 4 days?

(2) How much more did he sell on Tuesday than on Monday?

(3) How much more did he sell on Thursday than on Wednesday?

(4) How much did he receive for the sugar sold Monday at $6\frac{1}{2}$ cts. a pound?

(5) How much did he receive for the sugar he sold Tuesday at the same price?

3. Mr. Lawrence has 22 bu. of potatoes in bags. Each bag contains $2\frac{3}{4}$ bu. How many bags of potatoes has he?

4. He has 55 bu. of apples in barrels. Each barrel contains $2\frac{3}{4}$ bu. He sells the apples for \$3.25 a barrel. How much does he receive for the apples?

5. Mr. Johnson sold $15\frac{3}{4}$ doz. brooms to Mr. Fen, $18\frac{1}{2}$ doz. to Mr. Kay, $16\frac{2}{3}$ doz. to Mr. Mendell, and $9\frac{5}{8}$ doz. to Mr. Keach.

(1) How many dozen brooms did he sell?

(2) How many brooms did he sell?

(3) He sold them for \$3 per dozen. What does each man pay him?

(4) How much do they all pay him?

6. A grocer bought sugar at $3\frac{1}{2}$ ¢ a pound and sold it at the rate of 7 lb. for \$.35. On Monday he sold packages containing the following amounts: $3\frac{1}{2}$ lb., 14 lb., 7 lb., $17\frac{1}{2}$ lb., 21 lb., $24\frac{1}{2}$ lb., 28 lb., $10\frac{1}{2}$ lb.

(1) How many pounds of sugar did he sell?

(2) How much did he make on the sugar that he sold on Monday?

7. Mr. Bamberger bought 2 doz. ash cans for \$18 and sold them for $\$7\frac{7}{8}$ each. How much did he make on a dozen?

8. From a piece of cloth containing $40\frac{1}{2}$ yd. Mr. Strauss sold the following: $5\frac{3}{8}$ yd., $6\frac{1}{4}$ yd., $5\frac{1}{2}$ yd.

(1) How many yards remained unsold?

(2) He paid \$2.40 per yard. What is the remainder worth at that price?

(3) He sold the cloth for \$3 per yard. How much did he receive for it?

9. Mr. Leonard bought cheese for \$20. He paid $\$1\frac{1}{8}$ per pound and sold it for \$.15 per pound. How many pounds did he buy? How much did he make?

10. How many hats each requiring $6\frac{1}{4}$ yd. of ribbon can be trimmed from 100 yd.?

11. A dressmaker used $81\frac{3}{4}$ yd. of linen for 8 suits. How many yards were used in each suit?

12. If a seamstress can make an apron from $1\frac{1}{4}$ yd. of muslin, how many aprons can she make from $37\frac{1}{2}$ yd.?

13. Mr. Hofer bought remnants of silk in the following quantities: $45\frac{3}{8}$ yd., $8\frac{3}{4}$ yd., $16\frac{5}{8}$ yd., $38\frac{1}{2}$ yd. How many yards did he buy in all?

14. He sold $\frac{1}{3}$ of the silk to Mrs. Stevens and $\frac{1}{3}$ of it to Miss Haines. How many yards did he sell to each? How many yards remained?

GENERAL REVIEW

WRITTEN EXERCISES

161. Solve ;

1. Draw a rectangle 4 in. by 5 in. How many square inches does it contain?

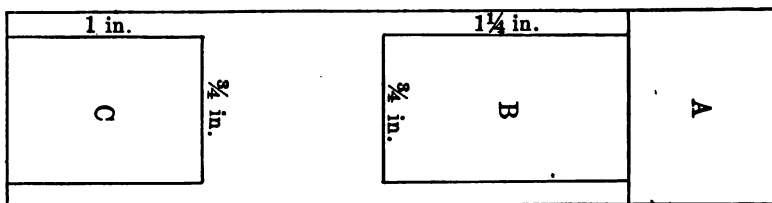
2. Let each inch represent 5 ft. How many feet will each long side represent? Each short side? The perimeter?

3. How many square feet would the rectangle represent, if it were drawn on a scale of $\frac{1}{2}$ in. to 5 ft.?

4. How many square feet would it represent, if drawn on a scale of $\frac{1}{4}$ in. to 5 ft.?

5. Draw a rectangle 6 in. by 8 in., on a scale of 1 in. to 6 ft. How many square feet does it represent?

6. How many square feet would it represent if drawn on a scale of $\frac{1}{2}$ in. to 6 ft.?



7. The rectangle represents a city lot. It is drawn on a scale of one inch to 25 ft.

The length of a city lot is called its depth.

The width of a lot facing a street is called its frontage.

What is the depth of the lot? Its frontage? How many square feet are there in the lot? How many feet in the perimeter?

8. The space A represents the front yard ; B, the house ; and C, the barn. Give the dimensions : Of A. Of B. Of C.

ORAL EXERCISES

162. Solve:

1. What is the ratio of 100 to 25? Of 100 to $12\frac{1}{2}$? Of 100 to $33\frac{1}{3}$? Of 1,000 to 500? Of 1,000 to 250? Of 1,000 to 125?

2. If $12\frac{1}{2}$ bu. of wheat are worth \$8.125, what are 100 bu. worth?

3. If $16\frac{2}{3}$ yd. of cloth are worth \$20, what are 100 yd. worth?

4. 125 A. of land were sold for \$687.50. What is the value of 1,000 A. at that rate?

5. Mr. Lindsley sold $33\frac{1}{3}$ T. of hay for \$508. His neighbor, Mr. Avery, has 100 T. to sell. What would he receive at the same rate?

6. Ned, the newsboy, buys *Records* at the rate of 5 for 3 cts. He bought 24 cts. worth and sold them for 1 ct. each. How many did he buy? How much did he make?

7. Joe buys *Tribunes* at the rate of 3 for 5 cts., and sells them at 2 cts. each. When he bought 25 cts. worth, how many did he buy? When he bought 40 cts. worth, how much did he make? When he bought 60 cts. worth? \$1.00 worth?

WRITTEN EXERCISES

163. Add:

1. $\frac{1}{2} + \frac{3}{4} + \frac{5}{8}$.

4. $12\frac{1}{2}, 18\frac{3}{4}, 25\frac{5}{8}$.

7. $18\frac{3}{8}, 21\frac{1}{4}, 28\frac{1}{8}$.

2. $\frac{2}{3} + \frac{3}{4} + \frac{7}{12}$.

5. $17\frac{2}{3}, 30\frac{7}{12}, 19\frac{1}{4}$.

8. $15\frac{1}{3}, 18\frac{1}{4}, 27\frac{1}{6}$.

3. $\frac{2}{4} + \frac{5}{8} + \frac{3}{16}$.

6. $25\frac{3}{8}, 40\frac{1}{8}, 75\frac{3}{8}$.

9. $54\frac{1}{9}, 28\frac{1}{2}, 47\frac{5}{9}$.

164. Subtract:

1. $\frac{7}{8} - \frac{3}{4}$.

3. $20 - 15\frac{5}{8}$.

5. $40 - 18\frac{9}{10}$.

2. $\frac{3}{8} - \frac{5}{8}$.

4. $20\frac{1}{2} - 16\frac{3}{4}$.

6. $36\frac{1}{8} - 24\frac{5}{10}$.

165. Solve:

1. $\frac{5}{8} + \frac{1}{2} - \frac{3}{4}$.

3. $1\frac{3}{4} + 2\frac{1}{2} - 1\frac{5}{8}$.

5. $8\frac{5}{8} + 9\frac{1}{4} - 3\frac{7}{10}$.

2. $\frac{1}{5} + \frac{7}{8} - \frac{1}{4}$.

4. $6\frac{3}{8} - 4\frac{1}{8} + 7\frac{1}{8}$.

6. $9\frac{3}{8} - 6\frac{1}{2} + 2\frac{5}{12}$.

WRITTEN EXERCISES

166. Solve:

1. How many acres are there in a lot 40 rd. long and 20 rd. wide?

2. How many acres in a field 80 rd. long and 41 rd. wide?

3. How much is the field worth at \$375 an acre?

4. How many minutes are 2 da. 20 hr. 40 min.?

5. How many hours in 4 wk. 3 da. 12 hr.?

6. If a person's income is \$4 an hour, how much is it a week?

7. An ox weighed 8 cwt. 45 lb. 12 oz. How many ounces did it weigh?

8. How many ounces in 5 cwt. 60 lb. 14 oz.?

9. How many ounces in 12 cwt. 36 lb. 8 oz.?

10. At $8\frac{1}{4}$ cts. a pound, what is the value of a beef weighing 9 cwt.?

11. If 7 lb. of sugar are worth \$0.42, what are 60 lb. worth?

12. If 8 A. of land are sold for \$2,560, what is the value at that price of a farm containing 60 A.?

13. Mr. Balcom sold 9 bbl. of flour for \$47.25. What is the value of 39 bbl. at the same rate? Of 45 bbl.? Of 55 bbl.?

14. If 12 lb. of butter are sold for \$3.72, how much are 66 lb. worth at the same rate?

15. If 3 doz. eggs cost 84 cts., how much will 18 eggs cost?

16. Find the areas of the following right triangles :

- (1) Base 12 in., altitude 15 in.
- (2) Base 25 ft., altitude 28 ft.
- (3) Base 36 rd., altitude 32 rd.
- (4) Altitude 15 in., base 18 in.
- (5) Altitude 21 ft., base 19 ft.
- (6) Altitude 18 rd., base 20 rd.

17. Draw a right triangle whose base is 6 in. and altitude 8 in.

18. What is the area represented by this triangle if drawn on a scale of $\frac{1}{2}$ in. to 4 ft.?

19. What is the area of the same triangle if drawn on a scale of $\frac{1}{4}$ in. to 6 ft.?

20. Mr. Davis had 200 bu. of cranberries and sold $145\frac{3}{4}$ bu. How many bushels had he left?

21. Take $48\frac{7}{12} + 36\frac{5}{8} + 51\frac{1}{6}$ from $225\frac{3}{4}$.

22. If there are 50 yd. of cotton on a spool, and it takes $2\frac{1}{2}$ yd. of thread to stitch a yard of cloth, how many spools will it take to stitch a dozen skirts, each $4\frac{1}{2}$ yd. around?

23. From a lot containing $80\frac{3}{4}$ A., $36\frac{3}{8}$ A. were sold. How many acres remained?

24. How many yards must be added to the sum of $350\frac{1}{2}$ yd. and $635\frac{1}{4}$ yd. to make a mile?

25. Mr. Lane is $50\frac{3}{8}$ yr. old. Mr. Durand is $36\frac{3}{4}$ yr. old. How much older is Mr. Lane than Mr. Durand?

26. Mr. Lawrence's age is 60 yr. and 6 mo., his son William's age is $15\frac{7}{12}$ yr., and his daughter Mary's age is $21\frac{3}{4}$ yr. How many years does the father's age exceed the age of each of his children?

27. How much older is Mary than William?

28. School begins at 9 o'clock and closes at a quarter before 12. Ten minutes are allowed for opening exercises, twenty minutes for recess, and five minutes to prepare for closing. How many hours are left for study?

29. $28\frac{3}{4} + 36\frac{3}{8} + 42\frac{3}{16}$ is how much less than $132\frac{3}{8}$?

30. The distance from New York to Albany is 160 mi. At 8 o'clock a train starts from New York for Albany, and at the same time a train starts from Albany for New York. The New York train runs at $45\frac{5}{8}$ mi. an hour; the Albany train runs at $44\frac{7}{12}$ mi. an hour. How far apart are the trains at 9 o'clock?

31. 105.5 acres $- 51\frac{3}{4}$ acres $+ 40\frac{3}{16}$ acres are how many acres?

32. A dealer buys $8\frac{3}{4}$ bbl. of apples at $\$1\frac{5}{8}$ a barrel. He sells them at $\$2\frac{1}{2}$ a barrel. How much does he pay for the apples? How much does he gain?

33. A train traveled for $13\frac{3}{8}$ hr. at the rate of $16\frac{3}{4}$ mi. an hour. How many miles did it go?

34. A boy had a garden to weed. The first day he worked $4\frac{1}{2}$ hr., the second day $2\frac{3}{4}$ hr., the third day $3\frac{1}{8}$ hr., and the fourth day $3\frac{5}{8}$ hr. How many hours did it take him to weed the entire garden?

35. He was paid at the rate of $\frac{1}{8}$ of a dollar for every hour that he worked. How much did he earn?

36. A dry goods merchant sold $6\frac{1}{2}$ yds. of cloth to one person, $5\frac{3}{8}$ to another, and $9\frac{3}{4}$ to a third. How many yards did he sell?

37. If the piece of cloth contained 24 yds. at first, how many were left after the sale?

38. At $\$1\frac{1}{4}$ per yard, what did each customer pay for her piece? How much did the merchant receive in all?

RATIO AND PROPORTION EQUATIONS

167. The expression $2 \times 6 = 3 \times 4$ is read "2 times 6 equals 3 times 4."

A statement of the equality of two quantities is called an equation.

ORAL EXERCISES

168. Read the following equations:

$$\begin{array}{ll} 6 + 7 = 8 + 5. & 9 + 6 = 10 + 5. \\ 10 - 8 = 9 - 7. & 8 \times 3 = 6 \times 4. \end{array}$$

Prove the following equations by performing the operations indicated by the signs:

1. $2 + 5 = 6 + 1.$

4. $\frac{4 + 4}{2} = \frac{7 + 5}{3}.$

2. $7 - 5 = 4 - 2.$

3. $6 \times 5 = 3 \times 10.$

5. $\frac{3 \times 8}{6} = \frac{6 \times 6}{9}.$

RATIO AND PROPORTION

169. How many men can Mr. Monroe hire for a day with \$36, if he pays each man \$4?

In this example \$36 is a quantity to be measured. It is measured by \$6, which is the **measuring unit**. 9 is the **ratio** of 36 to 4.

Ratio is the relation of a quantity to a measuring unit of that quantity. It is equal to the quotient found by dividing one quantity by the other.

Ratios are written in various ways. The ratio of 8 to 4 may be expressed $8 \div 4$, or $\frac{8}{4}$. It is written also $8 : 4$. What does this ratio equal?

Which of the following ratios are equal?

$\frac{8}{4}$; $12 \div 4$; $18 : 6$; $24 : 8$; $6 : 3$; $\frac{12}{6}$; $15 : 5$; $\frac{3}{2}$

170. An equality of ratios is called a **proportion**. $\frac{12}{4} = \frac{18}{6}$ is a proportion.

This proportion may be written: $12:4::18:6$.

This means that the ratio of $12 \div 4$ equals the ratio of $18 \div 6$. It is commonly read, "12 is to 4 as 18 is to 6." A better form is either $12:4=18:6$, or $\frac{12}{4} = \frac{18}{6}$. In this example, 12, 4, 18, and 6 are called the **terms** of the proportion.

The first and fourth terms are called the **extremes**.

The second and third terms are called the **means**. (Mean means "middle".)

In the proportion $12:4=18:6$, multiply the two extremes together. Multiply the two means together. What do you observe?

In a proportion the product of the means equals the product of the extremes.

ORAL EXERCISES

171. Prove the following proportions, by comparing the products of extremes and means:

1. $8:16=24:48$. 4. $8:4=22:11$. 7. $8:48=2:12$.
2. $15:45=25:75$. 5. $9:27=4:12$. 8. $13:39=5:15$.
3. $7:49=9:63$. 6. $16:4=32:8$. 9. $99:9=66:6$.

In the proportion $12:4=18:6$,

$$\frac{12 \times 6}{18} = 4; \quad \frac{12 \times 6}{4} = 18; \quad \frac{18 \times 4}{6} = 12; \quad \frac{18 \times 4}{12} = 6.$$

1. Which terms are used as divisors in the first two divisions? In the second two divisions?

2. In each division what term does the quotient give?

The product of the extremes divided by one of the means equals the other mean, and the product of the means divided by one of the extremes equals the other extreme.

172. In stating a problem it is quite common to put it into the form of an equation, taking some letter, as x , to indicate the unknown quantity; this quantity, when found, being the answer to the problem. For example:

1. $3 \times 4 = x$. What does x equal?

The answer, when found, takes the place of the x in the equation:

$$3 \times 4 = 12; \text{ therefore, } x = 12.$$

2. The ratio of 24 to 8 = the ratio of 15 to what number?

The ratio of 24 to 8 is 3. What is the number to which 15 bears the ratio 3? That is: Since one of the factors of 15 is 3, what is the other factor?

Let x stand for the missing factor,
and write the proportion thus: $24 : 8 = 15 : x$.

Dividing the product of the means
by one extreme, we have, $x = \frac{8 \times 15}{24}$;

$$\text{or, } x = \frac{\overset{5}{\cancel{15}} \times \underset{\cancel{3}}{8}}{\underset{\cancel{24}}{24}} = 5.$$

Substituting 5 for x , we have, $24 : 8 :: 15 : 5$.

Prove in two ways that the proportion is correct.

To find the missing term of a proportion, divide the product of the given extremes, or means, as the case may be, by the other given term.

ORAL EXERCISES

173. Find the value of x in each of the following proportions:

1. $25 : 5 = x : 2$.

6. $\frac{30}{15} = \frac{6}{x}$.

9. $\frac{1}{2} : x = \frac{8}{32}$.

2. $x : 3 = 15 : 5$.

7. $\frac{40}{8} = \frac{20}{x}$.

10. $\frac{28}{7} = x : 2$.

4. $36 : x = 12 : 4$.

8. $24 : 12 = \frac{x}{4}$.

11. $\frac{20}{5} = x : 2\frac{1}{2}$.

5. $21 : x = 9 : 3$.

WRITTEN EXERCISES

174. Find the value of x in each of the following proportions:

$$1. \frac{80}{60} = \frac{x}{25}.$$

$$2. x : 75 = 13 : 65.$$

$$3. 33\frac{1}{3} : x = \frac{2}{3}.$$

$$4. \frac{16}{5} = \frac{x}{9}.$$

$$5. 14 : 98 = 5 : x.$$

$$6. 156 : 780 = 230 : x.$$

$$7. 198 : 66 :: x : 100.$$

$$8. 540 : x = 7 : 70.$$

$$9. x : 315 = 210 : 3,150.$$

175. If fractions occur in the proportion, there is a way of first removing them before solving the problem.

1. In the proportion $\frac{1}{2} : \frac{2}{3} = 6 : x$, what is the value of x ?

SOLUTION: Reducing the fractions in the first ratio to a common denominator: $\frac{3}{6} : \frac{4}{6} = 6 : x$.

Multiplying both terms of a ratio by the same number does not affect the ratio. Hence, we may multiply both 2 and 4 by 6, and have as the resulting proportion, $3 : 4 = 6 : x$.

Hence, two fractions having a common denominator are to each other as their numerators are. So, in the example above, we can substitute the numerators for the fractions and proceed. $x = \frac{4 \times 6}{3} = 8$.

SUMMARY OF CHAPTER II

FACTORS.

Prime number, composite number, common divisor, greatest common divisor.

MULTIPLES.

Common multiple, least common multiple.

FRACTIONS.

Terms, numerator and denominator. Reduction. Addition, subtraction, multiplication, division.

CHAPTER III

DECIMAL FRACTIONS

READING AND WRITING DECIMALS

176. In the diagram how many small squares are there in a row?

Copy the diagram and complete the division into squares.

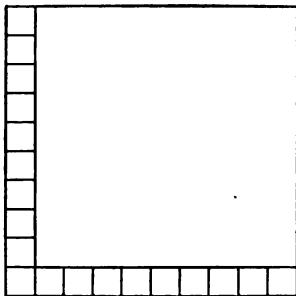
1. One square is what part of a row?

2. One row is what part of the large square?

3. Write one tenth as a common fraction.

$\frac{1}{10}$ may be written also .1.

When so written, it is called a decimal fraction.



The period is called the decimal point.

$\frac{2}{10}$ is written as a decimal .2.

$\frac{3}{10}$ is written as a decimal .3.

4. Write as decimals: $\frac{5}{10}$; $\frac{7}{10}$; $\frac{4}{10}$; $\frac{9}{10}$; $\frac{8}{10}$; $\frac{6}{10}$.

5. What number = $\frac{10}{10}$? What number = $\frac{11}{10}$?

$\frac{11}{10}$, or $1\frac{1}{10}$, is written as a decimal 1.1.

In writing mixed numbers as decimals, the decimal point is used to separate the whole number from the decimal fraction.

6. Write as decimals: $7\frac{5}{10}$; $28\frac{3}{10}$; $115\frac{2}{10}$; $36\frac{7}{10}$; $240\frac{2}{10}$.

7. In 1.1, in what place does the 1 to the left of the decimal point stand?

The 1 to the right of the decimal point stands in **tenth's** place.

8. In $\frac{1}{10}$ what does the 1 show? The 10?

9. In .1 what does the 1 show? What shows the denominator of the fraction?

10. In 7.5 what does the 7 show? The 5? Where does the 5 stand with relation to the decimal point? What does its position show?

177. Into how many of the small squares could you divide the large square on page 91?

1. Each small square is what part of the large square?

$\frac{1}{100}$ is written as a decimal .01.

$\frac{2}{100}$ is written as a decimal .02.

2. Write as decimals: $\frac{3}{100}$; $\frac{6}{100}$; $\frac{8}{100}$; $\frac{4}{100}$; $\frac{7}{100}$; $\frac{5}{100}$;
 $\frac{9}{100}$. $\frac{10}{100}$ is written as a decimal .10.

$\frac{11}{100}$ is written as a decimal .11.

3. Write as a decimal each of the hundredths from $\frac{12}{100}$ to $\frac{25}{100}$.

4. How many are $\frac{100}{100}$? How many are $\frac{101}{100}$?

$\frac{101}{100}$, or $1\frac{1}{100}$, is written as a decimal 1.01.

5. Write as decimals: $25\frac{25}{100}$; $150\frac{42}{100}$; $302\frac{50}{100}$; $32\frac{5}{100}$;
 $9\frac{55}{100}$; $9\frac{22}{100}$; $17\frac{88}{100}$; $70\frac{70}{100}$; $70\frac{7}{100}$.

6. In 1.01, what does the 1 at the left of the decimal point show? The 1 at the right? What does the 0 show? What is the decimal fraction? How is its denominator known?

The second place to the right of the decimal point is **hundredth's** place.

178. When anything is divided into 1,000 equal parts, what is one of the parts called?

$\frac{1}{1000}$ is written as a decimal .001.

1. Write as decimals each thousandth from $\frac{1}{1000}$ to $\frac{9}{1000}$.

50 $\frac{5}{1000}$ is written as a decimal 50.005.

2. Write as decimals: $75\frac{1}{1000}$; $160\frac{8}{1000}$; $300\frac{7}{1000}$.

$\frac{10}{1000}$ is written as a decimal .010.

$\frac{11}{1000}$ is written as a decimal .011.

3. Write as a decimal each thousandth from $\frac{12}{1000}$ to $\frac{25}{1000}$.

4. Write as decimals: $28\frac{16}{1000}$; $75\frac{35}{1000}$; $220\frac{82}{1000}$.

$\frac{100}{1000}$ is written as a decimal .100.

$\frac{110}{1000}$ is written as a decimal .110.

$\frac{111}{1000}$ is written as a decimal .111.

5. Write as decimals each thousandth from $\frac{125}{1000}$ to $\frac{138}{1000}$.

6. Write as a decimal: $1\frac{128}{1000}$; $10\frac{425}{1000}$; $105\frac{650}{1000}$;
 $1,248\frac{875}{1000}$; $3,000\frac{200}{1000}$; $23\frac{103}{1000}$; $49\frac{204}{1000}$; $97\frac{808}{1000}$; $106\frac{502}{1000}$.

7. How many places to the right of the decimal point are needed to express thousandths? Which place is **thousandth's place**?

179. Read: 3.2; 3.02; 3.002; 3.20; 3.200. In each case what is the fraction? What is its numerator? How is its denominator expressed?

A **decimal fraction** is one whose denominator is 10 or 10 multiplied by itself one or more times. Its denominator is not written, but is indicated by the position of a period called the **decimal point**.

The word *decimal* means "by tens." (It is from the Latin word *decem*, meaning "ten.")

ORAL EXERCISES

180. Read :

- | | | | |
|------------|-------------|--------------|-------------|
| 1. 75.056. | 6. 27.707. | 11. 14.93. | 16. 410.01. |
| 2. 40.428. | 7. 63.024. | 12. 86.60. | 17. 309.50. |
| 3. 362.08. | 8. 2000.2. | 13. 928.5. | 18. 15.068. |
| 4. 13.204. | 9. 24.0034. | 14. 603.29. | 19. 26.093. |
| 5. 14.013. | 10. 39.362. | 15. 913.006. | 20. 12.002. |

WRITTEN EXERCISES

181. Write in decimal form :

- | | | | |
|----------------------|-------------------------|---------------------------|----------------------------|
| 1. $4\frac{3}{10}$. | 5. $\frac{9}{10}$. | 9. $\frac{17}{1000}$. | 13. $\frac{1102}{1000}$. |
| 2. $9\frac{7}{10}$. | 6. $\frac{25}{100}$. | 10. $11\frac{1}{100}$. | 14. $\frac{1307}{1000}$. |
| 3. $2\frac{4}{10}$. | 7. $\frac{25}{1000}$. | 11. $30\frac{33}{1000}$. | 15. $\frac{4001}{1000}$. |
| 4. $\frac{6}{100}$. | 8. $\frac{136}{1000}$. | 12. $75\frac{50}{1000}$. | 16. $\frac{3076}{10000}$. |

182. Beginning with the highest order, read the figure of each order in the number below :

Millions	Hundred-thousands			Ten-thousands			Thousands			Hundreds			Tens	Units	.	Tenths	Hundredths	Thousandths	Ten-thousandths	Hundred-thousandths	Millionths
6,	1	2	3,	4	5	0	.	5	4	3	2	1	6								

To read a decimal, read it as if it were an integer and give it the name of its lowest, or right-hand, order.

Thus, the decimal fraction in the number above is read: "Five hundred forty-three thousand, two hundred sixteen millionths."

Read the number in full.

ORAL EXERCISES

183. Read :

1. 0.3645.	8. 4.5.	15. .85.	22. 95.05.
2. 0.0836.	9. 25.15.	16. .404.	23. 28.70.
3. 0.017.	10. 368.832.	17. 45.10.	24. .560.
4. 0.26345.	11. 8,006.806.	18. 18.36.	25. .002.
5. 0.705316.	12. .102.	19. .209.	26. .040.
6. 0.75.	13. 20.07.	20. 13.13.	27. .500.
7. 0.7.	14. 204.204.	21. 33.04.	28. 3.065.

WRITTEN EXERCISES

184. Write in figures :

- Seven hundred forty-two and three tenths.
- Three hundred and five hundredths.
- Four hundred and three thousandths.
- Two thousand one hundred seventy-five and ninety-nine thousandths.
- Four thousand forty and one hundred eight thousandths.
- Four thousand fifteen and sixteen thousandths.
- 4,798 ten-thousandths.
- 308 ten-thousandths.
- 25 ten-thousandths.
- 40,400 hundred-thousandths.
- 750,105 millionths.
- 16 hundredths.
- 9 tenths.
- 8 and 7 tenths.
- 75 and 36 hundredths.

185. Read :

$$\begin{array}{llll} \frac{1}{10} = .1 & (a) & \frac{1}{10000} = .0001 & (d) \\ \frac{1}{100} = .01 & (b) & \frac{1}{100000} = .00001 & (e) \\ \frac{1}{1000} = .001 & (c) & \frac{1}{1000000} = .000001 & (f) \end{array}$$

1. What is the ratio of a to b ?
2. What is the ratio of b to c ?
3. What is the ratio of c to d ?
4. What is the ratio of d to e ?
5. What is the ratio of e to f ?

186. Read :

$$\begin{array}{r} 1.1 \\ 10.01 \\ 100.001 \\ 1000.0001 \\ 10000.00001 \\ 100000.000001 \\ 1000000.0000001 \\ \hline 1111111.1111111 \end{array}$$

1. By what ratio does the value of a figure in a whole number increase when moved one place from right to left?

2. In the first line of the table, what is the ratio of the decimal to the integer?

1 at the right of a decimal point has the same ratio to 1 in the unit's place, as the 1 in unit's place has to the 1 in ten's place.

3. Why do we use the words "tenths," "hundredths," and so forth, for figures at the right of the decimal point, and "tens," "hundreds," and so forth, for figures at the left?

A method of naming numbers in a ratio of 10 is called a **decimal system**.

UNITED STATES MONEY

187. How many cents make \$1? One cent is what part of a dollar?

1. How many cents make 1 dime?
2. How many dimes make 1 dollar?
3. What is the ratio of 1 dollar to 1 dime?

Cents may be written decimally, as hundredths of a dollar; for example, \$0.01.

4. Write as decimals: 5 cents; 4 cents; 9 cents; 12 cents; 10 cents; 30 cents; 25 cents; 50 cents; 75 cents; 62 cents; 87 cents; 6 cents; 3 cents; 99 cents; 5 dollars and 25 cents; 8 dollars and 12 cents; 2 dollars and 2 cents.

188. United States money is made on a decimal system; that is, a system of tens.

The denominations are mills, cents, dimes, and dollars. They increase in tenfold ratio; thus:

10 mills = 1 cent.

10 cents = 1 dime.

10 dimes = 1 dollar.

10 dollars = 1 eagle.

One mill is one thousandth of 1 dollar and is so expressed decimally. Thus, 1 mill is written \$0.001. "2 dollars, 37 cents, and 5 mills" is written \$2.375.

189. The coins in use are: 1 of bronze—the cent; 1 of nickel—the nickel (5 cents); 4 of silver—the dime, the quarter dollar, the half dollar, the dollar; 3 of gold—the eagle (\$10), the half eagle (\$5), the double eagle (\$20).

There is no coin for the mill.

There is also paper money, or the government's promise to pay gold or silver on demand, in the following denominations: \$1, \$2, \$5, \$10, \$20, \$50, \$100, \$500, \$1,000.

ADDITION

190. Add:

1. 2 cents, 15 cents, 3 dollars and 20 cents, 7 dollars and 75 cents, 6 dollars.

$$\begin{array}{r}
 \text{SOLUTION: } \$ \quad .02 \\
 \quad \quad .15 \\
 \quad \quad 3.20 \\
 \quad \quad 7.75 \\
 \quad \quad 6. \\
 \hline
 \$17.12
 \end{array}$$

2. 16.25, 32.06, 17.50.

$$\begin{array}{r}
 \text{SOLUTION: } \quad 16.25 \\
 \quad \quad 32.06 \\
 \quad \quad 17.50 \\
 \hline
 \quad \quad 65.81
 \end{array}$$

To add decimals, write them so that the decimal points are in a column, and proceed as with whole numbers, placing the decimal point of the sum under those of the addends.

ORAL AND WRITTEN EXERCISES

191. Read and add the following:

1. 145.725	2. 28.92	3. 308.042	4. 16.6751
62.384	31.06	57.267	30.1475
647.038	74.00	184.150	67.0581
403.006	93.48	360.009	50.2533
5.125	45.05	250.905	9.1625
70.305	13.75	61.125	14.3875
32.056	42.97	34.333	35.2034
63.967	39.09	262.967	360.4963
212.875	38.64	148.203	618.4305
38.005	73.29	25.267	408.7214
<u>570.225</u>	<u>82.67</u>	<u>318.571</u>	<u>62.5496</u>

WRITTEN EXERCISES

192. Write in columns and add :

1. $5.7 + 14.8 + 25.05$.
2. $28.03 + 75.9 + 20.102$.
3. $85.125 + 62.25 + 9.7$.
4. $125.025 + 4.3 + 40.40$.
5. $26.5 + 84.75 + 100.001$.
6. $38.18 + 61.08 + 40.008$.
7. $96.9 + 84.009 + 25.09$.
8. $225.005 + 116.5 + 312.5$.
9. $236.084 + 360.840 + 60.848 + 842.3605 + 1,423.3264$.
10. $10,206.60201 + 487.015 + .0425 + 701.528 + .701528$.

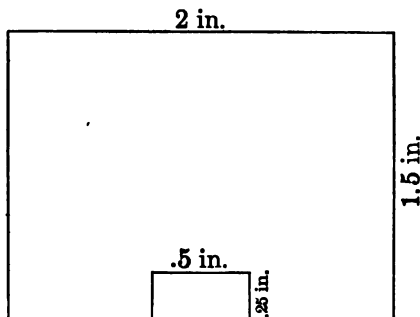
193. Solve :

1. A room is 24.5 ft. long and 18.25 ft. wide. What is the distance around the room ?

2. A picture is 18.25 in. long and 9.5 in. wide. How long is the perimeter of the picture ?

3. The sides of a triangle are 12.75 ft., 16.5 ft., and 20.7 ft., respectively. How long is the perimeter of the triangle ?

4. This is a diagram of a hall drawn on the scale of 1 in. to 25 ft.



Give, in feet and decimals of a foot :

(1) The perimeter of the hall.

(2) The perimeter of the platform.

5. The cyclometer on Harry's bicycle stood at 137.25 mi. when he started on a ride Saturday morning. He rode 19.05 mi. How did his cyclometer stand then ?

SUBTRACTION

194. Subtract:

1. \$4.85 from \$6.50.

$$\begin{array}{r} \text{SOLUTION: } \$6.50 \\ \quad \underline{4.85} \\ \$1.65 \end{array}$$

2. 218.316 from 320.5.

SOLUTION:

$$\begin{array}{r} 320.500 \\ 218.316 \\ \hline 102.184 \end{array}$$

Is there any difference in value between .5 and .500?
Why?
Why are the 00 written to the right of the 5?

To subtract decimals, write the numbers so that the decimal point of the subtrahend will be under that of the minuend, and so that like orders will be in column. When the decimal places in the minuend and the subtrahend are unequal, make them equal by annexing ciphers to the one that has the smaller number of decimal places, and then subtract as with whole numbers.

WRITTEN EXERCISES

195. Subtract:

- | | |
|-----------------------|--------------------------|
| 1. 342.42 — 35.78. | 8. 200.08 — 156.3. |
| 2. 425.6234 — 287.65. | 9. 831.7 — 245.645. |
| 3. 456.3 — 38.35. | 10. 86.83 — 29.0164. |
| 4. 304.78 — 165.728. | 11. 219.625 — 45.375. |
| 5. 23.75 — 12.7. | 12. 17.17 — 14.125. |
| 6. 45.75 — 36.375. | 13. 1 — 10 hundredths. |
| 7. 306.4 — 218.025. | 14. 20 — 15 thousandths. |
15. 15 and 15 hundredths — 275 ten-thousandths.
 16. 326 thousandths — 425 millionths.
 17. 1 and 7 tenths — 216 thousandths.
 18. 200 and 5 thousandths — 58 and 11 hundredths.
 19. 24 tenths — 15 millionths.

196. Solve:

Mr. Pell, a grocer, bought and sold goods as follows:

	BOUGHT	SOLD
2 bbl. flour	for \$ 9.25 ;	for \$ 10.05.
5 bu. corn	for \$ 3.75 ;	for \$ 4.00.
1 bbl. sugar	for \$ 9.375 ;	for \$ 10.50.
1 bbl. potatoes	for \$ 3.50 ;	for \$ 4.25.
1 tub butter	for \$ 11.20 ;	for \$ 13.45.

1. How much did he make on each sale? How much did he make in all?

2. From a coil of wire 525.5 ft. long, 240.75 ft. were cut. How many feet of wire remained?

3. From a pile of wood containing 325.4 cu. ft., 87.06 cu. ft. were taken. How many cubic feet of wood remained?

4. From a piece of cloth containing 40.25 yd., 25.5 yd. were sold. How much cloth was left?

5. The distance from John's house to the schoolhouse is 125.06 rd., and the distance from Minnie's house to the schoolhouse is 240.025 rd. How many rods farther from the school does Minnie live than John?

6. John jumped 10.3 ft., William, 9.75 ft., and Charles, 8.8 ft. How much farther did John jump than William? William than Charles? John than Charles?

7. Mr. Hunt had 150.5 acres of land and sold 42.375 acres. How many acres had he left?

8. Mr. Bentley had 215.75 acres of land. He sold Mr. Lee 35.5 acres and Mr. Delavan 75.725 acres. How many acres had he left?

9. On Monday Mr. Huyler had \$800.75 in the bank. On Tuesday he deposited \$65.75. On Thursday he drew \$500.50; on Friday, \$125.75; on Saturday, \$97.25. How much had he left in the bank?

ACCOUNTS

197. The following were the receipts and expenditures which John Wilson entered in his cash account, for the month of June, 1910:

On the 4th of June he paid \$3 for picking strawberries; he sold them on the 5th of June for \$20. June 7 he paid \$6 for picking strawberries; he sold them on the 8th of June for \$32. On the 15th of June he paid \$7.50 for picking strawberries; he sold them on the 16th for \$37.75. On June 20 he received for beet greens, \$2.85. On the same day he bought a suit of clothes for \$24.50 and a pair of shoes for \$3.25. On June 25 he received \$12.75 for peas. On June 30 he paid his hired man \$20.

Mr. Wilson kept his cash account in the following form:

1910 Receipts				1910 Expenditures			
June	5	Strawberries	20 00	June	4	Picking Strawberries	3 00
"	8	"	32 00	"	7	"	6 00
"	16	"	37 75	"	15	"	7 50
"	20	Beet Greens	2 85	"	20	Suit of Clothes	24 50
"	25	Peas	12 75	"	20	Shoes	3 25
				"	30	To hired man	20 00
						Balance	41 10
			\$ 105 35				\$ 105 35

At the end of the month Mr. Wilson counted the cash he still had on hand and found that it amounted to \$41.10. He entered this in his book as "Balance."

1. How much did he receive during the month?
2. How much did he spend?

3. What was the difference between his receipts and his expenditures?

4. How did this difference compare with the amount of the money he still had on hand?

5. How did he prove that his account was correct?

The amount that must be added to one side of an account to make it equal the other is called the **balance**. Finding this amount and thereby proving the account is called **balancing** the account.

6. Did Mr. Wilson gain or lose money for the month of June?

WRITTEN EXERCISES

198. Make and balance accounts for the following:

1. On Aug. 1, 1909, Elmer Hubbard sold a load of sweet corn for \$15.60. On the 2d he received \$18.75 for potatoes. He paid \$10.50 for labor on the 7th. On the 9th he received \$12.25 for sweet corn and \$8.60 for cabbages. Aug. 12 he paid \$11.25 for garden tools, and on the same date he paid \$7.50 for labor. Aug. 15 he received \$9.25 for peas. On the 13th he sold sweet corn for \$17.50. Aug. 20 he paid \$15 for groceries. He sold a load of cabbages on the 21st for \$25. On the 22d he paid \$14.75 for labor. He sold a load of sweet corn on the 25th for \$21.50. He received \$19.50 for potatoes on the 29th. Aug. 31 he paid \$17.75 for labor.

2. May 3, 1909, John Chase, who keeps a cigar store and newstand, paid \$6.50 for newspapers, and sold them for \$9.50. He received \$8.75 for cigars. He paid \$12 for magazines and sold magazines for \$10.25. He received for smoking tobacco \$6.65. He paid newsboys \$2.25 and a clerk \$1.50. He paid \$3.25 for postal cards and sold them for \$5.75. He paid \$1.25 for stationery and sold it for \$2.75.

MULTIPLICATION

199. How many times $\frac{1}{10}$ is 1? How many times $\frac{1}{10}$ is 100? Is 1000? Read:

$$(\frac{1}{10} \times 10) = \frac{10}{10} = 1 = .1 \times 10.$$

$$10 = .1 \times 100.$$

$$100 = .1 \times 1000.$$

$$2.7116 = .27116 \times 10.$$

$$27.116 = .27116 \times 100.$$

$$271.16 = .27116 \times 1000.$$

1. Compare .1 with 1. In multiplying by 10, in which direction was the decimal point moved? How many places?

2. Compare .1 with 10. In multiplying by 100, how many places was the decimal point moved, and in which direction?

3. When .27116 was multiplied by 10, how was the decimal point changed?

4. When .27116 was multiplied by 100, how was the decimal point changed?

5. When .27116 was multiplied by 1,000, how was the decimal point changed?

6. If 31.46 is changed to 314.6, what is the multiplier? If it is changed to 3,146?

To multiply a decimal by 10, we move the decimal point one place to the right.

7. In any decimal, how many places to the right should the decimal point be moved to multiply the number by 100? By 1,000?

8. What was the multiplier in changing:

.004 to .04?

13 to 13,000?

.013 to 13?

.22 to 220?

2.3506 to 2350.6?

333.33 to 33,333?

.31 to .310?

.1304 to 13.04?

5.203 to 520.3?

.0004 to .004?

.0101 to 1.01?

3.2003 to 320.03?

200. Multiply each of the following by 10; by 100; by 1,000.

- | | | |
|---------|-----------|----------------|
| 1. .8. | 5. 3.125. | 9. 14.2068. |
| 2. .18. | 6. .0375. | 10. 125.12305. |
| 3. .45. | 7. 1.427. | 11. 263.1428. |
| 4. .62. | 8. 3.062. | 12. 832.0606. |

201. Read :

$$1. 40 \times .3 = 40 \times \frac{3}{10} = \frac{120}{10} = 12. \quad 40$$

$$\begin{array}{r} .3 \\ \hline 12.0 \end{array}$$

$$2. 40 \times .03 = 40 \times \frac{3}{100} = \frac{120}{100} = 1\frac{20}{100}, \text{ or } 1.20 = 1\frac{2}{10}, \text{ or } 1.2.$$

$$\begin{array}{r} 40 \\ .03 \\ \hline 1.20 \end{array}$$

$$3. 4 \times .03 = 4 \times \frac{3}{100} = \frac{12}{100}, \text{ or } .12. \quad 4$$

$$\begin{array}{r} .03 \\ \hline .12 \end{array}$$

$$4. .4 \times .3 = \frac{4}{10} \times \frac{3}{10} = \frac{12}{100}, \text{ or } .12. \quad .4$$

$$\begin{array}{r} .3 \\ \hline .12 \end{array}$$

In multiplying decimals, multiply as with whole numbers, and point off in the product as many decimal places as are found in both multiplier and multiplicand together.

ORAL EXERCISES

202. Multiply :

- | | | | |
|--------------------|---------------------|----------------------|----------------------|
| 1. $30 \times .5.$ | 6. $8 \times .6.$ | 11. $40 \times .06.$ | 16. $24 \times .02.$ |
| 2. $20 \times .4.$ | 7. $9 \times .3.$ | 12. $70 \times .08.$ | 17. $80 \times .07.$ |
| 3. $60 \times .8.$ | 8. $12 \times .2.$ | 13. $20 \times .05.$ | 18. $12 \times .03.$ |
| 4. $90 \times .7.$ | 9. $6 \times .9.$ | 14. $50 \times .07.$ | 19. $10 \times .09.$ |
| 5. $82 \times .8.$ | 10. $8 \times .13.$ | 15. $60 \times .08.$ | 20. $12 \times .07.$ |

WRITTEN EXERCISES

203. Copy and multiply :

1. $\begin{array}{r} 364 \\ .3 \\ \hline \end{array}$	4. $\begin{array}{r} 783 \\ .9 \\ \hline \end{array}$	7. $\begin{array}{r} 2386 \\ .7 \\ \hline \end{array}$	10. $\begin{array}{r} 4089 \\ .8 \\ \hline \end{array}$	13. $\begin{array}{r} 3004 \\ .5 \\ \hline \end{array}$
2. $\begin{array}{r} 48 \\ .12 \\ \hline \end{array}$	5. $\begin{array}{r} 650 \\ .09 \\ \hline \end{array}$	8. $\begin{array}{r} 3845 \\ .27 \\ \hline \end{array}$	11. $\begin{array}{r} 9060 \\ .35 \\ \hline \end{array}$	14. $\begin{array}{r} 8018 \\ .08 \\ \hline \end{array}$
3. $\begin{array}{r} 207 \\ .006 \\ \hline \end{array}$	6. $\begin{array}{r} 42 \\ .008 \\ \hline \end{array}$	9. $\begin{array}{r} 726 \\ .018 \\ \hline \end{array}$	12. $\begin{array}{r} 2493 \\ .402 \\ \hline \end{array}$	15. $\begin{array}{r} 3604 \\ .075 \\ \hline \end{array}$

204. Solve :

1. A farm contains 3,200 acres. Three tenths of it is planted with wheat, two tenths of it is planted with corn, twenty-five hundredths of it is meadow land, fifteen hundredths of it is pasture, and one tenth of it is woodland.

- (1) How many acres of wheat are there? Of corn?
- (2) How many more acres of wheat than of corn?
- (3) How many acres of meadow land? Of pasture?
- (4) How many more acres of meadow than of pasture?
- (5) If the wheat averages 32 bu. to the acre, how many bushels were raised?
- (6) If the corn averages 36 bu. to the acre, what was the corn crop?
- (7) If the wheat is \$.98 a bushel, what is the value of the wheat crop?
- (8) If the corn is \$.65 a bushel, what is the value of the corn crop?

2. At the rate of \$2.40 for 8 lb. of butter, what is the value of a tub containing 75 lb.?

3. How much is a barrel of pork weighing 300 lb. worth at the rate of 15 lb. for \$1.95?

4. Mr. Miller had 44 tons of hay. He sold it at the rate of \$88.75 for $5\frac{1}{2}$ tons. How much did he receive for it?

SALES ON INSTALLMENTS

WRITTEN EXERCISES

205. Solve:

1. Kirche & Co. sold to Mr. Luther, Jan. 12, 1908: 1 parlor suit for \$75; 1 rug for \$35; 24 yd. brussels carpet, at \$1.50 per yard; and a rocking chair for \$11.50.

(1) How much did the purchases amount to?

Mr. Luther agreed to pay \$15.50 a month until the whole bill was paid.

(2) How much did Mr. Luther pay in three months?

(3) How much did he owe then?

(4) How much did he pay in seven payments?

(5) How much did he then owe?

2. Jan. 15, 1908, the same company sold to Mr. Monroe: 1 parlor suit for \$65; 30 yd. of carpet, at \$1.25 per yard; 6 dining room chairs, at \$2.25 each; an oak table for \$18.75; 1 writing desk for \$22; 1 rug for \$19.50; 1 stand for \$21; 25 yd. of linoleum, at \$1.40 per yard; 2 kitchen chairs, at \$0.75 each. The goods were bought on the installment plan. Mr. Monroe agreed to pay not less than \$15 per month. The payments were made as follows: Feb. 1, \$25; March 1, \$16.75; April 1, \$20; May 1, \$16.75; June 1, \$15; July 1, \$28.50; Aug. 1, \$21.50; Sept. 1, \$15.25.

(1) How much was due after each payment?

(2) If Mr. Monroe paid \$15 per month after September, how many months would it take to pay the bill?

(3) What would be the amount of the final payment?

3. Mar. 8, 1908, they sold Mrs. Monroe 1 rug for \$50; 2 chairs for \$48; 1 dining room set for \$73.60; and a library table for \$36.50. She paid for the goods at the rate of \$18 per month. How many months did it take to pay the entire bill?

DIVISION

206. What part of $\frac{1}{10}$ is $\frac{1}{100}$? What part of 1 is $\frac{1}{100}$? What part of 10?

How do we multiply a decimal by 10? By 100? By 1,000? Read:

$$10 \div 10 = 1.$$

$$1 \div 10 = .1.$$

$$1 \div 100 = .01.$$

$$1 \div 1,000 = .001.$$

$$271.16 \div 10 = 27.116.$$

$$271.16 \div 100 = 2.7116.$$

$$271.16 \div 1,000 = .27116.$$

$$271.16 \div 10,000 = .027116.$$

$$21.04 \div 100 = .2104.$$

$$.04 \div 1000 = .00004.$$

1. In dividing 1 by 10, in which direction is the decimal point moved? How many places? In dividing 1 by 100? By 1,000?

2. In dividing 1 by 100 in which direction is the decimal point moved? How many places?

3. In dividing 271.16 by 10,000, how many places is the decimal point moved? How many places are there to the left of the decimal point in the dividend? How is the additional place needed in the quotient supplied?

4. How many places was the decimal point moved when 271.16 was divided by 100?

5. How many places was the decimal point moved in dividing 21.04 by 100? In dividing .04 by 1000?

To divide a decimal by 10, 100, 1000, and so forth, move the decimal point as many places to the left as there are ciphers in the divisor, making as many places as may be needed in the quotient by prefixing ciphers if necessary.

WRITTEN EXERCISES

207. Divide:

1. By 10: 236.85; 82.5; 8.16; .728.
 2. By 100: 732.46; 95.274; .3724.
 3. By 1,000: 2,781.64; 812.654; .74186.
 4. By 10, 100, and 1,000 in turn, each of the following:
- (1) 721.864. (3) 3.86475. (5) 9,864.5. (7) 9,864.2753.
 (2) 14.25793. (4) .00864. (6) 7,354.8. (8) 4,000.20.

208. Divide 4,265.8 by 2,000.

SOLUTION: $4265.8 \div 1000 = 4.2658$; $4.2658 \div 2 = 2.1329$.

WRITTEN EXERCISES

209. Divide:

1. By 2,000: 246.6; 12.4; 3.26; 8,244; 468.4; 24.6.
2. By 300: 429.36; 843.69; 127.26; 3,300; 30; 3.
3. By 20: 600; 60; 66.60; 2,000; 22.20; 2.

210. Divide:

1. 25.5 by 5.

SOLUTION:
$$\begin{array}{r} 5.1 \\ 5 \overline{)25.5} \end{array}$$

2. .16 by 2.

SOLUTION:
$$\begin{array}{r} .08 \\ 2 \overline{).16} \end{array}$$

3. 126 by 24.

SOLUTION:
$$\begin{array}{r} 5.25 \\ 24 \overline{)126.00} \\ \underline{120} \\ 60 \\ \underline{48} \\ 120 \\ \underline{120} \end{array}$$

To divide a decimal by a whole number, place the decimal point of the quotient over that of the dividend and divide, writing ciphers in any vacant orders of the quotient after the decimal point. If there is a remainder, one or more ciphers may be added to the dividend and the division continued.

WRITTEN EXERCISES

211. Divide:

1. By 8: .00128; .04248; 26.4072.
2. By 6: .03246; 32.4880; 275.4426.
3. By 7: 14.9184; .081961; 4.58017.
4. By 9: 365.4126; 65.96213; .4361652.
5. 36.50 by 25. 8. 8.80 by 16. 11. 326 by 25.
6. 15.06 by 94. 9. 54.42 by 14. 12. 752 by 32.
7. 4.05 by 45. 10. 72.24 by 21. 13. 98.1 by 18.

212. Divide 30 by 6.

$$30 \div 6 = 5, \text{ or } \frac{30}{6} = 5.$$

Multiply both terms by 10.

$$300 \div 60 = 5, \text{ or } \frac{300}{60} = 5.$$

Divide both terms of "300 ÷ 60" by 10.

$$30 \div 6 = 5, \text{ or } \frac{30}{6} = 5.$$

1. In each of these operations what is the ratio of dividend and divisor?

2. Does multiplying or dividing both dividend and divisor by the same number affect the quotient? Tell why.

213. Divide 16.485 by 2.15.

$$\begin{array}{r} \text{SOLUTION:} \quad 7.644\frac{8}{15} \\ 215 \overline{)1643.500} \end{array}$$

By what can the divisor be multiplied to make it a whole number?

Multiplying the dividend also by 100, do we change the ratio of the terms?

Explain the appearance of the two ciphers in the dividend by working out the division. How can the result be tested?

To divide a decimal by a decimal, simplify the problem by multiplying both divisor and dividend by 10, or by such multiple of 10 as will make the divisor a whole number; then place the decimal point of the quotient over that of the dividend and divide.

ORAL EXERCISES

214. Divide:

- | | | |
|-----------------|------------------|-----------------|
| 1. 25 by .5. | 8. 7.5 by 7.5. | 15. 83 by 8.3. |
| 2. 75 by .25. | 9. 1.25 by 2.5. | 16. .5 by 2.5. |
| 3. 1.5 by 1.5. | 10. 8.3 by 8.3. | 17. .75 by .15. |
| 4. 1.25 by .25. | 11. 2.5 by 5. | 18. .83 by 83. |
| 5. 1.5 by 15. | 12. .75 by .05. | 19. 8.3 by 83. |
| 6. .5 by 25. | 13. .75 by .075. | 20. 83 by 83. |
| 7. 7.5 by .25. | 14. 1.25 by 25. | 21. 9.8 by 1.4. |

215. Solve:

1. If a bicycle goes 12.5 ft. at each revolution of its wheel, how many revolutions will it make in going 75 ft.? 37.5 ft.? 62.5 ft.? 25 ft.? 87.5 ft.? 50 ft.?

2. At 2.5 cts. each, how many lead pencils can be bought for 50 cts.? For $12\frac{1}{2}$ cts.?

3. There are 5.5 yd. in 1 rod. How many rods are there in 110 yd.? In 27.5 yd.?

4. The distance around a square field is 248.32 rd. What is the length of one of its sides?

5. Mary paid \$1.65 for 15 Christmas tree ornaments. How much did they cost apiece?

WRITTEN EXERCISES

216. Divide, carrying the division for three decimal places:

- | | | |
|-------------------|---------------------|-----------------------|
| 1. 743.2 by 8.15. | 7. 36 by 9.015. | 13. .62805 by 75. |
| 2. 49.83 by 7.43. | 8. 36 by .5214. | 14. .62805 by .065. |
| 3. 682.5 by .195. | 9. 12.05 by 3.5. | 15. .62805 by 34.46. |
| 4. 8.686 by .048. | 10. 12.05 by 4.15. | 16. .00375 by .008. |
| 5. 36 by 2.1. | 11. 12.05 by 5.345. | 17. .62928 by .009. |
| 6. 36 by 8.35. | 12. 12.05 by .0035. | 18. 2.6295 by 1.0201. |

WRITTEN EXERCISES

217. Dividing to 4 decimal places if necessary, find the quotient of:

1. 112 and 15 thousandths divided by 56 ten-thousandths.
2. 65 ten-thousandths $\div$ 1 and 3 tenths.
3. 146 thousand and 42 hundred-thousandths $\div$ 240 and 3 thousandths.
4. 3 thousand and 932 ten-thousandths $\div$ 108 ten-thousandths.
5. 31 and 5 tenths $\div$ 252 thousandths.
6. 36 and 624 thousandths $\div$ 12 ten-thousandths.
7. $686.25 \div 7.5$.
8. $5.136 \div 1.5$.
9. $3.92084 \div 280.06$.
10. $9.3 \div .378$.
11. $.00154 \div .4375$.
12. $7.2511 \div 6.5$.
13. $22.11 \div .11$.
14. $33.044 \div .011$.
15. $66.012 \div 1.1$.
16. $14.4288 \div .72$.
17. $288.576 \div .0154$.
18. $38.7268 \div 3.08$.
19. $3.23166 \div 83$.
20. $13.28664 \div .0561$.
21. $4,628.137 \div .00763$.
22. $13.8845 \div .0367$.
23. $.00465 \div .1478$.
24. $60.75 \div .02008$.

218. Solve:

1. A farmer had 550 bu. of potatoes in bags. Each bag contained 2.75 bu. How many bags of potatoes had he?

2. Mr. Davis bought sugar at \$.065 a pound wholesale. How many pounds could he buy for \$25.48?

3. How many pounds of coffee at \$.2275 wholesale could be bought for \$82.80?

4. How many pounds of raisins at \$.095 could be bought for \$7.22?

5. Mr. Ray's coal bill for 1901 was \$135.375. He paid \$4.75 per ton. How many tons did he purchase?

6. If steel is worth \$24.25 a ton, what number of tons is represented by a sale amounting to \$216.625?

7. A grocer paid \$19.235 for a barrel of sugar weighing 315 lb. What was the cost per pound?

8. At 5 cts. a spool, how many spools of thread can be bought for \$7?

9. At 73 cts. a bushel, how many bushels of wheat can be bought for \$25.815?

10. Since 2,000 lb. make 1 ton, what is the value of a pound of hay at \$16.50 per ton? At \$18.40? At \$22.60? At \$20.80?

11. Mr. Flint bought 4,260 lb. of steel at \$28.25 per ton. Find the cost.

12. Mr. Harring bought 3,240 lb. of coal at \$5.25 per ton. Find the cost.

13. Mr. Penrose bought 18,260 lb. of straw at \$8 per ton. Find the cost.

14. A coal dealer bought 86,240 lb. of coal at \$3.50 per ton, and sold it for \$93.50. How much did he lose?

15. John Doan bought 40,250 lb. of hay at \$18 per ton, and sold it for \$425. What was his gain?

16. Mr. Ladd bought 20,250 lb. of hay at \$20 per ton. Find the cost.

17. At \$4.25 per ton, what will 8,500 lb. of coal cost?

18. If 60 bushels of potatoes cost \$.60 a bushel, how much will 4.5 pecks cost?

19. What will .3 of an acre of land cost if $\frac{6}{10}$ of an acre costs \$400?

20. If .02 of a number equals 4, what will 3 times the number be?

REDUCTION

REDUCING A DECIMAL FRACTION TO A COMMON FRACTION

$$219. .5 = \frac{5}{10}. \quad .06 = \frac{6}{100}. \quad .125 = \frac{125}{1000}.$$

When a decimal fraction is written in the form of a common fraction, the denominator is 1 with as many ciphers annexed as there are places in the decimal. Tell why.

To reduce a decimal fraction to a common fraction, write the decimal number as a numerator over its denominator expressed in figures, and reduce the fraction to lowest terms.

WRITTEN EXERCISES

220. Reduce to common fractions:

- | | | | | |
|----------|-----------|----------|------------|-------------|
| 1. .25. | 4. .105. | 7. .7. | 10. .0019. | 13. .00128. |
| 2. .037. | 5. .3016. | 8. .095. | 11. .0815. | 14. .0103. |
| 3. .375. | 6. .08. | 9. .004. | 12. .32. | 15. .0023. |

REDUCING A COMMON FRACTION TO A DECIMAL FRACTION

221. $\$ \frac{3}{4}$ = how many cents? How many hundredths of \$1? Write $\frac{75}{100}$ as a decimal. Reduce $\frac{75}{100}$ to its lowest terms. What do you observe? How do you change $\frac{3}{4}$ to an equal fraction whose denominator is 100?

Express $\frac{3}{4}$ as a problem in division.

$$\frac{3}{4} = 3 \div 4.$$

Divide 3 by 4 as in division of decimals.

SOLUTION: .75

$$4 \overline{)3.00}$$

To reduce a common fraction to a decimal form, annex decimal places to the numerator and divide by the denominator.

WRITTEN EXERCISES

222. Reduce to decimals:

- | | | | | |
|--------------------|--------------------|----------------------|----------------------|------------------------|
| 1. $\frac{1}{2}$. | 5. $\frac{3}{8}$. | 9. $\frac{1}{10}$. | 13. $\frac{1}{16}$. | 17. $1\frac{5}{16}$. |
| 2. $\frac{1}{4}$. | 6. $\frac{5}{8}$. | 10. $\frac{7}{10}$. | 14. $\frac{3}{25}$. | 18. $4\frac{13}{16}$. |
| 3. $\frac{1}{8}$. | 7. $\frac{7}{8}$. | 11. $\frac{1}{25}$. | 15. $\frac{3}{16}$. | 19. $7\frac{9}{25}$. |
| 4. $\frac{1}{6}$. | 8. $\frac{2}{4}$. | 12. $\frac{3}{25}$. | 16. $\frac{1}{50}$. | 20. $9\frac{11}{16}$. |

223. Reduce:

- To the decimal of a foot: 6 inches; 3 inches; 9 inches.
- To the decimal of a peck: $\begin{cases} 1 \text{ peck; } 2 \text{ pecks; } 3 \text{ pecks.} \\ 1 \text{ quart; } 2 \text{ quarts; } 6 \text{ quarts.} \end{cases}$
- To the decimal of a gallon: 1 quart; 2 quarts; 3 quarts.
- To the decimal of an hour: 15 minutes; 30 minutes.

224. In the reductions above, the division is completed in three decimal places without a remainder.

Change $\frac{1}{6}$ to a decimal.

SOLUTION:
$$\begin{array}{r} .166\bar{6} \\ 6 \overline{)1.000} \end{array}$$

In this problem the quotient figure is 6 for each division after the first, however far the division is carried; and with each of these repeated quotients there is the remainder, 4. Such a decimal is called a **repeating decimal**.

In reducing a fraction to a decimal form, if there is a remainder after dividing to three decimal places, write as a common fraction the remainder over the divisor, and place it after the last decimal figure in the quotient.

WRITTEN EXERCISES

225. Change to decimals:

- | | | | | |
|--------------------|--------------------|---------------------|----------------------|-----------------------|
| 1. $\frac{1}{3}$. | 4. $\frac{6}{7}$. | 7. $\frac{5}{8}$. | 10. $\frac{5}{14}$. | 13. $\frac{11}{12}$. |
| 2. $\frac{2}{3}$. | 5. $\frac{4}{9}$. | 8. $\frac{2}{11}$. | 11. $\frac{5}{12}$. | 14. $\frac{3}{17}$. |
| 3. $\frac{3}{4}$. | 6. $\frac{5}{7}$. | 9. $\frac{3}{8}$. | 12. $\frac{7}{16}$. | 15. $\frac{9}{22}$. |

REVIEW

226. Write in decimal form and do as the signs indicate :

1. $\frac{8}{100} + \frac{3}{4} + \frac{9}{10} + \frac{3251}{1000} + \frac{14}{1000} + \frac{3}{8} + \frac{4285}{100000}$.
2. $\frac{375}{1000} - \frac{1}{8}$.
3. $4.25 \times \frac{3}{8}$.
4. 1.28×14.4 .
5. $2.15 \div .15$.
6. $3.065 \times .024$.
7. $.0075 \times \frac{3}{4}$.
8. $17.28 \div 1.2$.
9. $1.728 \div .12$.
10. $.125 \div .125$.
11. $6.25 \div .00125$.
12. $17\frac{3}{4} - .625$.
13. $25\frac{7}{8} + 34\frac{5}{8}$.
14. 875 ten-thousandths $- .0014$.
15. 1 thousand $\times$ 1 thousandth.

227. Solve :

1. Joseph bought a hat for \$2 $\frac{3}{4}$, a pair of shoes for \$3.62 $\frac{1}{2}$, a suit of clothes for \$12 $\frac{1}{4}$, cuffs for 75 cents, and collars for \$.37. He gave in payment two ten-dollar bills. How much change did he receive? Make out the bill in dollars and cents, and receipt it.

2. 24 sheets of paper make 1 quire ; 20 quires make 1 ream. At \$.12 $\frac{1}{2}$ a quire, how many reams of paper can be bought for \$9?

3. At \$.035 each, how many dozen oranges can be bought for \$15.12?

4. James paid \$.06 $\frac{1}{4}$ per quire for paper. How many reams could he buy for \$15?

5. By changing the position of the decimal point, multiply by 10, by 100, by 1000, each of the following numbers :

- | | | |
|--------------|----------------|----------|
| (1) .8365. | (3) 136.02754. | (5) .7. |
| (2) 48.2604. | (4) .00245. | (6) .15. |

6. By changing the position of the decimal point, divide by 10, by 100, by 1000, each of the following numbers :

- | | | |
|-------------|----------|-------------|
| (1) 364.28. | (3) .5. | (5) .014. |
| (2) 2.8526. | (4) .12. | (6) 63,005. |

PERCENTAGE

228. After a certain battle it was found that 2 out of every 100 soldiers had been killed. What per cent of the soldiers had been killed?

In naming a fraction whose denominator is 100, we may use the term **per cent** in place of the denominator 100.

Per cent means "by the hundred," or "per hundred," or "hundredths."

2 per cent means 2 in 100, 2 per hundred, or $\frac{2}{100}$. It is written as a decimal, .02; or, with the sign %, 2 %.

1. In the example given above, how many, out of every 100 soldiers, were killed? What per cent were killed? Write the decimal fraction of the whole number of soldiers that were killed. Write the number %.

25 per cent, or 25 %, = 25 per hundred = $\frac{25}{100} = .25$.

2. Write as above the different expressions that equal 50 per cent; 75 per cent; 30 per cent; 1 per cent.

3. Express in three other ways, "4 per hundred."

4. What part of \$200 is \$100?

How many hundredths of \$200 is \$100?

How many per cent of \$200 is \$150?

25 per cent of \$200 equals how many dollars?

$\frac{40}{100}$ of \$200 equals how many dollars?

In what other ways can $\frac{40}{100}$ be expressed?

In certain classes of problems the common fraction is used to indicate quantity: as $\frac{1}{2}$ of a dollar; $\frac{3}{4}$ of a mile.

In other problems the decimal form is used to indicate quantity: as .34 of an acre, .9 of a foot, .40 of a ton.

In other problems the quantity indicated is a certain per cent: as 20 per cent gain, 15 per cent interest. Per cent is used in business problems of gain, loss, taxes, etc.

TABLE OF EQUIVALENTS

229. This table shows the equivalent in per cent of each of the proper fractions and mixed numbers given.

10%	$\frac{1}{10}$								
20%		$\frac{1}{5}$							
100%, or 1									
50%, or $\frac{1}{2}$					50%, or $\frac{1}{2}$				
25%, or $\frac{1}{4}$		25%, or $\frac{1}{4}$							
		25%, or $\frac{3}{8}$		87½%, or $\frac{7}{8}$					
100%, or 1									
83⅓%, or $\frac{2}{3}$				83⅓%, or $\frac{2}{3}$					
16⅔%		$\frac{1}{6}$		66⅔%, or $\frac{2}{3}$					
8⅓%	$\frac{1}{12}$								

Supply the equivalent in each of the following :

1. $\frac{3}{4} = \text{---}$. $\frac{5}{8} = \text{---}$. $\frac{1}{10} = \text{---}$. $\frac{5}{10} = \text{---}$.
 $\frac{1}{100} = \text{---}$. 100 per cent = --- .

2. From the table write each common fraction and its equivalent per cent; as, $\frac{1}{5} = 20\%$.

3. Make a list of these in order, beginning with the smallest.

4. Memorize these equivalents.

5. $\frac{1}{100} + \frac{25}{100} = \text{---}$.

6. Write $\frac{1}{100}$ as a mixed number. As a decimal. As a per cent.

7. Give the equivalent per cent of $\frac{3}{4}$, $\frac{9}{10}$, $\frac{4}{5}$, $\frac{5}{8}$.

ORAL EXERCISES

230. What per cent equals each of the following:

- | | | | | |
|--------------------|--------------------|--------------------|---------------------|---------------------|
| 1. $\frac{1}{2}$? | 3. $\frac{3}{4}$? | 5. $\frac{7}{8}$? | 7. $1\frac{1}{3}$? | 9. $\frac{5}{6}$? |
| 2. $\frac{5}{4}$? | 4. $\frac{1}{8}$? | 6. $\frac{1}{3}$? | 8. $\frac{2}{5}$? | 10. $\frac{2}{3}$? |

231. Name an equivalent common fraction or mixed number for each of the following:

- | | | | |
|-----------|-----------------------|------------------------|-----------------------|
| 1. 25 %. | 3. $12\frac{1}{2}$ %. | 5. $187\frac{1}{2}$ %. | 7. $66\frac{2}{3}$ %. |
| 2. 150 %. | 4. $33\frac{1}{3}$ %. | 6. $137\frac{1}{2}$ %. | 8. $16\frac{2}{3}$ %. |

232. Find 125 % of 500.

SOLUTION:

$$125\% = 100\% + 25\%.$$

$$100\% \text{ of } 500 = 500.$$

$$25\% \text{ of } 500 = 125.$$

$$125\% \text{ of } 500 = 500 + 125 = 625.$$

Find 75 % of 600.

SOLUTION:

$$75\% = 100\% - 25\%.$$

$$100\% \text{ of } 600 = 600.$$

$$25\% \text{ of } 600 = 150.$$

$$75\% \text{ of } 600 = 600 - 150 = 450.$$

If 100 % of a ship's crew is 120 men, how many men would the following percents represent?

- | | | | |
|-----------------------|------------------------|------------------------|-----------|
| 1. 120 %. | 3. $112\frac{1}{2}$ %. | 5. $137\frac{1}{2}$ %. | 7. 25 %. |
| 2. $33\frac{1}{3}$ %. | 4. $16\frac{2}{3}$ %. | 6. 150 %. | 8. 200 %. |

WRITTEN EXERCISES

233. If 400 acres are 100 %, how many acres are equal to each of the following:

- | | | | |
|-----------------------|-----------------------|-------------------------|-------------------------|
| 1. 50 %? | 5. $62\frac{1}{2}$ %? | 9. 125 %? | 13. $87\frac{1}{2}$ %? |
| 2. 175 %? | 6. $12\frac{1}{2}$ %? | 10. $112\frac{1}{2}$ %? | 14. $37\frac{1}{2}$ %? |
| 3. 150 %? | 7. 75 %? | 11. $66\frac{2}{3}$ %? | 15. $137\frac{1}{2}$ %? |
| 4. $33\frac{1}{3}$ %? | 8. $16\frac{2}{3}$ %? | 12. 80 %? | 16. 150 %? |

FINDING THE PERCENTAGE

234. If, in the example given in Section 228, the whole number of soldiers in battle were 10,000, how many soldiers were killed?

SOLUTION:

10,000	Base
.02	Rate
200.00	Percentage

200 soldiers were killed in the battle.

The number found by taking a certain per cent of a number is called the **percentage**.

Thus, in the example given, 2 per cent of the soldiers were killed, the percentage being 200 soldiers.

The number per cent is called the **rate per cent**, the **rate**, or the **per cent**.

The number of which a percentage is found is called the **base**.

The base multiplied by the rate equals the percentage.

ORAL EXERCISES

235. Thinking of the cost as 100 %, answer each of the following:

1. If a book was sold for half of its cost, what per cent of the cost was received? What per cent was lost?

2. If it was sold for $\frac{1}{4}$ of its cost, what per cent of the cost was received? What per cent was lost?

3. If it was sold for $\frac{3}{4}$ of its cost, what per cent of the cost was received? What per cent was lost?

4. If it was sold for $\frac{1}{8}$ more than its cost, what per cent of the cost was gained? What per cent of the cost was received?

5. If it was sold for $\frac{3}{8}$ more than its cost, what per cent of the cost was gained? What per cent was received?

WRITTEN EXERCISES

236. Solve:

1. A cattle dealer shipped across the country 80 head of cattle. On the journey 5% died. How many died?

2. Of 1,200 candidates examined for a city's high schools, $87\frac{1}{2}\%$ were admitted. What percentage of the candidates were admitted?

3. On a steamer bound for Naples, 728 steerage passengers were listed. $62\frac{1}{2}\%$ of these were Italians. What percentage were Italians?

4. A farmer spent on his farm in one year \$1,600. $12\frac{1}{2}\%$ of this he spent for plants and fertilizers, 50% for labor, 25% for improved machinery, and the rest for other expenses. What percentage did he spend for each?

5. Mr. Lawson raised 4,280 bu. of corn, and sold 85% of it at \$.67 a bushel. How much did he receive for what he sold?

6. Mr. Horton bought 4,268.24 lb. of sugar. He sold .375 of it at \$.04 $\frac{1}{2}$ per pound. How much did he receive for what he sold?

7. Mr. Tompkins bought 82,360 ft. of hemlock boards, and sold $87\frac{1}{2}\%$ of them at \$.02 $\frac{1}{2}$ per foot. How much did he receive?

8. He sold the remainder for \$.03 per foot. How much did he receive for that?

9. Mr. Boughton has a salary of \$1,200 per year. He spends .85 of it. What per cent of his salary does he save each year? How much will he save in 5 years?

10. Mr. Early raised 325.25 bu. of corn in one field, 275 bu. in another field, 146.75 bu. in another field. He used 65% of his crop and sold the remainder for \$.73 per bushel. How much money did he receive?

11. Mr. Randall had \$7,685. He bought a farm and had 15 % of his money left. How much had he left?

12. A lot cost \$1,600. Find the per cent gained or lost, the money gained or lost, the money received, when it is sold for each of the following per cents of the cost:

187½ %.	87½ %.	150 %.	25 %.
125 %.	12½ %.	112½ %.	75 %.
162½ %.	37½ %.	137½ %.	62½ %.

13. Mr. Morse bought a lot for \$2,400. He sold it for 37½ % more than he paid. How much money did he gain? How much money did he receive for it?

14. If Mr. Morse had sold his lot for 37½ % less than it cost, what would he have received?

15. John had \$4,000 in the bank. He drew out 62½ % of it. How much money did he draw out? How much had he left in the bank?

16. A book that cost \$2.00 was sold for 25 % less than the cost price. How much money was received for it? How much was lost?

17. Mrs. Olds has an income of \$1,000 a year. She spends 75 % of it. How much money does she save?

18. A man bought a farm for \$16,000 and sold it for 25 % more than he gave. How much did he gain? How much did he receive for it?

19. If he had sold it for 25 % less than he gave, how much money would he have lost? How much would he have received?

20. A farmer had 160 acres of wheat. If it cost him \$22 per acre to produce it, how much would he gain if his per cent of gain was 50 % of the cost?

21. A boy bought papers for $\frac{1}{2}$ cent each and sold them for 100 % more than they cost. What was the selling price of each paper ?

INTEREST

237. Mr. Averill borrowed \$250 of Mr. Johnson. He agreed to repay after one year the \$250, and to pay Mr. Johnson in addition, for the use of the money, 5 % of the \$250 borrowed. At the end of the year he paid back the \$250 and \$12.50 in addition, making a total of \$262.50.

SOLUTION: \$250 Principal In practice write: \$250.

	.05 Rate		.05
\$12.50	Interest		12.50
			<u>250.</u>
			\$262.50

Principal + Interest = Amount

$$\$250 + \$12.50 = \$262.50$$

Money paid for the use of another person's house or lands is called **rent**.

Money paid for the privilege of selling goods is called **license**.

Money paid for the use of borrowed money is called **interest**.

The borrowed money on which interest is paid is called the **principal**.

Interest is usually reckoned by the year, as a certain percentage of the principal. The per cent at which interest is computed is called the **rate**.

To find the interest for any fixed time, multiply the principal by the rate per cent expressed as a decimal.

The principal plus the interest, or the total sum due at a stated time, is called the **amount**.

ORAL EXERCISES

238. Solve:

1. What is the interest on \$200, for 1 year, at 6%?
2. What is the interest on \$120 for 1 year at 6%?
3. James Hanson borrowed of Richard Dempsey \$120, and agreed to pay him interest at the rate of 6% for one year. How much interest did he owe at the end of the year? What was the total amount of his debt?
4. In the last example what is the principal? What is the rate? The interest? The amount?
5. Find the interest due at the end of 1 year on each of the following loans:

\$250 at 3%.	\$600 at 4%.	\$800 at 5%.
\$450 at 3%.	\$650 at 4%.	\$150 at 5%.
\$675 at 3%.	\$345 at 4%.	\$225 at 5%.
\$495 at 3%.	\$230 at 4%.	\$350 at 5%.
6. Find the amount due at the end of 1 year for each of the following loans:

\$500 at 5%.	\$800 at 4%.	\$150 at 3%.	\$400 at 6%.
--------------	--------------	--------------	--------------

WRITTEN EXERCISES

239. Find the amount payable at the end of 1 year, on each of the following loans:

- | | | |
|--------------------------------|----------------------------------|-----------------------------------|
| 1. \$175 at 4%. | 5. \$165 at 5%. | 9. \$8,000 at $6\frac{1}{2}\%$. |
| 2. \$308 at $4\frac{1}{2}\%$. | 6. \$2,845 at $5\frac{1}{2}\%$. | 10. \$3,076 at 3%. |
| 3. \$875 at $3\frac{1}{2}\%$. | 7. \$3,492 at $2\frac{1}{2}\%$. | 11. \$4,290 at $4\frac{1}{2}\%$. |
| 4. \$736 at $4\frac{1}{2}\%$. | 8. \$5,783 at 4%. | 12. \$5,832 at 6%. |

240. Find the interest on \$250 for 3 years at 4%.

SOLUTION: Interest for 1 yr. = $\$250 \times .04 = \$10.00$.Interest for 3 yr. = $\$10 \times 3 = \$30$.

WRITTEN EXERCISES

241. Find the interest on each of the following :

1. \$120, at 3 %, for 2 years.
2. \$180, " 4 %, " 3 years.
3. \$264, " 6 %, " 5 years.
4. \$300, " $4\frac{1}{2}$ %, " 10 years.
5. \$624, " $5\frac{1}{2}$ %, " 6 years.
6. \$720, " 6 %, " 4 years.
7. \$564.25, " $4\frac{1}{2}$ %, " 4 years.
8. \$76.85, " 3 %, " 10 years.
9. \$194.75, " $3\frac{1}{2}$ %, " 2 years.
10. \$327, " 7 %, " 3 years.
11. \$862.625, " 5 %, " 6 years.

242. Find the amount payable for each of the following at the close of the time stated :

1. \$450, at 6 %, for 2 years.
2. \$250, " 4 %, " 3 years.
3. \$600, " 6 %, " 4 years.
4. \$1,200, " 3 %, " 1 year.
5. \$600, " $4\frac{1}{2}$ %, " 3 years.
6. \$300, " 3 %, " 5 years.
7. \$225, " 4 %, " 10 years.
8. \$320.50, " 4 %, " 9 years.
9. \$450.25, " 5 %, " 6 years.

243. What fractional part of 1 yr. is 3 mo.? 9 mo.? 4 mo.? 8 mo.? 2 mo.? 10 mo.? 1 mo.? 11 mo.?

What is the interest on \$400 for 3 mo. at 6 %?

SOLUTION: Interest for 1 yr. = \$400 $\times$.06 = \$24.

Interest for 3 mo. = \$24 $\div$ 4 = \$6.

1. What is the interest of \$1200, at 6%, for 1 yr.? For 3 mo.? 9 mo.? 4 mo.? 8 mo.? 2 mo.? 10 mo.? 1 mo.? 11 mo.? 7 mo.? 5 mo.?

2. What is the interest on \$1200, at 5%, for 1 yr.? For 3 mo.? 9 mo.? 4 mo.? 8 mo.? 2 mo.? 10 mo.? 1 mo.? 11 mo.? 7 mo.? 5 mo.?

3. What is the interest on \$1200, at 4%, for 1 yr.? For 8 mo.? 4 mo.? 9 mo.? 3 mo.? 10 mo.? 2 mo.? 1 mo.? 7 mo.? 11 mo.? 5 mo.?

244. Find the interest on \$460, at 5%, for 2 yr. 6 mo.

SOLUTION: \$ 460

 .05

 \$23.00

 2

 \$46.00

 11.50

 \$57.50

\$460 $\times$.05 = \$23.00, interest for 1 yr.

\$23 $\times$ 2 = \$46, interest for 2 yr.

\$23 $\div$ 2 = \$11.50, interest for 6 mo.

\$46.00 + \$11.50 = \$57.50, interest.

WRITTEN EXERCISES

245. Find the interest on:

1. \$360, at 6%, for 3 yr. 6 mo.
2. \$180, " 5%, " 2 yr. 4 mo.
3. \$720, " 4%, " 4 yr. 3 mo.
4. \$150.50, " 6%, " 5 yr. 6 mo.
5. \$480.50, " 5%, " 2 yr. 3 mo.
6. \$600, " 4%, " 3 yr. 2 mo.
7. \$320.80, " 4%, " 3 yr. 3 mo.
8. \$691.60, " 5%, " 2 yr. 9 mo.
9. \$276.48, " 6%, " 1 yr. 8 mo.
10. \$248.12, " 6%, " 3 yr. 2 mo.

GENERAL REVIEW

ORAL EXERCISES

246. Solve:

1. What is the ratio of 5 to 15? To 20? To 25? To 50?
2. 5 is what per cent. of 15? Of 20? Of 25? Of 50?
3. What is the ratio of $\frac{3}{4}$ pound to 8 ounces?
4. What is the ratio of $\frac{1}{2}$ pound to 12 ounces?
5. What is the ratio of 4 ounces to .75 pounds?
6. What is the ratio of 500 pounds to a ton?
7. What is the ratio of 1 ton to 1500 pounds?

WRITTEN EXERCISES

247. Solve:

1. If 9 bu. of wheat cost \$14.40, what is the cost of 81 bu.? Of 27 bu.? Of 54 bu.? Of 108 bu.?

2. If 108 yd. of ribbon cost \$27, what will 54 yd. cost at the same price? 36 yd.?

3. If a case of bluing is worth \$1.60, what are $7\frac{3}{4}$ cases worth? $15\frac{1}{8}$ cases?

4. Mr. Smith had \$2,100 in the Madison Bank, and \$1,500 in the Morristown Bank. He drew out $\frac{2}{3}$ of the money that he had in the Madison Bank and put it in the Morristown Bank. How much had he then in each bank?

5. Mr. Wilson had \$11,500. He bought a farm of 75 acres at \$125 per acre. How much money had he left?

6. Mr. Wilson then bought a pair of horses for \$225, a carriage for \$120, harnesses for \$75, a harrow for \$15, a plow for \$10, and a mowing machine for \$80. With the remainder he bought cows at \$40 each. How many cows did he buy?

7. Mr. Alberts sold 36 cows for \$45 each, and bought horses for \$135 each with the money. How many horses did he buy?

8. John bought $6\frac{3}{4}$ lb. of beefsteak at 24 cts. a pound, and gave the butcher \$2. How much change did he receive?

9. Julia bought 3 pieces of ribbon. The first piece contained $7\frac{1}{4}$ yd., the second $5\frac{1}{2}$ yd., the third $10\frac{3}{8}$ yd. How many yards did Julia buy?

10. A grocer bought 25 chests of tea, each chest holding 46 lb., at \$.49 a pound, and sold it at \$.75 a pound. How much did he gain?

11. Draw a 4-inch square on a scale of $\frac{1}{2}$ inch to 5 ft. How many feet does its perimeter represent? How many square feet does the square represent?

12. Draw a 4-inch square on a scale of $\frac{1}{4}$ inch to 5 ft. How many feet does the perimeter represent? How many square feet does the square represent?

13. The Plaut Co. paid \$480 for a piano; they sold it for $166\frac{2}{3}\%$ of the cost. Was there gain or loss? What per cent was it? What fraction of the cost? How much money was gained or lost? What was received for the piano?

14. A carriage costing \$480 was sold for $133\frac{1}{3}\%$ of cost. What was the selling price?

15. A lot costing \$500 was sold for 75% of cost. What was the gain or loss?

16. A horse costing \$600 was sold for $66\frac{2}{3}\%$ of cost. What fraction of the cost was this?

17. What is the ratio of $1:\frac{1}{5}$? $\frac{1}{10}:\frac{1}{5}$? $\frac{9}{10}:\frac{1}{5}$? $1\frac{3}{10}:\frac{1}{5}$?

18. If $\frac{1}{5}$ of the cost of an acre is \$328, what will $\frac{1}{10}$ of an acre cost? $\frac{7}{10}$ of an acre? $2\frac{1}{5}$ acres? $2\frac{1}{10}$ acres?

19. Eisele & Co. paid \$234.80 for 135 bbl. of cement, which they sold at \$1.75 per barrel. How much did they make?

20. A farmer sold $\frac{3}{4}$ of a ton of hay at one time, 1,200 lb. at another, .375 of a ton at another time. How many pounds did he sell in all?

21. How many rods in 375 thousandths of a mile?
22. How many rods in $\frac{4}{5}$ of a mile?
23. How many rods in .875 of a mile?
24. What is the ratio of a rod to $5\frac{1}{2}$ ft.?
25. What is the ratio of $16\frac{1}{2}$ feet to 1 yd.?
26. What is the ratio of 1 foot 6 inches to 1 yd.?
27. What is the ratio of 320 rods to $\frac{1}{2}$ mile?
28. Ten cashmere goats furnish wool enough for a shawl 4.5 ft. square. How many square feet of cashmere are made from the wool of one goat?
29. One goat may produce 3 lb. of fiber worth 30 cts. a pound. What are the fleeces of 500 goats worth at this rate?
30. Thirty per cent of the fleece is often short hair, worth only 16 cts. a pound. Making this allowance, what is the real value of the fleece, or mohair, in Exercise 29?
31. The 8 mohair mills in the United States used in one year 1,119,465 lb. of imported mohair and 42,465 lb. of domestic mohair. How much did they use in all?
32. Adding 30 % for short hairs, how many pounds of fleece were produced to supply these mills?
33. What was the average per mill? How many goats did it take to produce this fleece?
34. In 1860 the average rate of wages of a woolen mill operated in Massachusetts was 87.3 cts. a day. In 1890 it was \$1.24. What was the average increase a year?
35. An expert sorter can examine 90 lb. of wool in one day. How long does it take him to examine 40 bales of 250 lb. each?
36. The highest price paid for raw wool is 30 cts. per pound. If the loss by scouring averages $2\frac{1}{2}$ % a bale, what is the loss?

37. A painter's wages for working 8 hours a day every working day of the week, for 40 weeks, were \$672. What were his wages per hour?

38. How many hours are there in a year? How many minutes? How many seconds?

39. Mr. Luke had 295 bu. 3 pk. of wheat and sold 168.75 bu. How many bushels had he left?

40. From 840 ft. 9 in. take 476.25 ft.

41. A grocer exchanged with a farmer $28\frac{1}{2}$ gal. of molasses, at 40 ct. a gallon, for butter at 32 cts. a pound. How many pounds did he receive?

42. A grocer sold 18 lb. 12 oz. of butter on Monday; $27\frac{3}{4}$ lb. on Tuesday; 36.375 lb. on Wednesday. He sold it for 26 cts. a pound. How much did he receive for it?

43. There are 272.25 sq. ft. in 1 sq. rd. How many square feet are there in 3.7 sq. rd.?

44. At \$.37 $\frac{1}{2}$ per gallon, how many gallons of vinegar can be bought for \$8.75?

45. A merchant spent \$180 for silk, at \$2.25 per yard. How many yards did he purchase?

46. For \$27 how many yards of broadcloth can be bought at \$4.50 per yard?

SUMMARY OF CHAPTER III

DECIMAL FRACTIONS.

Reading and writing decimals.

Reducing to common fractions and common fractions to decimals.

Addition, subtraction, multiplication, and division of decimals, with applications to United States money, to general business, to percentage, and to interest.

CHAPTER IV

DENOMINATE NUMBERS

ABSTRACT AND CONCRETE NUMBERS

248. Mention five numbers. Add names to them, telling of something numbered in each case.

"5, 7, 9" are numbers, but not of any particular things.

Numbers used without naming anything numbered are called **abstract numbers**. (*Abstract* means "separated from.")

"5 horses, 7 miles, 9 boys" are also numbers, but numbers of the things named — horses, miles, boys.

Numbers used with the names of the things numbered are called **concrete numbers**. (*Concrete* means "put together with.")

249. When you say that a certain book contains 300 pages, a page is the unit by which you measure the size of the book.

When you say each page is 5 inches long, an inch is the unit by which you measure the length of the page.

If you divide 12 apples into groups of three apples each, 3 apples is the unit by which you measure the 12 apples. 12 apples contains the unit, 3 apples, 4 times.

If you are counting abstract numbers by ones, one is the unit.

If you are counting by twos or threes or tens, two or three or ten is the unit.

UNITS OF MEASUREMENT

250. A quantity used to measure another quantity is called a **unit of measurement** or simply a **measure**.

Measuring a quantity is finding out how many times the unit is contained in the quantity measured.

Many units of measurement are fixed by law, as the exact length of a yard, the weight of a pound, the capacity of a gallon. In buying or selling, it is illegal to use false or incorrect measures.

In most towns there is an official called the Sealer of Weights and Measures. He keeps in his office a metal bar with a yard exactly measured upon it, an exact pound weight, an exact gallon, and other exact measures. He obtains these exact measures from an official in the state capitol, and this official obtains them from another in the national capitol at Washington, where exact measures are very carefully kept.

Any one desiring to know whether his measures or his scales are correct, or not, can find out by asking the sealer to compare them with those in his office. If they are correct, the sealer will put a stamp, or seal, upon them to attest the fact.

251. Concrete numbers expressed in terms of fixed measures, as pounds, feet, quarts, are called **denominate numbers**.

Recall that the word *denominator* means "namer." *Denominate numbers* are *naming numbers*. The measure, as *cent*, *ounce*, *pint*, always gives a *name* to the number used.

A denominate number consisting of only one term or denomination is called a **simple denominate number**. 8 miles is a simple denominate number.

A denominate number consisting of more than one term or denomination is called a **compound denominate number**. 8 miles, 6 rods, 2 feet is a compound denominate number.

MEASURES OF CAPACITY

252.

LIQUID MEASURE

4 gills (gi.)	= 1 pint (pt.)
2 pints	= 1 quart (qt.)
4 quarts	= 1 gallon (gal.)

This table is used in measuring liquids, such as molasses, oil, milk.

The barrel is used as a measure of liquids in large quantities. Unless otherwise stated, it contains $31\frac{1}{2}$ gallons.

ORAL EXERCISES

253. Solve:

1. How many quarts are there in 3 gal. 2 qt.? In 5 gal. 1 qt.? In 6 gal. 3 qt.?
2. How many pints are there in 3 qt. 1 pt.? In 7 qt. 1 pt.? In 8 qt. 1 pt.?
3. How many gills in 3 qt. 1 pt.? In 4 qt. 1 pt. 3 gi.?
4. How many pints in 2 gal. 1 qt.? In 2 gal. 3 qt. 1 pt.?
5. How many gills in 50 % of 1 gal.? In 25 %?
6. How many gills in $\frac{1}{8}$ of 2 gal.? In .25 of 2 gal.? In 50 % of 2 gal.? In 25 % of 2 gal.? In $37\frac{1}{2}$ % of 3 gal.?
7. If a jug of vinegar contains $2\frac{1}{2}$ gal., what is its value at 8 cts. a quart?
8. If a can of milk contains $10\frac{3}{4}$ gal., what is its value at 6 cts. a quart?
9. A keg of molasses containing 15.5 gal. was sold for $12\frac{1}{2}$ cts. per quart. How much was received for it?
10. A keg of syrup containing $12\frac{1}{2}$ gal. was sold at the rate of 25 cts. per quart. How much was received for it?
11. What is the value of 4 gal. 2 qt. of alcohol, at \$1.25 per quart?

254.

DRY MEASURE

2 pints (pt.)	= 1 quart (qt.)
8 quarts	= 1 peck (pk.)
4 pecks	= 1 bushel (bu.)
32 quarts	= 1 bushel

This table is used in measuring dry substances, chiefly grains, fruits, and vegetables.

ORAL EXERCISES

255. Solve:

1. How many pecks in 2 bu.? In 5 bu. 1 pk.? In 6 bu. 2 pk.?

2. At 25 cts. per peck, what is the value of 2 bu. 1 pk. of potatoes? Of 5 bu. 2 pk.? Of 6 bu. 2 pk.?

3. How many quarts in 3 pk.? In 2 pk. 5 qt.? In 5 pk. 7 qt.? In 8 pk. 6 qt.?

4. At 5 cts. per quart, what is the value of 1 pk. 2 qt. of beans? Of 2 pk.? Of 1 pk. 5 qt.? Of 2 pk. 4 qt.?

5. At 3 cts. a quart, what is the value of 1 bu. of beans? Of 1 bu. 1 pk.? Of 2 bu.? Of 2 bu. 1 pk.? Of 1 bu. 2 pk.?

6. What will be the cost of 2 bu. 3 pk. of apples at 10 cts. per quart? Of 3 bu. 2 pk.? Of $1\frac{1}{2}$ bu.?

7. What will be the cost of $\frac{5}{8}$ of a bushel of peaches at 15 cts. per quart? Of $1\frac{3}{8}$ bu.? Of $2\frac{1}{2}$ pk.? Of $12\frac{1}{2}\%$ of a bushel? Of $16\frac{2}{3}\%$ of a bushel? Of 75% of a peck?

8. At 4 cts. per quart, what is the value of 5 bu. of beans? Of 25% of a bushel? Of $12\frac{1}{2}\%$ of a bushel? Of $37\frac{1}{2}\%$ of a bushel? Of $37\frac{1}{2}\%$ of 1 bu. 1 pk.? Of $62\frac{1}{2}\%$ of 1 bu. 1 pk.? Of 75% of 1 bu. 1 pk.? Of $87\frac{1}{2}\%$ of 1 bu. 1 pk.? Of .1, .3, .7, .8, and .9 of 1 bu. 1 pk.? Of $\frac{2}{4}$, $\frac{3}{8}$, $\frac{2}{5}$, $\frac{1}{3}$, $\frac{5}{6}$ of a bushel; of a peck?

MEASURES OF LENGTH, AREA, AND VOLUME

256. LONG (LINEAR) MEASURE

12 inches (in.)	= 1 foot (ft.)
3 feet	= 1 yard (yd.)
$5\frac{1}{2}$ yards	= 1 rod (rd.)
$16\frac{1}{2}$ feet	= 1 rod
320 rods	= 1 mile (mi.)
1,760 yards	= 1 mile
5,280 feet	= 1 mile

ORAL EXERCISES

257. Solve:

1. How many inches in 1 ft. 10 in.? In 2 ft. 11 in.? In 5 ft. 8 in.? In 9 ft. 3 in.? In 12 ft. 6 in.? In 11 ft. 9 in.? In 7 ft. 7 in.?

2. Express in feet and decimals of a foot: 150 in.; 87 in.; 99 in.; 138 in.; 123 in.; 64 in.; 100 in.

3. How many feet in 10 yd. 2 ft.? In 12 yd.? In 9 yd. 2 ft.? In 11 yd. 1 ft.?

4. How many inches in 1 yd. 1 ft. 6 in.? In 2 yd. 2 ft.? In 3 yd. 1 ft. 9 in.? In 5 yd.?

5. How many yards in 3 rd.? In 6 rd.? In .05 rd.? In .8 rd.? In .12 rd.? In $7\frac{1}{2}$ rd.?

6. How many rods in $16\frac{1}{2}$ yd.? In 33 yd.? In 44 yd.? In 11 yd.?

7. How many rods in 1 mi.? In 4 mi.? In 5 mi.? In 10 mi.? In 20 mi.? In 40 mi.?

8. What part of a mile is 40 rd.? 80 rd.? 160 rd.? 32 rd.? 16 rd.? 8 rd.? 4 rd.?

9. How many rods in $\frac{1}{2}$ of 1 mi.? In $\frac{1}{4}$? In $\frac{1}{8}$? In $\frac{1}{16}$?

10. What part of a yard is 18 in.? 9 in.? $4\frac{1}{2}$ in.? 6 in.?

11. What part of a yard is 2 ft. 6 in.? 1 ft. 8 in.? $\frac{2}{3}$ ft.?

258. SQUARE (SURFACE) MEASURE

144 square inches (sq. in.)	= 1 square foot (sq. ft.)
9 square feet	= 1 square yard (sq. yd.)
$30\frac{1}{4}$ square yards	= 1 square rod (sq. rd.)
160 square rods	= 1 acre (A.)
640 acres	= 1 square mile (sq. mi.)

259. Solve: ORAL EXERCISES

1. What is the ratio of a square foot to a rectangle 3 in. by 4 in.? 4 in. by 6 in.? 3 in. by 8 in.? 6 in. by 6 in.?

2. What part of a square foot is 72 sq. in.? 36 sq. in.? 18 sq. in.? 9 sq. in.? 24 sq. in.?

3. Draw a square yard on the blackboard or the floor. Divide it into square feet. How many square feet are there in a row, and how many rows of square feet are there in a square yard? How many square yards are there in a table 1 yd. square? What is the ratio of 2 sq. yd. to a 2-yard square? How much larger is a rug 3 yd. square than one containing 3 sq. yd.?

4. How long and how wide is a square rod? Into how many rows of how many square yards each may a square rod be divided?

260. Solve: WRITTEN EXERCISES

1. How wide must a plot of land 20 rd. long be to contain an acre? A plot 16 rd. long? One 32 rd. long?

2. What is an acre of land worth at the rate of \$75 for 16 sq. rd.? At \$300 for 32 sq. rd.? At \$50 for 20 sq. rd.?

3. What is the value of a square mile of land, at the rate of \$8,000 for 160 acres? At \$15,000 for 320 acres? At \$9,000 for 80 acres? At \$16,000 for 120 acres? At \$12,000 for 40 acres?

261.

CUBIC MEASURE

$$\begin{aligned}
 1,728 \text{ cubic inches (cu. in.)} &= 1 \text{ cubic foot (cu. ft.)} \\
 27 \text{ cubic feet} &= 1 \text{ cubic yard (cu. yd.)}
 \end{aligned}$$

ORAL EXERCISES

262. Solve:

1. Describe a 4-inch cube by stating the following:

The number of cubic inches in a row; the number of rows in a layer; the number of cubic inches in a layer; the number of layers in the cube; the number of cubic inches in the cube. If necessary, build the cube with blocks.

2. Describe a 5-inch cube.
3. Describe an 8-inch cube.
4. Describe a 12-inch cube.

5. State the cubic contents of each of the following solids:

	(1)	(2)	(3)	(4)	(5)	(6)
Length:	8 in.	12 in.	10 in.	12 in.	1 ft. 3 in.	1 ft. 4 in.
Width:	5 in.	6 in.	8 in.	6 in.	6 in.	5 in.
Height:	4 in.	5 in.	6 in.	6 in.	4 in.	3 in.

6. Describe a cubic yard.

7. How many cubic yards are there in a cube whose sides are each 2 yd.?

8. How many cubic feet are there in $\frac{2}{3}$ of a cubic yard?
In $\frac{4}{5}$? In $\frac{7}{8}$? In $\frac{5}{6}$?

9. How many cubic feet in a stone 8 ft. long, 2 ft. wide, and 1 ft. thick?

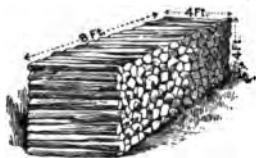
10. If the stone were only half as long and half as thick, how many cubic feet would it contain?

11. How many cubic inches in a block of wood each of whose sides measures 8 in.?

263.

MEASURE OF WOOD

128 cubic feet (cu. ft.) = 1 cord



264. Wood that is cut in 4-foot lengths is called *cord wood*. A *cord* is a pile of 4-foot wood, 4 ft. high and 8 ft. long.

Stove wood is cut into short lengths, commonly 16 inches. A pile of such wood 4 ft. high and 8 ft. long is called a **stove wood cord**.

ORAL EXERCISES

265. Solve:

1. What part of a cord of wood is 64 cu. ft.? 32 cu. ft.?
2. How many cords of wood are there in a pile 8 ft. long, 4 ft. wide, 4 ft. high?
3. Mr. Alberts has a pile of cord wood 120 ft. long and 8 ft. high. At \$6 a cord, for how much could he sell it?

266. Lumber is bought and sold by the board foot. A **board foot** is 1 foot square and 1 inch or less thick.

In measuring boards, thickness of *less* than 1 inch is treated as 1 inch. Thickness of more than 1 inch is computed exactly.

1. How many board feet are there in a board 15 ft. long, 8 in. wide, and $\frac{5}{8}$ in. thick?

SOLUTION: 8 in. = $\frac{2}{3}$ ft. $15 \times \frac{2}{3} \times 1$ board ft. = 10 board ft.

2. How many board feet are there in a plank 20 ft. long, 15 in. wide, and $1\frac{1}{2}$ in. thick?

SOLUTION: 15 in. = $\frac{5}{4}$ ft. $20 \times \frac{5}{4} \times \frac{3}{2}$ board ft. = $\frac{75}{2}$ = $37\frac{1}{2}$ board ft.

To find the number of board feet in a piece of lumber, multiply the length expressed in feet by the width in feet, and the product by the thickness in inches.

267.

AVOIRDUPOIS WEIGHT

16 ounces (oz.)	= 1 pound (lb.)
100 pounds	= 1 hundredweight (cwt.)
20 hundredweight (2000 lb.)	= 1 ton (T.)

There are 2,240 lb. in a long ton. Weights are computed by long tons when large quantities of bulky articles, such as coal, are bought and sold.

ORAL EXERCISES

268. Solve:

1. How many ounces are there in 2 lb.? In 2 lb. 8 oz.? In 5 lb. 12 oz.? In 10 lb. 9 oz.? In 25 % of a pound?

2. How many pounds are there in 32 oz.? In 48 oz.? In 80 oz.? In 88 oz.? In 100 oz.?

3. What is the value of 3 lb. 8 oz. of butter at 20 cts. a pound? At 25 cts.?

4. What is the value of 5 lb. 4 oz. at 20 cts.? At 25 cts.?

5. What is the value of 4 lb. 12 oz. at 20 cts.?

6. How many pounds are there in a ton? In 4 cwt.? In 4 T. 5 cwt. 25 lb.? In $6\frac{1}{2}$ T.? In $8\frac{1}{4}$ T.? In $5\frac{1}{8}$ T.? In $12\frac{1}{2}$ % of a ton? In $37\frac{1}{2}$ %? In 75 %? In $87\frac{1}{2}$ %? In 80 %? In 40 %? In 5 %?

7. How many hundredweight in 2 T.? In $2\frac{1}{2}$ T.? In $4\frac{1}{4}$ T.? In $5\frac{3}{4}$ T.? In $\frac{1}{8}$ T.? In $\frac{3}{8}$ T.? In $\frac{5}{8}$ T.? In 75 % of a T.? In $87\frac{1}{2}$ % of a T.?

8. What is the value of a ton of hay at \$1.25 a hundredweight? Of $2\frac{1}{2}$ T.? Of $4\frac{1}{2}$ T.? Of 8.4 T.? Of $6\frac{1}{10}$ T.?

9. How many tons in 2,500 lb.? In 3,000 lb.? In 8,000 lb.? In 4,500 lb.? In 9,000 lb.?

10. How many pounds in $5\frac{1}{2}$ long tons? In $8\frac{3}{8}$ long tons? In $62\frac{1}{2}$ % of a long ton? In $87\frac{1}{2}$ % of a long ton?

269.

MEASURES USED IN COUNTING

12 units	= 1 dozen (doz.)
12 dozen	= 1 gross (gro.)
12 gross	= 1 great gross (gt. gro.)
20 units	= 1 score
24 sheets of paper	= 1 quire
20 quires (480 sheets)	= 1 ream

ORAL EXERCISES

270. Solve:

1. How many eggs in $3\frac{1}{2}$ doz. eggs? In $2\frac{1}{2}$ doz.?
2. How many pairs of scissors in $15\frac{1}{2}$ gro. of scissors? In 26 gt. gro.?
3. How many units in a gross? In a great gross.
4. How many dozen in 1,600 eggs? In 2,400 pencils?
5. At $12\frac{1}{2}$ cts. a dozen, what is a gross of screws worth?
6. If a gross of pens is worth \$1.25, what is a great gross worth?
7. At 2 cts. each, what is a gross of lead pencils worth?
8. When eggs are \$.45 a dozen, what are 3,600 worth?
9. If a gross of pens cost 24 cts., how many can be bought for 2 cts.? For 5 cts.?

If three are sold for a cent, is there a loss or a gain?

10. Mr. A. is 3 score and 10 years old and his wife is 2 score and 15 years old. What is the sum of their ages? How old is each? How much older is Mr. A. than his wife?

11. A man bought 2 doz. oranges for 24 cts. a dozen. He sold them at the rate of 5 cts. for 2. Did he gain or lose, and how much?

12. A man bought $\frac{1}{2}$ gross of drawing pencils at the rate of \$10 a gross. He sold them at 8 cts. each. How much did he gain?

271.

MEASURE OF TIME

60 seconds (sec.)	= 1 minute (min.)
60 minutes	= 1 hour (hr.)
24 hours	= 1 day (da.)
7 days	= 1 week (wk.)
$365\frac{1}{4}$ days	= 1 year (yr.)
12 months (mo.)	= 1 year
100 years	= 1 century

Years are numbered from the year of the birth of Christ. Those before his birth are marked B.C. ("Before Christ"). We say Rome was founded 752 B.C. Years since his birth are marked A.D. (*Anno Domini*, "in the Year of our Lord"). The years since the birth of Christ are called the *Christian Era*.

Years are grouped in *centuries*. The first century of the Christian Era began with the year 1 A.D. and ended with the year 100 A.D. We live in the 20th century, the 19th having ended with the year 1900, and the 20th having begun with the year 1901. It will end with the year 2000. Hence the year-number ending in two ciphers indicates the last year of the century.

The number of days in a month varies: April, June, September, and November have each thirty days. The others, except February, have each 31. February has 28 days, but in leap years 29.

A leap year has 366 days. In leap year the month of February, which usually has 28 days, has 29 days. Every year whose number is exactly divisible by 4 is a leap year, excepting that the last year of a century, as 1900, is a leap year only when divisible by 400. 1824 was a leap year. Why? 1200 was a leap year. Why? Was the year 600? Name the leap years in your own life. Will the year 2000 be a leap year? Why?

ORAL EXERCISES

272. Solve :

1. How many seconds are there in 5 min.? In 10 min.? In 12 min.? In 8 min.? In 30 min.? In 60 min.? In $2\frac{1}{2}$ min.? In $5\frac{1}{4}$ min.? In $10\frac{3}{4}$ min.? In 4.5 min.? In 6.1 min.?

2. How many minutes are there in 240 sec.? In 300 sec.? In 150 sec.? In 180 sec.? In 200 sec.? In 330 sec.?

3. How many hours are there in 2 da.? In 4 da.? In 6 da.? In 3 da.? In 5 da.? In 10 da.? In 20 da.? In 7 da.?

4. A mason is paid \$30 a week for working 6 da. of 8 hr. each. At that rate what should he receive for working 15 hr.? 20 hr.? 25 hr.? $3\frac{1}{2}$ da.? 40 hr.? $4\frac{1}{2}$ hr.? 5 da.?

5. What is the interest on \$600 at 4% for 6 months? 4 months? 8 months? 9 months? 1 year 6 months? 1 year 4 months? 1 year 3 months? 2 years? 2 years 4 months?

6. A man employed two masons to work for him. He agreed to pay one of them 60 cts. an hour, and the other \$20 a week. Which will earn the more money, if they work 24 da. of 8 hr. each? How much more?

7. If a man receives 30 cts. for $\frac{1}{2}$ hr. work, how many dollars will he earn in $5\frac{1}{2}$ da. of 6 hr. each?

8. Weston, the veteran walker, averaged 5 mi. an hour. At this rate how far would he travel in 6 wk. if he walked 10 hr. a day for 6 da. in the week?

9. How many minutes are there from Friday noon until 8 o'clock Monday morning?

10. If Washington had lived until to-day, how old would he be?

REDUCTION

273. Changing a denominate number to a number of a different denomination is called **reduction**.

274. Reduce 5 gallons, 3 quarts, 1 pint, 3 gills to gills.

SOLUTION :

$$\begin{array}{r}
 5 \\
 \times 4 \\
 \hline
 20 \\
 + 3 \\
 \hline
 23 \\
 \times 2 \\
 \hline
 46 \\
 + 1 \\
 \hline
 47 \\
 \times 4 \\
 \hline
 188 \\
 + 3 \\
 \hline
 191
 \end{array}$$

$$5 \text{ gal.} = 20 \text{ qt.}$$

$$20 \text{ qt.} + 3 \text{ qt.} = 23 \text{ qt.}$$

$$23 \text{ qt.} = 46 \text{ pt.}$$

$$46 \text{ pt.} + 1 \text{ pt.} = 47 \text{ pt.}$$

$$47 \text{ pt.} = 188 \text{ gi.}$$

$$188 \text{ gi.} + 3 \text{ gi.} = 191 \text{ gi.}$$

$$5 \text{ gallons, } 3 \text{ quarts, } 1 \text{ pint, } 3 \text{ gills} = 191 \text{ gills.}$$

The process of reducing a denominate number to one of lower denomination is called **reduction descending**.

To reduce a compound denominate number to one of lower denomination, beginning with the highest denomination, multiply each number by its ratio to the next lower denomination and add to the product the number of units given of the lower denomination. Continue until the denomination required is reached.

WRITTEN EXERCISES

275. Reduce:

1. 36 gal. 3 qt. 1 pt. 3 gi. to gills.
2. $12\frac{3}{4}$ gal. to gills.
3. $25\frac{1}{2}$ qt. to gills.
4. 5 bu. 3 pk. 7 qt. to quarts.

5. 24 bu. 2 pk. 5 qt. to quarts.
6. 75 bu. 0 pk. 6 qt. to quarts.
7. 38 bu. 3 pk. to quarts.
8. 47 bu. to quarts.
9. $\frac{3}{8}$ sq. mi. to acres.
10. $\frac{5}{16}$ A. to square rods.
11. $\frac{7}{32}$ A. to square rods.
12. $\frac{3}{4}$ sq. rd. to square yards; to square inches.
13. 25.5 sq. mi. to acres.
14. $38\frac{1}{2}$ A. to square rods.
15. 48 sq. rd. to square yards.
16. 14.25 sq. yd. to square feet.
17. 15 A. 24 sq. rd. 20 sq. yd. to square yards.
18. 3 T. 15 cwt. to ounces.
19. 20 lb. 12 oz. to ounces.
20. 36 T. to ounces.
21. 15 da. 8 hr. 15 min. 30 sec. to seconds.
22. 2 yr. 80 da. 9 hr. 10 min. 20 sec. to seconds.
23. By one multiplication, 12.25 mi. to feet.
24. By one multiplication, 36.5 mi. to yards.
25. By one multiplication, $76\frac{1}{2}$ mi. to yards.

276. Find and memorize :

1. 1 mi. = how many feet? How many yards? How many rods?
2. 1 acre = how many square rods?
3. 1 bu. = how many quarts? How many pints?
4. 1 gal. = how many pints? How many quarts?
5. 1 cu. yd. = how many cubic inches?
6. 1 day = how many minutes?
7. 1 sq. mi. = how many square rods?
8. 1 gro. = how many ones?

WRITTEN EXERCISES

277. Solve:

1. What is the value of 15 gal. 3 qt. 1 pt. 2 gi. of molasses, at 2 cts. per gill?

2. Find the cost of 16 bu. 1 pk. 3 qt. of beans, at 8 cts. per quart.

3. Find the cost of 36 bu. 3 pk. 5 qt. of chestnuts, at 6 cts. per quart.

4. At $12\frac{1}{2}$ cts. a half peck, find the cost of 16 bu. 3 pk. of potatoes. Of 12 bu. 2 pk. Of 75 bu. 1 pk.

5. What is the length of one side of a square rod? How many square feet in a square rod? In 24 sq. rd.? In 36 sq. rd.?

6. What is the value of 8 T. 4 cwt. 35 lb. of iron ore, at $3\frac{1}{2}$ cts. a pound?

7. Fast trains go from New York to Chicago in 18 hours. In how many minutes? In how many seconds?

8. Mr. Bleriot's airship crossed the Channel from Calais to Dover, a distance of 23 miles, at the rate of 45 miles an hour. How many minutes was it in crossing?

9. An Indian reservation opened for settlement was 100×150 mi. square. How many 160-acre farms did it make?

10. How many cubic feet of wood are there in a pile containing 180 cords?

11. The principal of a school having an enrollment of 720 pupils ordered 50 gross of pens. How many pens was that per pupil?

12. If a mason's wages are 1 ct. a minute, how much will he earn in 4 da. 5 hr. 40 min., working 8 hr. a day?

13. Mr. Hudson sold 2 A. and 40 sq. rd. of land, at \$1.25 per square rod. How much did he receive for it?

278. 1. In 191 gills, how many pints, quarts, and gallons are there?

SOLUTION	4	191 gi.		
	2	47 pt. + 3 gi.	In 191 gi. there are 47 pt. and	
	4	23 qt. + 1 pt.	8 gi. In 47 pt. there are 23 qt.	
		5 gal. + 3 qt.	and 1 pt. In 23 qt. there are 5 gal.	

191 gills = 5 gallons, 3 quarts, 1 pint, 3 gills.

Changing a denominate number to one of higher denomination is called **reduction ascending**.

To reduce a simple denominate number to one of higher denominations, divide the number of units of the given denomination by its ratio to the next higher denomination. Divide the quotient in like manner, and continue until the highest denomination is reached, or until no further division is possible. The remainder after each division will be of the same denomination as the dividend.

WRITTEN EXERCISES

279. Reduce to integers of higher denominations and prove:

- | | | |
|-----------------|------------------|--------------------|
| 1. 4,286 gills. | 5. 3,764 quarts. | 9. 521,488 inches. |
| 2. 2,875 " | 6. 7,640 " | 10. 782,232 " |
| 3. 3,127 " | 7. 18,854 " | 11. 391,116 " |
| 4. 5,284 " | 8. 13,764 " | 12. 1,564,464 " |

280. Solve:

- What is the cost of 3 bu. 1 pk. 4 qt. of potatoes, at \$.80 per bushel?
- A farmer sold 5 bu. 3 pk. 6 qt. of barley, at \$.64 a bushel. How much did he receive for it?
- What must be paid for 8 bu. 1 pk. 4 qt. of onions, at \$1.75 per bushel?
- What is the ratio of the number of feet in a mile to the number of yards in a mile? To the number of feet in a rod? To the number of feet in a yard?

5. Change 1,105 sq. yd. to square rods.

SUGGESTION: The divisor, $30\frac{1}{4} = 121$ fourths, and the dividend, 1,105 sq. yd. = 4,420 fourths of a square yard; therefore, after dividing, the remainder is fourths of a square yard.

6. How many square rods in 2,311 sq. yd.? In 4,644 sq. yd.? In 9,828 sq. yd.?

7. How many acres in 20,464 sq. yd.? In 61,392 sq. yd.? In 30,696 sq. yd.? In 15,248 sq. yd.?

8. What is the value of 3 A. 80 sq. rd. of land, at \$850 an acre?

9. A plot of land 40 rd. square was sold for \$1,750 per acre. How much was received for it?

10. Mr. Frost sold 4 T. 10 cwt. of hay for \$24 a ton. How much did he receive?

11. How many years are there in 2,150,400 minutes?

12. How many seconds are there in a day?

13. The income of a certain rich man is said to be \$1 a minute. How much is it a year?

14. Draw a diagram 5×6 in. If the scale is 1 in. to 12 rd., how large a field will the diagram represent?

15. The potato crop raised on the field averages 120 bu. per acre. How many bushels does the field produce?

16. The owner kept 50 bu. for his own use and sold the remainder in bags, each bag holding 2 bu. 3 pk. How many bags of potatoes did he sell?

17. He sold them at \$1.75 per bag. How much per bushel did he receive for them?

18. What was the entire crop worth at that rate?

19. If the seed cost \$4 per acre and the ploughing \$3 per acre, and if the other expenses were \$25, how much did the value of the potatoes exceed the expenses?

REDUCTION, USING DECIMALS

281. 1 quart, 1 pint equals $1\frac{1}{2}$ quarts. Write it as a decimal.

Change 2 quarts, 1 pint, 3 gills to the decimal of a gallon.

SOLUTION: $4 \overline{) 3.00}$ gi.

3 gi. = .75 pt. ; .75 pt. + 1 pt. = 1.75 pt.

.75

$2 \overline{) 1.750}$ pt.

1.75 pt. = .875 qt. ; .875 qt. + 2 qt. =

.875

2.875 qt.

$4 \overline{) 2.87500}$ qt.

2.875 qt. = .71875 gal.

.71875

It is customary not to carry the division beyond three places, but to use for the last figure, if the next figure is larger than 5, the next higher figure. Thus, the last quotient above would be written .719.

To reduce a compound denominate number to a simple number of higher denomination, arrange the numbers vertically with the lowest denominations at the top ; then, in order, divide each by the number required to make the next higher denomination, and add the quotient to the number of given units of that denomination.

WRITTEN EXERCISES

282. Solve:

1. Change 2 gal. 1 qt. 3 gi. to a decimal of a gallon. Prove the result.

2. Change 8 gal. 1 qt. 1 gi. to a decimal of a gallon. Prove the result.

3. What is the value of 6 gal. 1 qt. 1 pt. of vinegar, at 35 cts. a gallon.

4. What is the value of 35 gal. 2 qt. 1 pt. of syrup, at $\frac{3}{4}$ of a dollar a gallon?

5. What is the value of 8 gal. 3 qt. 1 pt. 2 gi. of alcohol, at \$3.75 per gallon?

6. Change to a decimal of a mile 150 rd. 5 yd. 2 ft. 3 in.

7. Change 70 A. 24 sq. yd. 6 sq. ft. to a decimal of a square mile.

SUGGESTION. In changing a compound denominate number to a decimal of a higher denomination, if the divisor contains a fraction write it as a decimal; as 30.25 for $30\frac{1}{4}$.

8. Change to a decimal of a ton 8 cwt. 14 lb. 4 oz.

9. What decimal of a day do 18 hr. 40 min. 30 sec. equal?

10. Mr. Beers sold 2 T. 8 cwt. 25 lb. of hay, at \$18.50 per ton. How much did he receive for the hay?

11. Mr. Sales sold 18 bu. 3 pk. 4 qt. of cranberries, at \$1.75 per bushel. How much did he receive?

12. Mr. Rhodes sold 16 gal. 3 qt. 1 pt. of vinegar, at 65 cts. a gallon. What did he receive for the vinegar?

13. A man's salary is \$3,600 a year. How much is it for 2 yr. 6 mo. and 15 da.?

283. Change .71875 gallons to a compound denominate number.

SOLUTION: .71875

$$\begin{array}{r} 4 \\ \hline 2.87500 \end{array}$$

$$4 \text{ qt.} \times .71875 = 2.875 \text{ qt.}$$

$$.875$$

$$\begin{array}{r} 2 \\ \hline 1.750 \end{array}$$

$$2 \text{ pt.} \times .875 = 1.75 \text{ pt.}$$

$$.75$$

$$\begin{array}{r} 4 \\ \hline 3.00 \end{array}$$

$$4 \text{ gi.} \times .75 = 3 \text{ gi.}$$

.71875 gallons = 2 quarts, 1 pint, 3 gills.

To change a decimal of a compound denominate number to one or more integers of a lower denomination, multiply by the ratio of that denomination to the next lower, and point off as in multiplication of decimals. Then reduce the decimal of the product to the next lower denomination.

ORAL EXERCISES

284. Solve:

1. How many feet in .75 of a yard? How many in 2.25 yd.?

2. Change the following fractions to decimals and change the decimals to integers of lower denominations: $\frac{3}{4}$ gal.; $\frac{2}{3}$ pk.; $\frac{5}{8}$ mi.; $\frac{7}{8}$ hr.; $\frac{1}{16}$ A.; $\frac{5}{16}$ T.; $\frac{1}{16}$ cu. ft.; $\frac{1}{16}$ cord.

3. How many minutes in $33\frac{1}{3}\%$ of an hour? In 125% of an hour?

4. How many ounces in $\frac{3}{8}$ of a pound? In $\frac{9}{16}$ of a pound? In $\frac{3}{4}$ of a pound?

5. Change the following to integers of lower denominations: $\frac{2}{3}$ yd.; $\frac{1}{2}$ bu.; $\frac{7}{12}$ hr.; $\frac{5}{8}$ T.

WRITTEN EXERCISES

285. Solve:

1. Express $\frac{7}{16}$ miles as rods.

2. Express $\frac{9}{16}$ miles as feet.

3. How many feet are there in 8% of a mile?

4. How many yards are there in 26% of a mile?

5. How many inches in 35% of a mile?

6. Reduce to integers of lower denominations: .6825 sq. rd.; .8455 A.; .4265 sq. mi.

7. Change the following decimals to integers of lower denominations; .307875 of a ton; .80625 of a ton; .407125 of a ton; .7925 of a ton.

8. Change to a lower denomination: .365 days; .730 days; .366 days.

9. Change to integers of lower denominations: .476 days; .3225 lb.; .24125 rd.; .296 hours.

10. Change to integers of lower denominations: $\frac{4}{5}$ cu. ft.; $\frac{1}{2}$ mi.; $\frac{3}{8}$ A.

ADDITION

286. Add: 4 gal. 3 qt. 1 pt. 3 gi.
 6 gal. 1 qt. 2 gi.
 3 gal. 2 qt. 1 pt. 2 gi.

Arrange as below, with like denominations in columns, and add, beginning at the right:

SOLUTION:	GAL.	QT.	PT.	GI.
	4	3	1	3
	6	1	0	2
	3	2	1	2
	14	3	1	3

$$3 \text{ gi.} + 2 \text{ gi.} + 2 \text{ gi.} = 7 \text{ gi.} = 1 \text{ pt.} + 3 \text{ gi.}$$

Add the 1 pt. to pints' column.

$$1 \text{ pt.} + 0 \text{ pt.} + 1 \text{ pt.} + 1 \text{ pt.} = 3 \text{ pt.} = 1 \text{ qt.} + 1 \text{ pt.}$$

Add the 1 qt. to quarts' column.

$$3 \text{ qt.} + 1 \text{ qt.} + 2 \text{ qt.} + 1 \text{ qt.} = 7 \text{ qt.} = 1 \text{ gal.} + 3 \text{ qt.}$$

Add the 1 gal. to gallons' column.

$$4 \text{ gal.} + 6 \text{ gal.} + 3 \text{ gal.} + 1 \text{ gal.} = 14 \text{ gal.}$$

WRITTEN EXERCISES

287. Find the sum of:

1. 2 mi. 36 rd. 12 ft., 15 mi. 48 rd. 15.5 ft., 21 mi. 6.25 ft.

2. 36 gal. 0 qt. 1 pt. 0 gi., 18 gal. 2 qt. 0 pt. 1 gi., 12 gal. 3 qt. 1 pt. 3 gi.

3. 20 lb. 1 oz., 15 lb. 3 oz., 1 T. 2 lb. 15 oz., 16 T. 3 cwt.

288. Instead of adding compound numbers as shown above, it is much more usual first to reduce each addend to one denomination, using decimals or common fractions.

For example: Add 25 lb. 8 oz. and 30 lb. 10 oz.

SOLUTION: 25 lb. 8 oz. = $25\frac{1}{2}$ lb.

30 lb. 10 oz. = $30\frac{5}{8}$ lb.

$56\frac{1}{8}$ lb. = 56 lb. 2 oz.

WRITTEN EXERCISES

289. Solve, using either method :

1. 80 mi. 150 rd. 12 ft. 10 in. + 36 mi. 200 rd. 13 ft. 9 in. + 75 mi. 65 rd. 8 ft. 11 in.

2. 15 sq. yd. 8 sq. ft. 36 sq. in. + 21 sq. yd. 6 sq. ft. 94 sq. in. + 18 sq. yd. 3 sq. ft. 78 sq. in.

3. 18 cu. yd. 12 cu. ft. 450 cu. in. + 14 cu. yd. 16 cu. ft. 1200 cu. in. + 20 cu. yd. 21 cu. ft. 800 cu. in.

4. 8 T. 15 cwt. 35 lb. 14 oz. + 12 T. 5 cwt. 65 lb. 8 oz. + 23 T. 18 cwt. 45 lb. 12 oz.

5. 65 da. 15 hr. 45 min. 15 sec. + 35 da. 18 hr. 30 min. 45 sec. + 75 da. 22 hr. 50 min. 30 sec. + 28 da. 12 hr. 15 min. 25 sec.

6. Mr. and Mrs. Rand have three children: Minnie, Jerome, and baby Lottie. Mr. Rand weighs 175 lb. 8 oz.; Mrs. Rand weighs $137\frac{3}{4}$ lb.; Minnie weighs 105 lb. 12 oz.; Jerome weighs $92\frac{1}{2}$ lb.; and Lottie weighs 18 lb. The whole family was weighed together on a platform scale. What was the total weight?

7. Mr. Rand's age is 46 yr. 8 mo. 12 da.; Mrs. Rand's, 40 yr. 11 mo. 6 da.; Minnie's, 14 yr. 3 mo. 15 da.; Jerome's, 11 yr. 10 mo. 3 da.; the baby's 1 yr. 7 mo. 9 da. What is the sum of all their ages?

8. William, Thomas, Henry, Cyrus, and George each made a standing jump. William jumped 8 ft. 1 in.; Thomas, 7 ft. $11\frac{1}{2}$ in.; Henry, 7 ft. 6 in.; Cyrus, 6 ft. $10\frac{1}{2}$ in.; George, 6 ft. 8. in. What is the sum of the distances jumped by the five boys?

9. One day in October the boys went chestnutting. William gathered 1 bu. 1 qt.; Thomas, $3\frac{1}{2}$ pk.; Henry, $\frac{1}{2}$ bu.; Cyrus, 3 pk. 4 qt.; and George, 2 pk. 2 qt. How many did they all gather?

10. They sold the nuts for 5 cts. per quart. How much did each boy receive?

11. Mr. Fairchilds has a dairy. He bottles the milk and sells it at the following prices: sterilized milk, quart bottles \$.15, pint bottles \$.08; milk not sterilized, \$.10 per quart, \$.05 per pint. He delivers, in Newark, on one block, every morning, as follows: No. 125, 2 qt. 1 pt. sterilized milk; No. 27, 3 qt. not sterilized; No. 128, 1 qt. 1 pt. sterilized; No. 129, 4 qt. 1 pt. not sterilized; No. 132, 3 qt. 1 pt. not sterilized. How much milk does he deliver on the block?

12. How much money does he collect weekly from each house? From the five houses?

13. Mr. King sold in one day 3 cwt. 30 lb. 8 oz. of beef; 1 cwt. 80 lb. 9 oz. of pork; 75 lb. 4 oz. of mutton; 3 cwt. 40 lb. of veal. How much meat did he sell?

14. Isadore gathered 2 bu. 1 pk. 4 qt. of chestnuts; Benjamin, 3 pk. 6 qt. 1 pt.; Samuel, 2 bu. 3 qt.; and Reuben, 1 bu. 5 qt. 1 pt. How many chestnuts did they all gather?

15. An oil dealer sold 9 gal. 2 qt. 1 pt. of oil to one customer; 4 qt. 1 pt. to a second customer; 14 gal. 5 qt. 3 pt. to another; and 6 gal. 3 qt. 2 pt. to a fourth. How much oil did the dealer sell to all four customers?

16. From a piece of ribbon was sold the following: 2 yd. 15 ft. 9 in.; 1 yd. 2 ft. 5 in.; 3 yd. 2 ft. 11 in.; 4 yd. 3 ft. 7 in. How much was sold from the piece?

17. A merchant sold to one customer 3 yd. 2 ft. 9 in. of cloth; to another 5 yd. 1 ft. 11 in.; to another 7 yd. 1 ft. 8 in.; to another 9 yd. 2 ft. 5 in. How many did he sell in all?

18. The first day out at sea, a ship traveled 112 mi. 13 rd. 2 yd. 1 ft.; the next day it went 246 mi. 3 yd. 2 ft. 6 in.; the third day, 296 mi. 6 rd. 1 ft. 8 in.; and the fourth day, 332 mi. 5 rd. 2 ft. 11 in. How far did it go in the four days?

SUBTRACTION

290. Subtract 10 gal. 3 qt. 1 pt. from 25 gal. 2 qt. 1 pt.

GAL.	Qt.	Pr.
25 (24 gal. 4 qt.)	2	1
10	3	1
14	3	0

3 qt. cannot be taken from 2 qt. Therefore we "borrow" 1 of the 25 gal. and change it to quarts, making 24 gal. 4 qt.

Adding the 4 qt. to the 2 qt. gives 6 qt. 6 qt. - 3 qt. = 3 qt.

24 gal. - 10 gal. = 14 gal.

WRITTEN EXERCISES

291. Subtract:

	Bu.	Pk.	Qt.	Pr.
1. From	14	3	1	3
Take	3	2	1	2

2. From the remainder in Exercise 1 take 6 bu. 1 pk. 0 qt. 2 pt.

	GAL.	Qt.	Pr.	Gr.
(1)	8	2	1	2
(2)	5	2	0	2
(3)	7	1	1	2
(4)	4	3	1	1
(5)				
	26	2	0	3

3. Test the addition above by subtracting (1) from (5); (2) from the remainder, (3) from this remainder. If the last remainder equals (4), the addition is correct. Tell why.

292. Subtract 20 gal. 1 qt. 1 pt. from 40 gal. 3 qt. 1 pt.

SOLUTION: 40 gal. 3 qt. 1 pt. = 40.875 gal.

20 gal. 1 qt. 1 pt. = 20.375 gal.

25.5 gal. = 20 gal. 2 qt.

293. In the following reduce the lower denominations to common or decimal fractions of the highest denomination before subtracting:

1. Mr. Ray had 75 gal. 3 qt. of molasses on Monday morning. During the day he sold 8 gal. 2 qt. 1 pt. On Tuesday he sold 9 gal. 3 qt.; on Wednesday, 12 gal. 1 pt.; on Thursday, 6 gal. 3 qt. 1 pt.; on Friday, 10 gal.; and on Saturday, 13 gal. 2 qt. How much had he left?

2. Mr. Travis had a tank containing 156.375 gal. of oil. He sold on an average 18 gal. 2 qt. 1 pt. a day. If the tank was full Monday morning, how much was left Saturday night?

3. Mr. Redding bought 45 bu. of potatoes at \$.72 a bushel and sold them for \$.03 $\frac{1}{2}$ a quart. How much did he make?

4. On the north field of his farm, Mr. Howard raised 215 bu. 3 pk. 6 qt. of tomatoes; on the east field, 175 bu. 2 pk. 4 qt. of potatoes; on the south field, 150 bu. 3 pk. of corn; and on the west field, 125 bu. 7 qt. of buckwheat. How many bushels of produce did he raise on the four fields?

5. Mr. Thatcher bought 25 A. of land and sold from it 6 building lots 50 ft. by 100 ft., and 16 lots 100 ft. by 100 ft. How many square feet remained unsold?

6. Mr. Shearer bought 18 cwt. 75 lb. of butter, at \$18.75 a cwt. What did it cost him? He sold it for 25 cts. a pound. How much did he make?

7. Mr. Frost bought 25 $\frac{1}{2}$ T. of hay for \$21.50 a ton, and retailed it in small quantities at \$1.50 a cwt. How much did he make?

8. Mr. Luke bought a tub of butter containing 56 $\frac{1}{2}$ lb. He paid \$.28 a pound. He sold 24 lb. 8 oz. for \$.32 a pound and the remainder for \$.35 a pound. How much did he make?

9. He bought a barrel of molasses containing $45\frac{1}{2}$ gal., at \$.64 per gallon. He sold 25 gal. 2 qt. at \$.70 per gallon and the remainder for \$.60 per gallon. How much did he make on the molasses?

10. Mr. Harris bought 12 T. 10 cwt. of hay for \$18.50 per ton. He sold $5\frac{1}{2}$ T. for \$21.25 per ton and the remainder for \$24.75 per ton. How much did he make?

11. Mr. Perine bought 25 bu. 2 pk. of peanuts at \$2.15 per bushel. He sold 9 bu. 16 qt. at \$2.50 per bushel and the remainder at \$2.40 per bushel. How much did he make?

12. In a plot of land 80 rd. by 40 rd., how many schools can each have a garden 120 ft. by 22 ft.?

13. A man bought $\frac{1}{2}$ A. of land. He sold $\frac{1}{2}$ of what he bought. How many square rods did he sell? What part of an acre remained?

14. If 16 lb. 8 oz. of butter cost 16 cts. a pound, for how much must I sell it to gain $\frac{1}{2}$ cent an ounce? How many pounds must I sell to gain \$4?

15. If one half quart of milk cost 3 cts., what will 4 gal. 3 qts. and 1 pt. cost?

16. It takes $5\frac{3}{4}$ hr. to go from New York to Boston by one train and 5 hr. 15 min. by another. What is the difference in minutes?

FINDING THE TIME BETWEEN TWO DATES

294. What months have 30 days each? What months have 31 days? What month has 28 days? When does it have 29 days?

Find the number of days from March 20th to June 18th.

SOLUTION: There are 11 da. in March after the 20th, 30 da. in April, 31 in May, and 18 in June. These make 72 days.

To find the exact number of days between two dates, add all the days, beginning with that following the first given and including the last name.

WRITTEN EXERCISES

295. Find the exact number of days between the following dates :

1. January 16, 1903, and July 4, 1903.
2. February 24, 1896, and August 14, 1906.
3. April 15, 1900, and November 16, 1900.
4. September 16, 1902, and July 22, 1903.
5. November 20, 1903, and January 5, 1904.

296. We call 30 days a month in finding the difference between two dates more than one year apart.

The months are indicated by numbers. Beginning with January, the 1st month, October is the 10th month, July the 7th.

Find the time, in years, months, and days, between October 21, 1492, and July 4, 1903.

Yr.	Mo.	Da.		Yr.	Mo.	Da.
1903	7	4	=	1902	18	34
1492	10	21		1492	10	21
410	8	13		410	8	13

WRITTEN EXERCISES

297. Solve:

1. Money borrowed August 10, 1900, was paid April 1, 1903. How long had the money been loaned?

2. What was the date of your birth? How long ago was that?

3. Henry sailed for Europe June 21, 1906. He returned to America September 3 of the same year. How many months and weeks was he away?

4. Fritz came to America from Europe April 8, 1902, and Antonio came May 29, 1906. How much longer has Fritz been in America than Antonio?

MULTIPLICATION

298. Multiply 12 gal. 3 qt. 1 pt. 3 gi. by 6.

	GAL.	Qt.	Pt.	Gi.
SOLUTION:	12	3	1	3
				6
	77	3	0	2

3 gi. $\times 6 = 18$ gi. = 4 pt. 2 gi. Write the 2 gi. under gills' column, and add the 4 pt. to the pints.

1 pt. $\times 6 = 6$ pt. 6 pt. + 4 pt. = 10 pt. = 5 qt. Add the 5 qt. to the quarts.

3 qt. $\times 6 = 18$ qt. 18 qt. + 5 qt. = 23 qt. = 5 gal. 3 qt. Write 3 qt. under quarts' column, and add the 5 gal. to the gallons.

12 gal. $\times 6 = 72$ gal. 72 gal. + 5 gal. = 77 gal.

To multiply a compound denominate number, multiply the numbers in order, beginning with the lowest denomination, and reduce each product to higher denominations, when possible.

WRITTEN EXERCISES

299. Solve:

1. A milkman sells 35 gal. 3 qt. of milk daily. How much does he sell in a week? If he pays $3\frac{1}{2}$ cts. per quart for the milk and 3 cts. a quart expenses, how much does the milk cost him? If he sells it at 8 cts. a quart, how much does he make?

2. Mr. Randall's average daily sale of vinegar is 6 gal. 2 qt. 1 pt. The cost is 27 cts. a gallon, the selling price 35 cts. What did he make in a week?

300. In the following problems, before multiplying, reduce the lower denominations to decimal or common fractions of the highest denomination:

1. Mr. Ames had 15 cans of milk, each containing 10 gal. 3 qt. 1 pt. He sold the milk in bottles at 6 cts. a pint. How much did he receive?

2. The length of an oblong tank is 4 ft. 6 in. ; the width, 3.25 ft., and the depth $2\frac{3}{4}$ ft. How many gallons will it hold ?

(1 gallon contains 231 cubic inches.)

3. How many gallons will a similar tank hold whose length is $6.83\frac{1}{2}$ ft. ; width, $5\frac{3}{4}$ ft., and depth, 4 ft. 9 in.

4. Mr. Leonard has 3 grain bins. The one filled with shelled corn is 10 ft. long, 8 ft. high, and 6 ft. wide. The one filled with wheat is 12.5 ft. long, 6 ft. high, and 5 ft. wide. The one filled with barley is 9 ft. long, 7 ft. high, and 4 ft. wide. How many bushels of each kind of grain are there in the bins ?

(There are 2,150.44 cubic inches in a bushel.)

5. The two adjacent sides of a rectangular lot are 13 yd. 2 ft. 9 in., and 18 yd. 1 ft. 10 in. What is the perimeter of the lot ?

6. The side of an equilateral triangle is 40 yd. 12 ft. 9 in. What is the perimeter of the triangle ?

7. The base of a triangle is 6 yd. 2 ft. 8 in. Each of the other sides is 8 yd. 1 ft. 6 in. What is the perimeter of the triangle ?

8. How much does the sum of the sides of the triangle (Exercise 7) exceed the base ?

9. There are approximately 7.5 gallons in a cubic foot of water. Find the approximate number of gallons that the following cisterns will hold :

	LENGTH	WIDTH	DEPTH
(1)	8 ft.	6 ft.	10 ft.
(2)	10 ft.	8 ft.	12 ft.
(3)	8 ft. 6 in.	8 ft.	9 ft.
(4)	8 ft. 3 in.	6 ft.	11 ft.
(5)	9 ft. 9 in.	8 ft.	10 ft.

10. Mr. Moore purchased 5 pieces of silk, each containing 10 yd. 1 ft. 6 in. How much did he purchase? He sold 2 dress patterns of 15 yd. 1 ft. 6 in. each. How much had he left?

11. What is the value of 5 cu. yd. of marble, at \$2.25 a cubic foot?

12. How many cubic yards in a wall 36 ft. long, 2 ft. thick, and 12 ft. high?

13. How many cubic yards in a wall 24 ft. long, 1 ft. 6 in. thick, and 16 ft. high?

14. Find the cubic feet in the walls of buildings with dimensions as follows:

	LENGTH	WIDTH	THICKNESS OF WALLS	HEIGHT
(1)	40 ft.	24 ft.	3 ft.	19 ft.
(2)	36 ft.	22 ft.	3 ft.	16 ft.
(3)	42 ft.	34 ft.	2 ft. 6 in.	21 ft.

15. How many cubic inches are there in 8 gal. 3 qt. 1 pt. 2 gi.?

16. The length of a square cistern is 8 ft. 6 in.; the depth, 12 ft. How many gallons of water will it contain?

17. How many gallons will a similar cistern hold, whose length is 9 ft. 3 in. and depth 10 ft.?

18. A cube is 6 ft. on a side. Another is 2 ft. on a side. How many cubic feet does each contain? How many times greater is the larger one than the smaller one?

19. A water tank is 10 ft. long, $2\frac{1}{2}$ ft. wide, and 18 in. deep. How many gallons of water will it contain?

20. How many feet thick is a wall 20 ft. long and 8 ft. high if it contains 320 cu. ft.?

21. A cistern contains 400 gal. of water. How deep is the water, if the cistern is 8 ft. long and 4 ft. deep?

DIVISION

301. Divide 36 gal. 1 qt. 1 pt. 1 gi. by 5.

SOLUTION:

$$\begin{array}{r} 7 \quad 1 \quad 0 \quad 1 \\ 5 \overline{) 36 \quad 1 \quad 1 \quad 1} \\ \hline \end{array}$$

36 gal. $\div$ 5 = 7 gal., and 1 gal. not divided. 1 gal. = 4 qt.

4 qt. + 1 qt. = 5 qt. 5 qt. $\div$ 5 = 1 qt.

1 pt. $\div$ 5 = 0, and 1 pt. not divided. 1 pt. = 4 gi.

4 gi. + 1 gi. = 5 gi. 5 gi. $\div$ 5 = 1 gi.

To divide a compound denominate number, divide the numbers in order, beginning with the highest denomination. Write each quotient over the number divided and reduce any remainder to the next lower denomination, adding it to the given number of that denomination.

WRITTEN EXERCISES

302. Multiply, and prove by division:

$$\begin{array}{r} 1. \quad \text{GAL.} \quad \text{QT.} \quad \text{PT.} \quad \text{GI.} \\ 10 \quad 2 \quad 1 \quad 2 \\ \hline \quad \quad \quad 9 \end{array}$$

$$\begin{array}{r} 2. \quad \text{GAL.} \quad \text{QT.} \quad \text{PT.} \quad \text{GI.} \\ 13 \quad 0 \quad 1 \quad 3 \\ \hline \quad \quad \quad 5 \end{array}$$

303. In the following, reduce the compound numbers to one denomination before dividing:

1. A wholesale grocer sold 241 gal. 3 qt. 1 pt. of molasses to 5 customers, each receiving the same quantity. How much did each receive?

2. How many times are 3 qt. 1 pt. contained in 35 gal. 2 qt.?

SUGGESTION: First reduce both compound numbers to pints.

3. A druggist has a barrel of alcohol, containing 45 gallons. If he sells 1 gal. 2 qt. 1 pt. a day, how long will it take him to sell all the oil?

4. A grocer has a cask containing 175.5 gal. of oil. His average daily sale is 6 gal. 3 qt. 1 pt. How long will it take him to sell all the oil?

5. Measure your desk top in inches and find the number of square inches in it. How many dimensions must you measure? Reduce your answer to square feet.

6. A certain car is 50 ft. long and has a seat along each side. Allowing 2 ft. 6 in. seating space for one person, how many persons will the car seat?

7. How many cubic feet of ice are there in a block 3 ft. $\times$ 2 ft. $\times$ $1\frac{1}{2}$ ft.?

8. How many such cakes will an ice house hold that measures, on the inside, 60 ft. $\times$ 40 ft. $\times$ 25 ft.?

9. How many such cakes will an ice wagon box 8 ft. long, 4 ft. high, and 4 ft. wide hold?

10. How many books, averaging $1\frac{1}{4}$ in. in thickness, will a 5-ft. bookshelf hold?

11. A hay press was 12 ft. $\times$ 4 ft. $\times$ 3 ft. How many cubic feet of hay did it contain when full?

12. When pressed, the bale of hay formed was $2\frac{1}{2}$ ft. $\times$ 3 ft. $\times$ 4 ft. How many cubic feet were there in a bale?

13. Mr. Benedict, a dairyman, sends 6 ten-gallon cans of milk to his dealer every day. The dealer puts one half of it into quart bottles and one half into pint bottles. How many bottles of each size does he fill? How much does Mr. Benedict receive for the milk at $3\frac{1}{2}$ cts. a quart?

14. Mr. Wilson built a solid stone wall to support his terrace. The wall was $2\frac{1}{2}$ ft. thick, $3\frac{1}{2}$ ft. high, and containing 350 cu. ft. of stone. How long was the wall?

15. Shepherd ran $\frac{1}{4}$ mi. in 5 sec. How many feet did he run in a second?

16. How many feet of boards 1 ft. wide are required to put a floor on a porch 18 ft. $\times$ 8 ft. 3 in.?

17. A house has 24 windows 3 ft. $\times$ 5 ft. Allowing 3 in. in width and 4 in. in length for each window frame, how many panes of glass 11 in. $\times$ 14 in. are required for the house?

18. A class bought 100 pictures 4 in. $\times$ 8 in. They mounted them on cards, allowing a 3-in. margin on each side. How many square feet of cardboard were used?

19. A kindergarten has 150 blocks, each a 6-inch cube. What must be the cubical contents, in feet, of a closet to hold all the blocks?

20. How long a shelf is required to hold 40 Arithmetics each $\frac{3}{4}$ in. thick?

21. If the Arithmetics are $5\frac{1}{2}$ in. high and $4\frac{1}{2}$ in. wide, what must be the inside dimensions of a box to hold them all?

22. Twenty-five children were riding on a load of hay. The load weighed 1 T. 1250 lb. After the children had all jumped down, the load weighed 1750 lb. What was the average weight of the children?

23.	Bu.	Pk.	Qt.	Pr.
	36	3	5	1
	36	3	5	1
	36	3	5	1
	36	3	5	1
	36	3	5	1
	184	2	3	1

Prove this example by multiplication.

24. Prove Example 23 by division.

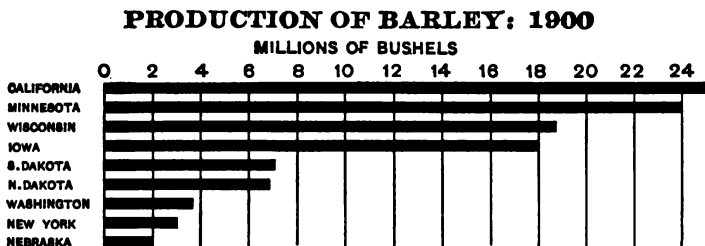
25. Multiply the following, and prove by addition:

	Bu.	Pk.	Qt.	Pr.	
(1)	28	2	6	1	$\times$ 7.
(2)	43	1	7	1	$\times$ 9.
(3)	20	3	3	1	$\times$ 11.

GRAPHS

304. Diagrams, called **graphs**, are sometimes used to show numbers or quantities to be compared with one another.

The following graph shows the comparative amounts of barley that were raised in certain states in 1900.



From this we see that California raised about 25,000,000 bushels.

1. How many bushels did Nebraska raise? How many did Iowa raise?

2. Read the amounts raised by other states named in the graph.

WRITTEN EXERCISE

305. Following is the number of bushels of rye produced in the same year in each of the states named:

Wisconsin	5 $\frac{1}{8}$ millions,
Pennsylvania	4 millions,
New York	2 $\frac{1}{8}$ millions,
Michigan	2 millions,
Nebraska	1 $\frac{5}{8}$ millions,
Minnesota	1 $\frac{3}{4}$ millions.

Make a graph with upright parallel lines, like the one given, representing by heavy horizontal lines the number of bushels of rye produced in each of the states named above.

GENERAL REVIEW

306. Solve:

1. Mr. Low raised 40 bu. 3 pk. 4 qt. of wheat an acre. What was the yield on 1 sq. rd.?

2. Mr. Ward bought 6 T. of butter on Monday. He sold 3 cwt. 15 lb. 4 oz. on Tuesday; twice as much on Wednesday; three times as much on Thursday as he sold on Wednesday; one half as much on Friday as he sold on Thursday; and as much on Saturday as he sold on the four days previous. How much butter had he left?

3. Mr. Latham does piecework, by which he earns an average of \$2.50 for 10 hr. work. His time for the first week in May was: Monday, 9 hr. 15 min.; Tuesday, 8 hr. 40 min.; Wednesday, 6 hr. 50 min.; Thursday, 12 hr. 10 min.; Friday, 11 hr. 5 min.; Saturday, 8 hr. How much did he earn during that week?

4. How many board feet are there in 12 beams, 24 ft. long, and 6 in. by 5 in.?

5. A corridor 40 feet long and 8 ft. wide is paved with tiles 8 in. by 6 in. How many tiles are there in the corridor?

6. What is the ratio of an area 40 rd. square to one of 40 sq. rd.?

7. A farmer sold 5 loads of hay, each weighing 18 cwt. 75 lb., at \$30 per ton. How much did he receive for it?

8. Mr. Reed has a room 15 ft. long and 12 ft. wide. The wainscoting around the room is 3 ft. high. At 17 cts. a square foot, what is the cost of the wainscoting? Illustrate by drawing.

9. How many steps, each 2 ft. 6 in., will a man take in going around a lot 5 rd. square?

10. John rode his bicycle $55\frac{3}{4}$ mi. in one day, and $1\frac{1}{4}$ times as far the next day. How far did he ride in both days?

11. If 16 suits of clothing can be made from 95 yd. of cloth, how many yards will be needed for 32 suits?

12. What is the ratio of 240 lb. to 2,000 lb.?

13. What per cent. does a coal dealer gain who buys 2,240 lb. for a ton and sells 2,000 lb. for a ton?

14. How many tons of 2,000 lb. each in 120 long tons? In 150 long tons? In 125.5 long tons?

15. Mr. Brewster bought 240 T. of pea coal, 2,240 lb. each, at \$3.25 per ton, and sold it at \$3.75 per ton of 2,000 lb. How much did he make?

16. He bought 200 T. of nut coal, 2,240 lb. each, at \$4.25 per ton, and sold it at \$5 per ton of 2,000 lb. What was his profit?

17. He also bought 150 T. of stove coal, 2,240 lb. each, at \$4.60 per ton, and sold it at \$5.25 per ton of 2,000 lb. What was his profit?

18. Eight men bought a mow of hay containing 35 T., each paying the same. How many tons, hundredweights, and pounds in each man's share?

19. In a regiment of 1,000 men, each man is allowed 3 oz. of butter per day. How much butter will supply the regiment for 30 da.?

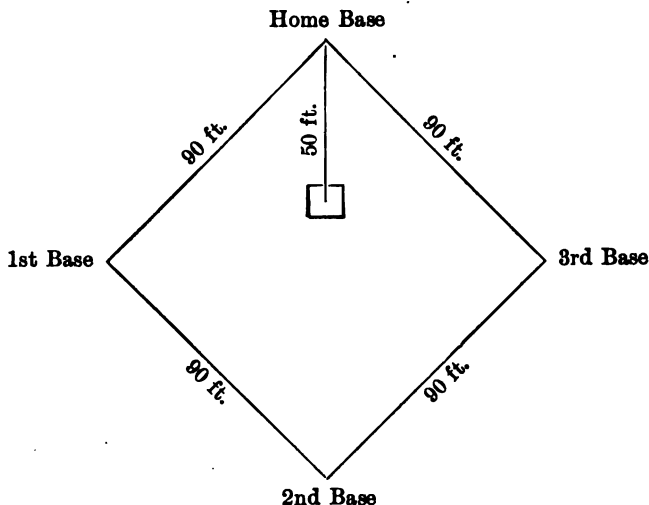
20. How many tons in 4,000 lb.? In 8,000 lb.? In 16,000 lb.? In 8,600 lb.? In 12,800 lb.? In 10,468 lb.? In 6,342 lb.?

21. Mr. Putnam bought 7,500 lb. of hay at \$15.75 per ton. What did he pay for it?

22. Mr. Russell sold 8,250 lb. of rye straw for \$14.25 per ton. How much did he receive for it?

23. In the fall of 1902, anthracite coal was sold for \$20 per ton. How much did 10,500 lb. cost?

24. The figure below represents a baseball "diamond." The bases are 90 ft. apart. How many yards apart are they?



25. The pitcher's position is 50 ft. from the home plate. How many yards from the home plate is it? (Use decimals.)

26. The diagonal of the diamond is approximately 127 ft. 3 in. How many yards and feet does it measure?

27. Jack made a home run. How many feet did he run?

28. Harry made a 3-base hit, Ray a 2-base hit, and Oakley a 1-base hit. They were all put out before going farther. What was the combined distance that the three ran? The distance that Oakley ran was what per cent. of the combined distance that Harry and Ray ran?

29. Patrick made three home runs in one game. How many feet less than a mile did he run?

30. Ole was at the bat three times. 8 strikes were called on him. What per cent. of his chances did he make?

31. Tony was at bat 4 times, three balls being pitched to him each time. He hit the last ball of the second and fourth times. What per cent. of his chances did he miss?

32. Ole batted the ball to the pitcher, who caught it and threw it to the second baseman. The second baseman caught it and put Tom, the runner, out, and then threw it to the catcher at the home plate. How far did the pitcher throw the ball? How many feet and inches was the ball thrown in this play?

33. How many square feet are there in the diamond? How many square yards? How many square rods?

34. A roll of wall paper is 24 ft. long and 1 ft. 6 in. wide. What will it cost to paper the walls and ceiling of a room 12 ft. long, 9 ft. wide, and 8 ft. high, with paper costing 15 cts. a roll, if it costs 20 cts. a roll to hang it and no allowance is made for matching?

35. Find the cost of papering rooms with the following dimensions at the costs given :

LENGTH	WIDTH	HEIGHT	COST PER ROLL	COST OF HANGING
(1) 18 ft.	12 ft.	9 ft.	20 ct.	25 cts.
(2) 15 ft.	10 ft.	8 ft.	35 ct.	25 cts.
(3) 24 ft.	18 ft.	10 ft.	50 ct.	25 cts.
(4) 20 ft.	15 ft.	9 ft.	25 ct.	20 cts.
(5) 21 ft.	16 ft.	9 ft.	15 ct.	20 cts.

36. Find the cost of covering floors of the following dimensions, with linoleum, at 75 cts. per square yard.

LENGTH	WIDTH
(1) 21 ft.	15 ft.
(2) 18 ft.	13 ft. 6 in.
(3) 24 ft.	16 ft. 6 in.
(4) 30 ft.	25 ft.
(5) 16 ft. 6 in.	18 ft.

37. Mary's father has a garden 8.25 rd. long and 5.04 rd. wide. How many square rods in the garden?

38. There are 272.25 sq. ft. in a square rod. How many square feet in the garden?

39. Mr. Brown built, in front of his house, a stone wall 132 ft. long, 5 ft. wide, and 4 ft. high. How many cubic feet are there in the wall?

40. He hired Mr. Wentworth to dig a cellar 30 ft. long, 25 ft. wide, and 7.5 ft. deep, and paid him $2\frac{1}{2}$ cts. a cubic foot. What did it cost Mr. Brown to have the cellar dug?

41. How many cords of wood in a pile 32 ft. long, 4 ft. wide, and 6 ft. high?

42. What is the pile worth at \$7.25 per cord? At \$8.60 per cord?

43. Find the following ratios: $10\frac{1}{2}$ ft. to $3\frac{1}{2}$ ft.; 22 ft. to $5\frac{1}{2}$ ft.; 16.5 ft. to 5.5 ft.; 30 ft. to $2\frac{1}{2}$ ft.

44. Find the cost of $16\frac{1}{2}$ yd. of muslin, at the rate of 66 cts. for $5\frac{1}{2}$ yd.

45. If \$375.50 is the value of $2\frac{1}{2}$ acres of land, what is the value of $7\frac{1}{2}$ acres? Of 30 acres?

46. Mr. Ellis bought plots of land, each 100 ft. deep, having respectively the following frontages: 50 ft., 75 ft., 125 ft., 200 ft., 150 ft., 175 ft. How many acres were there in the whole purchase? How many square feet were there in each plot?

47. How many building lots 25 ft. wide could be laid out on each plot?

48. Taking as your scale 1 in. = 50 ft., draw a diagram of adjoining plots, each 100 ft. deep, the frontages being respectively $62\frac{1}{2}$ ft., 125 ft., $87\frac{1}{2}$ ft.

49. What is each plot worth at the rate of \$1,500 a building lot of 25 ft. frontage?

50. Draw a diagram on a scale of 1 in. to 8 ft. of a room 18 ft. long and 12 ft. wide with walls 8 ft. high. How many feet of picture molding would it take to go around the room?

51. How many square yards are there in each side wall? In each end wall? In the ceiling?

52. At 48 cts. per square yard, what will it cost to plaster the walls and ceiling of the room, no allowance being made for doors and windows?

53. What will it cost if 15 % is allowed for doors and windows?

54. The house contains 8 rooms. The walls have in all 10 times as much space as that of the room just measured. Mr. Mason secured the contract for lathing and plastering at 50 cts. a square yard.

(1) Laths cost \$4.50 per M. It takes 1,000 laths for 55 sq. yd. How many laths are used?

(2) Two men are employed to drive the lath at \$3 per M. What will be the cost of labor for lathing?

(3) One man can drive 1,400 laths in 1 day. How long will it take the two men to do the lathing? How much will they earn a day?

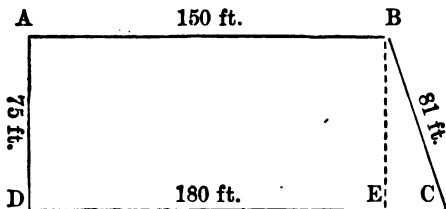
(4) Plaster costs 35 cts. a square yard. What will the plaster cost?

(5) A plasterer is paid \$4.50 a day, and lays 75 sq. yd. What will the plastering cost?

(6) Will Mr. Mason make or lose on the contract? How much?

55. Mr. Mason has a garden 45 ft. square, surrounded by a board fence 6 ft. high. The boards cost 5 cts. per board foot, and the painting 15 cts. a square yard. What did the boards cost? What did it cost to paint the fence?

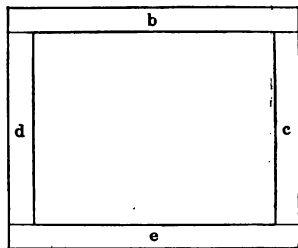
56. Mr. Brandt has a field of the dimensions shown in the diagram.



He fenced off a portion of it, as shown by the line BE. How many square feet are there in the rectangular part of the field? How many in the triangular end of the field? How many in the whole field?

57. This diagram represents the foundation walls of a building. It is drawn on a scale of 16 ft. to an inch.

What is the outside measurement around the walls? How thick are the walls? What is the perimeter of the space inclosed? If the walls are built 12 ft. high, how many cubic feet will they contain?



58. The difference between the perimeter and the length of the walls is how many times the thickness of the walls? Is that always so? Can you give the reason?

NOTE. The total length of an inclosing wall, or path, is found by adding 4 times the thickness of the wall or path to the inside perimeter, or subtracting 4 times the thickness from the outside perimeter.

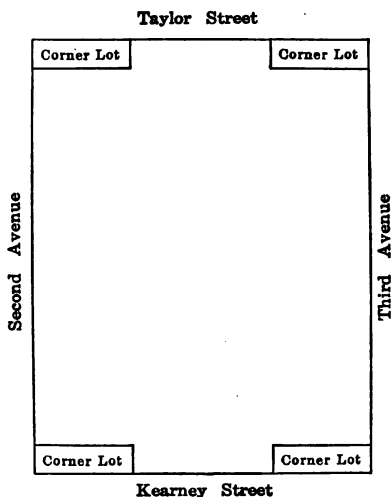
59. The circumference of Daniel's bicycle wheel is 12 ft. 8 in. How many times will it revolve in going a mile?

60. If the length of a passenger car is 25 ft. 6 in., how many cars can be stored on a mile of tracks?

61. Mr. Ames has a square lot, one side of which is 24 rd. in length. Draw a diagram of the lot on the scale of $\frac{1}{4}$ in. to 1 rd. How many rails 11 ft. long will it take to reach around the lot?

62. If Mr. Ames builds the fence 5 rails high and pays 3 cts. a foot for the rails, what will they cost him?

63. He uses posts 4 in. $\times$ 4 in. and 8 ft. high, and places one at every joint of the rails. How many will he use? How much will they cost him at 15 cts. a board foot?



64. The diagram represents a block in a city. It is drawn on a scale of 200 ft. to an inch. How long is it? How wide?

65. The houses on the block are built on lots each with a 25-foot frontage. How many houses are there on each of the avenues? How many on each of the streets?

66. The cost of paving both avenues and streets was \$1.25 a front foot. How much did the paving cost the owner of each corner lot? How much did the paving of the avenues and streets cost altogether?

67. A cement walk 4 ft. wide was put entirely around the block. What was the perimeter of the outside of the walk?

68. Draw a diagram of the block with the walk around it, and find how many square feet there were in the walk.

69. A sixth-grade manual training class made a window box for flowers and a shelf for it, supported by brackets. The box was 3 ft. long, 9 in. wide, and 6 in. deep. The shelf was of the same width as the box and 2 in. longer. The brackets were triangular, the sides about the right angle being each 9 in.



(1) Draw the ends, sides, and bottom of the box, also the shelf and brackets, to a scale of 1 in. to 1 ft.

(2) How many feet of boards were required for it all?

(3) At \$0.06 a foot how much did the boards cost?

(4) If boards 1 ft. wide were bought, how much was wasted?

SUMMARY OF CHAPTER IV

DENOMINATE NUMBERS.

Tables.

Reduction, ascending and descending, including the use of decimal and common fractions.

Addition, subtraction, multiplication, and division.

Problems in practical mensuration.

Manual training problems.

Graphs.

CHAPTER V

PERCENTAGE — APPLICATIONS OF PERCENTAGE

FINDING PERCENTAGE

307. What does the term per cent mean?

The whole of a number equals what per cent of it? To what per cent is $\frac{1}{2}$ of a number equal? $\frac{1}{2}$ of it? $\frac{1}{6}$ of it? $\frac{1}{10}$ of it?

By what sign is per cent expressed?

1. Express 6 %, 20 %, $37\frac{1}{2}$ %, and 25 %, each in three different ways.

2. Express as a decimal: $3\frac{1}{2}$ %; $\frac{1}{2}$ %; $4\frac{1}{4}$ %; $5\frac{1}{8}$ %; $\frac{1}{8}$ %; $\frac{3}{8}$ %; $\frac{7}{8}$ %; $\frac{3}{4}$ %.

TO FIND A PERCENTAGE

308. Find 5 % of \$375.

SOLUTION:	\$ 375
	.05
	\$ 18.75

What does the term percentage mean?

What does the term base mean?

What does the term rate mean?

In the above problem what is the 5 % called? The \$375? The result?

How do you find a percentage of a number?

Find 6 % of \$450 and apply your rule.

$$\text{Base} \times \text{Rate} = \text{Percentage.}$$

ORAL EXERCISES

309. Find:

- | | |
|----------------------|----------------------------------|
| 1. 5 % of 500 yd. | 9. 16 % of \$400. |
| 2. 2 % of 400 yd. | 10. 1 % of \$200. |
| 3. 4 % of \$250. | 11. $\frac{1}{2}$ % of 400. |
| 4. 6 % of 350 acres. | 12. $\frac{1}{4}$ % of 800 ft. |
| 5. 8 % of 4,000 mi. | 13. $\frac{3}{4}$ % of 1,200 yd. |
| 6. 9 % of 6120. | 14. $12\frac{1}{2}$ % of \$96. |
| 7. 15 % of \$600. | 15. $\frac{1}{8}$ % of \$96. |
| 8. 14 % of \$500. | 16. $33\frac{1}{3}$ % of \$260. |
16. With 144 as a base: $12\frac{1}{2}$ %, 75 %, $62\frac{1}{2}$ %, $16\frac{2}{3}$ %, 50 %, $33\frac{1}{3}$ %, $66\frac{2}{3}$ %.

WRITTEN EXERCISES

310. Find:

- | | |
|-------------------------------|-----------------------------------|
| 1. 16 % of 2,865. | 4. 12 % of 7,624. |
| 2. 38 % of 465.5. | 5. $14\frac{3}{4}$ % of \$860.25. |
| 3. $4\frac{1}{2}$ % of 786.5. | 6. $\frac{3}{4}$ % of \$320.40. |

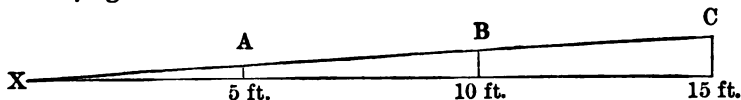
311. Solve:

1. Mr. Drew had \$5,000 on deposit in the National City Bank on Monday morning. On Tuesday he withdrew 25 % of it. On Wednesday he deposited \$1,000. On Thursday he withdrew 20 % of what he then had on deposit. On Friday he deposited \$1,900. On Saturday he withdrew $33\frac{1}{3}$ % of his balance. How much money had he in the bank at the end of the week?

2. Mr. Holman's salary of \$2,500 was cut 15 %. It was afterwards increased 15 %. What was his salary after the cut? After the increase? Why was his salary less than it was before the decrease, although the rate per cent of increase and decrease was the same?

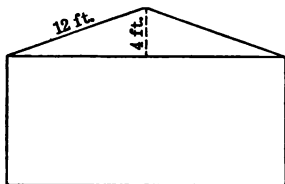
3. In three schools there are 3,840 pupils; $33\frac{1}{3}\%$ of the pupils are in School No. 1; 25 % of them are in School No. 2, and the remainder in School No. 3. How many pupils are there in each school? What per cent of the entire number of pupils are in School No. 3?

4. The line XC represents a road on a hillside. At A it is 5 ft. above the level; at B, 10 ft.; at C, 15 ft. We say that the *grade* is 5 ft. in 100 ft., or a 5 % grade. What is a 6 % grade? A 1 % grade? A 10 % grade? A 12 % grade? A 15 % grade?



5. How many feet does a 1 % grade rise in 1 mi.? In $1\frac{1}{2}$ mi.? In 40 rd.? In 150 rd.? In 1 mi. 160 rd.? In 2.25 mi.?

6. What is the grade, or slant per cent, in the roof in the diagram?



7. If the roof were 40 ft. long, what would be the per cent of grade? If it were 48 ft. long?

8. If it were 20 ft. long and 5 ft. high, what would be the per cent of grade? If it were 60 ft. long and 10 ft. high? If it were 80 ft. long and 20 ft. high?

9. The road from Norwalk to Ridgefield averages a 2 % grade. The distance is 10 mi. How much higher is Ridgefield than Norwalk?

10. John walks 1 mi. from the river up an incline to the top of the Palisades, 500 ft. above the river level. What is the grade?

TO FIND THE RATE PER CENT

312. In buying and selling, the per cent of gain or loss is usually estimated as a part of the cost.

1. Land bought for \$150 was sold for \$180. What was the gain per cent?

SOLUTION: \$180 (selling price) - \$150 (cost) = \$30 (gain).

The ratio of 30 (gain) to 150 (cost) = $\frac{1}{5}$ = 20 %.

2. Land bought for \$180 was sold for \$150. What was the loss per cent?

SOLUTION: \$180 (cost) - \$150 (selling price) = \$30 (loss).

The ratio of \$30 to \$180 = $\frac{30}{180} = \frac{1}{6}$ = 16 $\frac{2}{3}$ %.

To find the gain or loss per cent, find the ratio of the gain or loss to the cost, or base (or, divide the gain or loss by the cost), and express the quotient in hundredths as a decimal.

Percentage ÷ Base = Rate.

ORAL EXERCISES

313. Find the gain or loss per cent in each of the following:

COST	SELLING PRICE	COST	SELLING PRICE	COST	SELLING PRICE
1. \$15	\$18	9. \$1.20	\$1.50	17. \$ $\frac{5}{8}$	\$.75
2. 18	15	10. 1.50	1.20	18. $\frac{4}{5}$	1.20
3. .21	.14	11. 1.00	.50	19. .62 $\frac{1}{2}$	$\frac{7}{8}$
4. 4.00	4.50	12. 2.00	.25	20. .00 $\frac{1}{2}$	.01
5. .50	.62 $\frac{1}{2}$	13. $\frac{3}{4}$	1.00	21. .05	.075
6. .62 $\frac{1}{2}$	1.25	14. 1.00	$\frac{3}{4}$	22. .20	$\frac{3}{10}$
7. .375	.50	15. $\frac{1}{2}$	$\frac{3}{4}$	23. $\frac{9}{10}$	1.20
8. .50	.37 $\frac{1}{2}$	16. $\frac{1}{4}$	.50	24. 1.40	$\frac{7}{10}$

WRITTEN EXERCISES

314. Solve :

1. Mr. Luther bought a carriage for \$125 and sold it for \$135. What per cent did he make?
2. Mr. Arnold paid \$140 for a ring and sold it for \$120. What per cent did he lose?
3. Harry Rose bought a bicycle for \$27.50 and sold it for \$33. What per cent did he make?
4. Mr. Harvey bought a house for \$6,250. For repairs he paid a carpenter \$200, a mason \$175, a painter \$125. He sold the house for \$7,500. What per cent did he gain?
5. Mr. Dixon bought a ton of graphite for $3\frac{1}{4}$ cts. a pound and sold it for \$85. What per cent did he make?
6. A newsdealer bought 150 *Tribunes* at the rate of \$1.60 a hundred, and sold them at 3 cts. each. What per cent did he make?

FINDING THE BASE

315. There are 660 ft. in $12\frac{1}{2}\%$ of 1 mile. How many feet are there in a mile?

SOLUTION: 600 ft. the percentage = $12\frac{1}{2}\%$, the rate $\times$ the base.

Therefore the base = 600 ft. $\div 12\frac{1}{2}\%$.

The percentage divided by the rate equals the base.

$$\text{Percentage} \div \text{Rate} = \text{Base.}$$

WRITTEN EXERCISES

316. Solve :

1. In five classes of a school there are 225 pupils, which is 25% of all the pupils in the schools. 25% is what part of the whole number of pupils? How many pupils are there in the school?

2. John Dutcher paid \$25.75 for a suit of clothes, which is 20 % of his salary for a month. How many 20 %'s of his salary are there in his whole salary? What is his yearly salary?

3. In 5 % of a mile there are 3,168 inches. How many 5 %'s of a mile are there in a mile? How many inches are there in a mile?

4. Mr. Clarke sold Mr. Jones 12.50 acres of land, Mr. Fenton 34.5 acres, and Mr. Benson 65.76 acres. The number of acres that he sold was $33\frac{1}{3}$ % of his farm. How many acres were there in the farm?

5. A man sold 500 quarts of strawberries. This was 25 % of his entire crop. How many quarts were there in his entire crop?

6. A train traveled 20 miles an hour after an accident. This was 80 % of the rate before the accident. What was the rate before the accident?

7. An automobile cost \$750. This was 60 % of the selling price. For how much was it sold?

8. In the year 1900, the population of a city was 89,600, which is 80 % of its present population. How many people were there in the city in 1909?

9. Because of a fire, a merchant was obliged to sell his goods for \$2,160. This was 48 % of their cost. What did they cost?

10. A class numbered 35. This was 10 % of the entire school. How many were there in the school?

11. A teacher saves \$12 a month, which is 20 % of her salary? Find her salary for the month. How much does she save in a year?

12. 320 is 25 % of what number? 150 is $33\frac{1}{3}$ % of what number? 270 is $12\frac{1}{2}$ % of what number?

TO FIND THE AMOUNT OR DIFFERENCE

317. In calculating profit and loss, a selling price at a gain (cost + gain) is called the **amount**.

A selling price at a loss (cost - loss) is called the **difference**.

1. Mr. Shepherd sold his farm, which cost him \$7,200, at a loss of $16\frac{2}{3}\%$. What did he receive for it?

SOLUTION: $100\% - 16\frac{2}{3}\% = 83\frac{1}{3}\%$, selling price.

$83\frac{1}{3}\%$ of \$7,200 = \$6,000, selling price.

2. Mr. Copper sold silk that cost him \$16,800, at a profit of 15% . How much did he receive for it?

The cost = what per cent?

The cost + the gain = what per cent.?

115% of \$16,800 = \$16,800 $\times 1.15$ = \$19,320.

To find the amount or difference, rate per cent and cost being given, multiply the cost by 100% plus the gain, or minus the loss per cent.

Base + Percentage = Amount.

Base - Percentage = Difference.

ORAL EXERCISES

318. Find the selling price :

COST	GAIN OR LOSS %	BASE	GAIN OR LOSS %
1. \$4	25 % gain	11. 7.50 ft.	100 % gain
2. \$5	20 % loss	12. 3.75 ft.	100 % loss
3. \$3	$33\frac{1}{3}\%$ gain	13. 9 yd.	300 % gain
4. \$12	$16\frac{2}{3}\%$ gain	14. 72 yd.	$62\frac{1}{2}\%$ gain
5. \$16	$12\frac{1}{2}\%$ loss	15. 36 mi.	$11\frac{1}{3}\%$ gain
6. \$21	$14\frac{2}{7}\%$ gain	16. 28 mi.	$14\frac{2}{7}\%$ gain
7. \$14	$14\frac{2}{7}\%$ loss	17. .60 bu.	40 % loss
8. \$200	10 % gain	18. .40 bu.	60 % gain
9. \$400	20 % loss	19. 32 qt.	$6\frac{1}{4}\%$ loss
10. \$40	$37\frac{1}{2}\%$ gain	20. .48 qt.	$8\frac{1}{3}\%$ gain

REVIEW

319. Solve:

1. Mr. Webster's corn crop in 1902 was 15,480 bu. In 1903 it was $16\frac{2}{3}\%$ more. What was his crop in 1903?

2. In 1901 Mr. DeMott sold 150,000 peach baskets. If he sold $12\frac{1}{2}\%$ less in 1903, how many did he sell? At \$4.50 per hundred, what did he receive for the baskets he sold in both years?

3. Mr. Swan sold 1,250 ft. of pine boards and 20% more hemlock boards. How many feet of hemlock boards did he sell?

4. James Marvin has \$36.80 in the bank. His brother William has 30% more money. How much has William?

5. A wholesale grocer sold 1,680 lb. of butter and $8\frac{1}{3}\%$ more pounds of cheese. He sold his cheese for $9\frac{1}{2}$ cts. a pound. How much did he receive for it?

6. Mr. Brant bought 1,200 baskets of peaches for \$1.10 per basket and sold them at a profit of 10%. For how much did he sell them?

7. Mr. Oliver bought 825 bu. of apples at \$.90 a bushel and sold them at a loss of $11\frac{1}{3}\%$. How much did he receive for his apples?

8. A merchant invested \$8,000 in business and gained $37\frac{1}{2}\%$. He paid \$600 rent, and his living expenses were \$2,000. How much had he left?

9. At what price must cloth that cost \$4.80 a yard be marked that $6\frac{1}{4}\%$ may be made?

10. A milk dealer bought 48 qt. of cream at 30 cts. a quart, and sold it at \$1.50 a gallon. How much did he make? What per cent did he make?

11. Mr. Merrill bought 144 doz. lead pencils at 35 cts. a dozen and sold them for 5 cts. apiece. What per cent did he make?

12. Mr. Currier bought a lot for \$1,800 and built a house on it. He paid the carpenter \$3,500; the mason \$1,700; the plumber \$800; and the painter \$300. He sold the house and lot for \$7,200. What was his gain or loss per cent?

13. Mr. Vreeland bought a lamb which weighed 80 lb. for \$8, and sold it for $12\frac{1}{2}$ cts. a pound. What per cent did he make?

14. Mr. Rollin's wheat crop, in 1901, averaged 35 bu. an acre; in 1902 it averaged 40 bu. 2 pk. an acre. What per cent was the increase in 1902?

15. A grocer sold 5 bu. 2 pk. of apples on Friday, and 8 bu. on Saturday. Saturday's sale was what per cent larger than Friday's?

16. From a barrel of molasses containing 40 gal., 15 gal. 2 qt. were drawn. What per cent of the molasses was drawn? What per cent remained in the barrel?

17. There are 1,050 pupils on roll in a school, 50 of whom are absent. What per cent is present?

18. In the schools of a city there are 43,000 pupils enrolled. One of its schools has 1,720 pupils on its roll. What per cent of the pupils in the city are enrolled in this school?

19. From an acre of land a plot 10 rd. long and 5 rd. wide was sold. What per cent of the land was sold?

20. Mr. Ramsley contracted to dig a cellar 30 ft. long, 24 ft. wide, and $6\frac{1}{2}$ ft. deep. How many cubic feet of dirt were in the cellar? When an average depth of 2 ft. had been excavated, what per cent of the dirt had been removed?

INTEREST

320. What is the interest on \$1.00 for 1 year at 2% ? 3% ? 4% ? 5% ? 6% ? 7% ? 8% ? 9% ? 10% ?

321. There are many methods of computing interest. One of the simplest is shown by the following examples :

1. Find the interest on \$150 for 2 yr. 6 mo. at 5%.

SOLUTION: $\$150 \times .05 = \$7.50$, interest for 1 yr.

$\$7.50 \times 2 = \$15.00$, interest for 2 yr.

$\$7.50 \div 2 = \$3.75$, interest for 6 mo.

$\$15.00 + \$3.75 = \$18.75$, interest for 2 yr. 6 mo.

2. Find the interest on \$375 for 6 mo. 15 da. at 7%.

SOLUTION: $\$375 \times .07 = \$27.25$, interest for 1 yr.

$\$27.50 \div 2 = \$13.75$, interest for 6 mo.

$\$13.75 \div 6 = \$2.875$, interest for 1 mo.

$\$2.875 \div 2 = \$1.4875$, interest for $\frac{1}{2}$ mo. (15 da.).

$\$13.75 + \$1.49 = \$15.25$, interest for 6 mo. 15 da.

It is customary in writing interest or amount to omit mills and use the nearest number of cents. In writing the rate, however, the exact figures must be given.

In ordinary business money is borrowed commonly for even months, or for simple fractional parts of months; as for 15 da. = $\frac{1}{2}$ mo.

3. Find the interest on \$200 for 5 mo. at 4%.

SOLUTION: $\$200 \times .04 = \$8.00$.

$\frac{5}{12}$ of $\$8.00 = \$3.33\frac{1}{3}$ = interest for 5 mo.

Another method is to find the interest for \$1.00 for the given time and rate, and to multiply the principal by it.

4. Find the interest on \$175 for 1 yr. 3 mo. at 5%.

SOLUTION: Interest on \$1.00 for 1 yr. = \$.05.

Interest on \$1.00 for 3 mo. = $\frac{1}{4}$ of \$.05 = \$.0125.

$\$.05 + \$.0125 = \$.0625$.

$\$175 \times .0625 = \$10.94$.

WRITTEN EXERCISES

322. Find interest as follows :

1. On \$250 at 6 % for 2 yr. 4 mo.
2. On \$380.50 at 8 % for 4 yr. 6 mo.
3. On \$2,175 at 7 % for 5 yr. 5 mo.
4. On \$936.75 at 4 % for 6 yr. 7 mo. 6 da.
5. On \$480 at 3 % for 7 yr. 3 mo.
6. On \$716.25 at 5 % for 1 yr. 1 mo. 15 da.

323. Solve :

1. On Sept. 5, 1907, Mr. Peter Gray loaned \$3,000 to Mr. Earl Randall. He gave his note for 3 years at 7 % interest to be paid annually. How much interest did he pay in all? What was the total amount that he had to pay?

2. A man bought a span of horses for \$375. He gave his note for six months at 6 % per annum for the entire amount. How much was the interest when due? The entire debt?

3. On Sept. 1, 1908, Alfred Swift bought of James Hunter, 2 cows at \$42; 3 horses at \$225; and 6 hogs at \$15. He gave in payment his note, bearing interest at 6 %. He paid the note on Dec. 31, 1909. How long did it run? How much interest did he pay?

4. On May 15, 1906, Mr. Baker bought a farm for \$15,000. He paid \$10,000 and gave his note for the balance. On June 5, 1906, he paid the interest then due and one half the principal. How much did he pay? How much did he then owe? On July 20, 1907, he paid the interest due and \$1,000 of the principal. How much did he then owe? How much did he pay? On Aug. 30, 1908, he paid the interest due and the remainder of the principal. How much did he pay?

BANKS

For convenience and safety, many people keep their money in **banks**.

To put money in a bank is to **deposit** it.

Those who put money in banks are **depositors**.

A bank takes care of the money of depositors and pays it out as ordered by them.

The order of a depositor to a bank to pay out money deposited is called a **check**.

No. <u>234</u>	NEW YORK <u>May 1st</u> 19 <u>10</u>
THE SEVENTH NATIONAL BANK OF NEW YORK.	
PAY TO THE ORDER OF <u>H. S. Berwick</u>	\$ <u>150⁷⁵/₁₀₀</u>
<u>One hundred and fifty + ⁷⁵/₁₀₀</u>	DOLLARS
<u>John Doe</u>	

WRITTEN EXERCISES

324. Solve:

1. On Jan. 1 Mr. Jones deposited \$500 in the First National Bank. On Jan. 2 he gave checks as follows:

To Alfred Smith, \$ 5.25.

To Henry Evans, 32.16.

To Avery Bros., 41.63.

To Huyler & Co., 16.17.

How much remained in the bank to Mr. Jones's credit after the checks had been paid? The amount which remains in the bank to Mr. Jones's credit is called a **balance**.

2. On Monday Mr. Corson deposited \$1000 in the Second National Bank. On Tuesday he deposited \$150 more and drew out by check \$230. On Wednesday he deposited \$410.50 and drew out by check \$275.83. On Thursday he deposited \$476.18 and drew out \$125. On Friday he deposited \$116.50 and drew out \$487. On Saturday, he deposited \$567 and drew out \$750.65. What was his balance at the end of the week?

Some banks pay **interest** on money deposited. The banks that receive small amounts from depositors who wish to leave it for a long time are called **savings banks**. Such banks pay interest if the deposit is left for six months or more. If the depositors do not take their interest when it is due, the bank adds it to the deposit and pays interest on the sum.

WRITTEN EXERCISES

325. Solve:

1. Henry's grandfather put \$100 in the savings bank for him on his tenth birthday. The bank paid 2 per cent interest every half year. What was the interest at the end of the first half year?

2. Henry did not draw out his interest, and the bank added it to the deposit. How much did Henry then have in the bank?

3. At the end of the second half year what interest was due?

4. The interest due was added to the amount already in the bank. How much did Henry then have on deposit?

5. If he kept the money in the bank, till he was 21 years old, how much did he have on deposit at the end of each half year?

6. How much did he have on deposit at the end of the last half year?

MONEY ORDERS

It is often necessary to pay money in some distant place.

One way is to send bank checks. Another is to pay the money to an express company, who pays it over to the proper person. For this a fee is charged by the company.

One of the most common ways is by a post-office money order.

Dollars	Cents	Stamp of Issuing Office	
\$			(Form No. 600)
The Postmaster will insert			No. _____
here..... the office drawn on, when the office named by the remitter in the body of this application is not a Money Order Office. Spaces above this line are for the Postmaster's record, to be filled in by him.			
Application for Domestic Money Order			
Spaces below to be filled in by purchaser, or, if necessary, by another person for him			
Amount			
Twenty-five Dollars ten Cents			
Pay to Order of: Samuel West			
(Name of person or firm for whom order is intended)			
Whose Address is: No. 45 Arch Street			
Post Office: Philadelphia			
State: Penn.			
Sent by: John R. Saxton			
(Name of Sender)			
Address of sender: No. 18 Pearl Street			
PURCHASER MUST SEND ORDER (ON BLUE PAPER) TO PAYEE			

For a small fee the Government receives the money at a post office and agrees to pay it to the proper person at a post office in the distant place.

The charges for doing this for different sums are as follows :
For orders from :

\$ 0.01 to \$ 2.50		3 cents.
2.51 " 5.00		5 "
5.01 " 10.00		8 "
10.01 " 20.00		10 "
20.01 " 30.00		12 "
30.01 " 40.00		15 "
40.01 " 50.00		18 "
50.01 " 60.00		20 "
60.01 " 75.00		25 "
75.01 " 100.00		30 "

WRITTEN EXERCISES

326. Solve :

1. Mrs. Simons bought of Marshall Field & Co., dry goods to the amount of \$49.75; of the McClure Co., books and stationery for \$34.68; of The Fair, goods for \$31.60; of Spaulding Co., jewelry for \$16.25. She sent P. O. orders for the amount due. How much did she pay for each order?

2. The postmaster at Lansing made out the following money orders :

\$46.75	\$31.50	\$32.26	\$43.48
27.98	16.27	5.43	

How much did he receive for each ?

SUMMARY OF CHAPTER V

PERCENTAGE.

Finding gain or loss per cent ; amount and difference.

APPLICATIONS OF PERCENTAGE.

Interest. Banking. Money Orders.

CHAPTER VI

FORM STUDY

MEASUREMENT OF LINEAR FIGURES

327. Two straight lines meeting form an **angle**. The point where they meet is called the **vertex**. An angle measures the difference in direction of two lines.

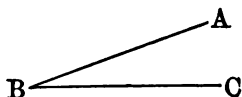


FIG. 1.

The lines BA and BC form an angle whose vertex is B.

The angle may be read, "the angle ABC," "the angle CBA," or "the angle B."

When two straight lines meet so that they make two equal angles, the angles are called **right angles**, and the lines are said to be **perpendicular** to each other.

The angles ACB and ACD are equal, and are both right angles; and the lines BD and AC are perpendicular to each other.

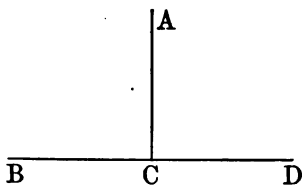


FIG. 2.

1. Fold a circular piece of paper into fourths. Open it and observe the creases. How many right angles are made by the folds?

2. Draw a circle on paper and divide it with lines that cross at the center as the creases do on the folded paper. How many angles are made by the crossing of the lines? What kind of angles are they?

3. Letter and read them. What part of a circle is a right angle?

Circles are divided into 360 equal parts called **degrees**.

4. How many degrees are there in a right angle? In $\frac{1}{2}$ of a right angle?

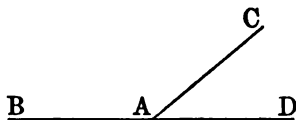


FIG. 3.

An angle greater or less than a right angle is called an **oblique angle**.

5. In Fig. 3, read the angle that is greater than a right angle.

An angle greater than a right angle is called an **obtuse angle**.

6. Read the angle that is less than a right angle.

An angle less than a right angle is called an **acute angle**.

7. Name each angle in the letters V, X, L, M, telling whether it is right, acute, or obtuse.

8. Find and show acute, right, and obtuse angles in your schoolroom.

328. A figure drawn on a plane or flat surface is called a **plane figure**.

Plane figures have two **dimensions**. That is, they can be measured in two ways: for length; and for breadth, or width. They have no thickness.

1. How many angles has this plane figure? What kind of angles are they?

2. How many sides has the figure?

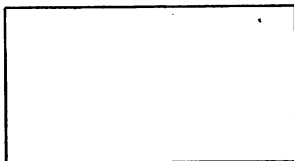


FIG. 4.

A plane figure with four sides and all its angles right angles is called a **rectangle**.

A rectangle with all of its sides equal is called a **square**.

3. How can the area of a rectangle be found without measuring it?

4. Prove it by drawing a rectangle and dividing it into square inches. (If the dimensions are not of the same denomination, change them to the same denomination before multiplying.)

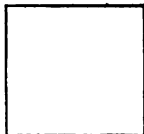


FIG. 5.

A plane figure with three sides and three angles is called a **triangle**.

A triangle which contains one right angle is called a **right triangle**.

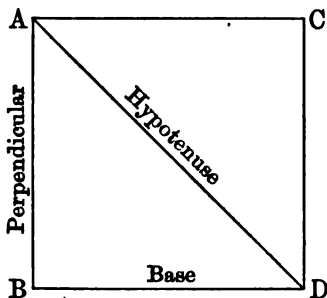


FIG. 6.

6. In Fig. 6 what kind of angles are ABD and ACD? Name two right triangles in the figure.

One side of a right triangle is called the **base**; the other, the **altitude** or **perpendicular**.

The side of the right triangle opposite the right angle is called the **hypotenuse**.

329. Cut a 3-inch square. Fold it so that the diagonally opposite corners meet, and cut through the fold. What kind of figures have you?

1. How many square inches are there in the square you have cut? How many triangles does it contain?

2. Each of the right triangles is what part of a square? How many square inches are there in one of the right triangles?

3. Draw a diagram of a square garden plot, 20 ft. on each side, to the scale of 1 in. to 5 ft. Divide it by a diagonal line. What is the shape of each part? What is the area of each?

4. Cut a rectangle 3 inches long and 2 inches wide. Draw a line connecting opposite corners and cut through this line. What kind of triangles have you?

5. The area of each triangle is what part of the rectangle?

6. What is the area of the rectangle? Of each triangle?

To find the area of a right triangle multiply the base by the altitude, and take half the product; or, multiply the base by $\frac{1}{2}$ the altitude or the altitude by $\frac{1}{2}$ the base.

7. How many square inches in a right triangle whose base is 8 inches and perpendicular 12 inches?

SOLUTION: $12 \times 8 = 96$; $96 \div 2 = 48$; therefore the area contains 48 square inches.

WRITTEN EXERCISES

330. Solve:

1. Find the square contents or area of right triangles with bases and altitudes as follows:

(a) Base 9 in., altitude 20 in.

(b) Base 8.6 in., altitude 14.25 in.

(c) Base 16.25 in., altitude 18.6 in.

2. Mr. Seymour has a garden shaped like a right triangle, the base of which is 12 yd. 2 ft. and the altitude is 45 ft. How many square feet are there in the garden?

3. Mr. Daly's flower bed is a right triangle whose base is 12.5 ft. and altitude 18.4 ft. There is a plant in each square foot of the bed. Find the cost of the plants at \$.06 per plant.

4. Mr. Newton has a right triangular field whose base is 24 rd. and whose altitude is 40 rd. Find the value of the field at \$187.50 per acre.

5. Each of the 20 pupils of a sixth grade class made a label for the bed of his garden in the shape of the label in the cut, which is $\frac{1}{2}$ the actual size. The dotted lines show the corners before they were cut off.

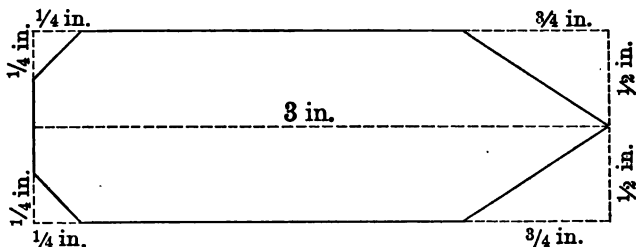


FIG. 7.

- (a) How many square inches of board were needed?
- (b) How many square inches were cut off each label?
- (c) How much was wasted in this way, in making 20 labels?
- (d) If the wood cost 8 cts. a square foot, what was the cost of the parts wasted?

6. The drawing of a plant label is 6 in. long and $1\frac{1}{2}$ in. wide. It is placed in the center of a sheet of paper which measures 9 in. by 6 in. What is the width of the long margins? Of the short margins?

7. The wood from which the label is made is $6\frac{1}{4}$ in. by $2\frac{1}{2}$ in. How many boys could be supplied with wood to make labels from a piece of board 1 ft. 3 in. wide and 2 ft. 1 in. long?

8. A class of 18 boys each made a match striker $5\frac{1}{2}$ in. long and 2 in. wide. An oblong piece of sandpaper was placed upon each match striker, 1 in. from each end and $\frac{1}{4}$ in. from each side. Would it require more than one sheet of sandpaper 9 in. by 11 in. to supply the class? What would be the most economical way of cutting the paper?

9. Mr. Ames built a corner closet in his kitchen, 30 in. wide on each side of the corner and 5 ft. tall. How much lumber was used in the base and the top of the closet?

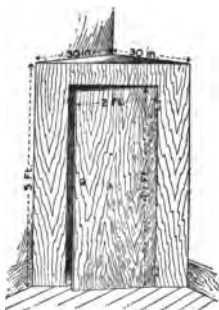


FIG. 8.

10. The door of the closet in Exercise 9 was 2 ft. $\times$ 4 $\frac{1}{2}$ ft. How many square inches were there in the door?

11. Mr. Acton had for his farm a quarter section of land; that is, a piece of land $\frac{1}{4}$ mi. on each side. How far was it around the farm? How many square rods did it contain?

331. The triangle ABC has *two equal sides*, AC and CB. It is called an **isosceles triangle**.

1. Read the angles of the isosceles triangle ABC.

The angle opposite the base of an isosceles triangle is called its **vertex**.

2. Draw a straight line from the vertex C perpendicular to the base. Cut out the triangle and fold it over the altitude. Cut through the fold.

3. Place together the two triangles that you have made, so that they form a rectangle.

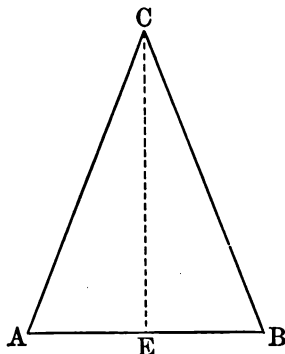


FIG. 9.

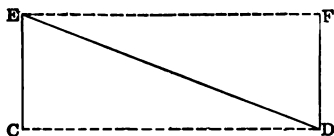


FIG. 10.

4. What part of the isosceles triangle does EF, the length of the rectangle, equal? What part does EC, the width of the rectangle, equal?

To find the area of an isosceles triangle, multiply the altitude by half the base; or, multiply the base by the altitude and take half the product.

WRITTEN EXERCISES

332. Find the areas of the following isosceles triangles:

1. Base 8 in., altitude 10 in.
2. Base 14 in., altitude 20 in.
3. Base 5 in., altitude 7 in.

333. The triangle ABE has all its sides equal.

It is called an **equilateral triangle**.

1. What is its altitude? Its vertex?

To draw an equilateral triangle:

Draw a straight line, as AB.

Using a compass, from the middle point C of the line AB draw the line CD perpendicular to the line AB; from

A draw a line equal to the base AB, meeting the perpendicular at some point, as E; connect the point E with B and so complete the triangle.

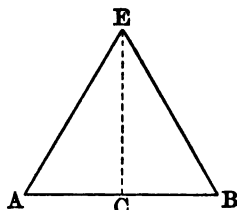


FIG. 11.

2. Draw equilateral triangles with the following bases: 2 inches, 3 inches, 4 inches, 6 inches, 8 inches, 12 inches.

3. If each of the equilateral triangles were drawn on the scale of 6 ft. to an inch, what would be the perimeter of each?

4. The perimeter of the triangle whose base is 2 inches is what per cent of the perimeter of the triangle whose base is 3 inches?

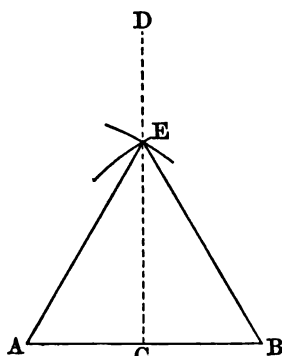


FIG. 12.

334. The area of an equilateral triangle equals the base multiplied by the altitude, divided by 2; or:

$$\text{The area of a triangle} = \frac{1}{2} \text{ base} \times \text{altitude.}$$

WRITTEN EXERCISES

335. Solve:

1. Find the area of the following triangles:

(a) Altitude 12 in., base 16 in.

(b) Altitude 15 in., base 18 in.

(c) Altitude 18 in., base 22 in.

(d) Altitude 20 in., base 24 in.

(e) Altitude 25 in., base 40 in.

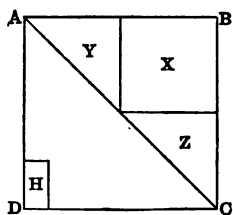


FIG. 13.

2. If each triangle were drawn on a scale of 1 inch to 5 rods, what fraction of an acre would each triangle represent?

3. This square ABCD represents Mr. Williams's farm. It is drawn on the scale of 1 inch for 40 rd. Measure it and find how many acres it contains.

The rectangle *H* was set apart for the house plot. How many acres did it contain?

4. A wire fence was built across the farm diagonally. How many acres were there in the triangular portion ABC? In ADC?

5. Mr. Williams fenced in the rectangular piece of land *H* for house, barn, and garden. The remainder of the piece ADC he sowed with wheat. How many acres of wheat did he have?

6. The wheat averaged $21\frac{1}{2}$ bu. to the acre. He sold it for 92¢ a bushel. How much did he receive for his wheat?

7. The land ABC he divided into three fields, X, Y, and Z. How many acres were there in each field?

8. The field X he set out with peach trees, 1 rd. apart, and none nearer the outside than $8\frac{1}{4}$ ft. How many trees did he set out?

9. The field Z he used as a pasture. If an acre can sustain 1 cow, how many cows could he keep?

10. On the field Y he raised hay. It yielded $1\frac{1}{2}$ T. an acre. Half of the hay he sold at \$18 a ton. How much did he receive for it?

11. How many tons of hay did he keep?

12. This is a diagram of Mr. Terry's garden, drawn on a scale of 16 ft. to an inch. He told Tom and Helen that they could have $\frac{1}{2}$ of it to divide equally and plant as they chose. Helen and Tom divided their half into two equal parts by a diagonal line.

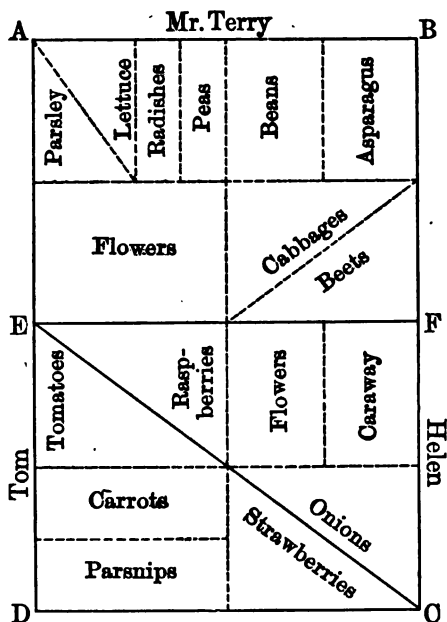


FIG. 14.

(a) How many square feet did each one have?

(b) How many square feet were there in each bed?

(c) Tom picked on an average four quarts of strawberries to every square foot of the bed. How many quarts did he pick in all?

(d) These he sold at \$.10 a quart. How much did he make?

(e) Which bed did he make the larger, the one for tomatoes or the one for parsnips?

13. Draw a diagram of a farm making it 3×5 in. on a scale of 1 inch for 90 rods. Find how many acres there are in your farm.

14. Divide it into four or five fields as you prefer. Make at least one in the shape of a rectangle. Find the number of acres in the rectangle.

15. If you raise corn on one of your fields, how many bushels will you have if it averages 60 bu. to an acre? How much will you receive for the corn if you sell it at \$.82 per bushel?

16. Build a fence around your farm. How many feet long will it be?

17. How much will it cost you if you have to pay \$.20 per foot for the fence?

18. The Principal's office measures 10×16 ft. He has a desk 6×3 ft. and a table 5×4 ft. How much floor space is covered by the desk? How much floor space remains?

MEASURING CIRCLES

336. A circle is a plane figure bounded by a curved line, every part of which is equally distant from a point within called the center.

The **circumference** is the line that bounds the circle.

The **diameter** is a straight line, passing through the center in any direction, its ends terminating in the circumference.

The **radius** (plural *radii*) is a straight line from the center to the circumference.

337. Measure the radius and the diameter of the circle. How do they compare in length?

The diameter is twice the radius, or $d = r \times 2$. Or $r = \frac{d}{2}$.

Draw and cut out a circle having a 3-inch radius.

With a piece of flexible wire or a string, measure the circumference. Compare the length of the circumference with that of the diameter. Calling the length of the circumference c ,

c = very nearly $3\frac{1}{7}$ times d , or $\frac{22}{7}$ of d .

$d = \frac{7}{22}$ of c .

WRITTEN EXERCISES

338. In each of the following supply the missing terms in place of the dash :

DIAMETER	RADIUS	CIRCUMFERENCE	DIAMETER	RADIUS	CIRCUMFERENCE
1. 14 in.		—	6. —		22 in.
2. 35 in.		—	7. —	—	88 in.
3. —	7 in.	—	8. —	51 in.	
4. —	14 in.	—	9. 110 in.	—	
5. —		44 in.	10. —	—	66 in.

11. The circular running track in the school gymnasium is $\frac{1}{10}$ mi. in circumference. What is its diameter?

12. On a merry-go-round the distance from the center to the horses was 25 ft. In riding around once, how far did Fred go?

13. Henry has a button tied to a string $8\frac{1}{2}$ in. long. What is the largest circumference that can be described by the button as he swings it around in a circle? Illustrate by a diagram.

14. A race course is half a mile in circumference. How many feet is it across it? Illustrate by a diagram.

339. The more exact relation of circumference and diameter is:

$$\text{circumference} = d \times 3.1416, \text{ or } 2r \times 3.1416.$$

3.1416 is represented by the Greek letter π (pronounced π).

$$\pi \times d = 2\pi \times r; \text{ or, } \pi d = 2\pi r.$$

Use this equation in solving the following problems.

WRITTEN EXERCISES

340. Solve:

1. The radius of a circle is 53 in. Find the diameter.
 2. The diameter of a circle is 112 in. Find the radius.
 3. The diameter of a circle is 21 in. Find the circumference.
 4. The diameter of a circle is 14 in. Find the circumference.
 5. The circumference of a circle is 22 in. Find the diameter.
 6. A tree in Ravenna Park, Seattle, is 44 ft. in circumference. What is its diameter?
 7. The steam pipes in the basement are 4 in. in diameter. They are wrapped in asbestos cloth. How many square feet of asbestos cloth are required to wrap a pipe 15 ft. long? To wrap a pipe $30\frac{1}{2}$ ft. long?
 8. A circular geranium bed is 8 ft. in diameter. How many feet of wire netting will be required to inclose it? Draw to scale of $\frac{1}{2}$ in. to 1 ft.
 9. The radius of a spider's web was $3\frac{1}{2}$ in. What was its outer circumference?
 10. A circular lily pond measured 35 ft. diameter. What did it cost to inclose it with wire fencing at 25 cts. per yard?
 11. A dog is tied to a stake by a rope that is 9 ft. long. What is the largest circle the dog can make around the stake?
-

SUMMARY OF CHAPTER VI

FORM STUDY:

Linear Figures — rectangles, triangles.

General Review — Mensuration.

Circles — measuring circumference, diameter, or radius.

GENERAL REVIEW

341. Solve:

1. The time passed since noon is 55 % of the time from noon to 6 o'clock P.M. How many minutes have passed since noon? What time is it?

2. The time passed since 6 o'clock P.M. is 80 % of the time from 6 o'clock P.M. to 40 minutes past 2 A.M. What time is it?

3. Mr. Mendell sold $8\frac{1}{3}$ % of his oats to one customer, $16\frac{2}{3}$ % to another, 20 % to another, 25 % to another, 16 % to another, and had 420 bu. left. How many bushels had he at first? What was its value at $\frac{5}{8}$ of a dollar per bushel?

4. On a certain day the sun rose at 43 minutes after 5, and set at 25 minutes after 6. What per cent of the day was sunlight?

5. Mr. Fiske's salary as a teacher is \$1,600. His real estate tax is \$90; his water tax, \$25; his gas bill, \$15; coal bill, \$45; life insurance, \$250; fire insurance, \$8; other expenses, \$825. The remainder he puts into the bank. What per cent of his salary does he put into the bank?

6. A merchant buys sugar at an average of 4 cts. a pound, and sells it at a profit of 8 %. How many pounds must he sell to make \$640?

7. There are 100 sq. rd. in .625 of an acre. How many square rods in $\frac{1}{8}$ of an acre? In 25 % of an acre?

8. In $\frac{7}{8}$ of a mile there are 280 rd. How many rods in .5 of a mile? In $62\frac{1}{2}$ % of a mile?

9. How long is a pile of wood 4 ft. high and 4 ft. wide that contains a cord?

10. If 1 ft. be added to the height of the pile, what is the fractional increase of the whole pile?

11. If 2 ft. be added to the height, what is the per cent of increase in the pile?

12. If 6 in. be added to the height, what is the per cent of increase?

13. What is a pile of wood 8 ft. long, 5 ft. high, 4 ft. wide worth, at \$6.40 per cord?

14. A pile of oak wood 8 ft. long, 6 ft. high, 4 ft. wide was sold for \$8.40 per cord. How much was received for it?

15. Mr. DeMott sold a load of wood 8 ft. long, 4 ft. 6 in. high, 4 ft. wide, for \$5.25 per cord. How much did he receive for it?

16. If a foot be added to the length of a cord, what fractional part will the pile be increased? If 2 ft. be added?

17. Find the value of a pile of wood 9 ft. long, 4 ft. high, and 4 ft. wide, at \$6.40 per cord.

18. James paid \$18.60 for a suit of clothes; Harry paid $16\frac{2}{3}\%$ more for his suit. What did Harry's suit cost?

19. A telegraph pole standing near a tree is 45 ft. high. The tree is $11\frac{1}{3}\%$ higher than the pole. What is the height of the tree?

20. Mr. Putnam had 300 bu. of apples. He sold $83\frac{1}{3}\%$ of them for 50 cts. a bushel. How much did he receive for them?

21. From a coil of rope 50 ft. were sold, which was $\frac{5}{8}$ of the coil. How long was the coil?

22. In a gale $\frac{5}{12}$ of Mr. Greene's fence was blown down; 140 ft. remained standing. How long was the fence?

23. A pair of shoes marked \$3.60 was sold at $16\frac{2}{3}\%$ less than the marked price. What was received for the shoes?

24. What is the quotient of 20% of 80, divided by $\frac{2}{3}$ of 12?

25. $\frac{2}{3}$ of 24 is 10 % of what number? $\frac{5}{9}$ of 45 is $12\frac{1}{2}$ % of what number?

26. 75 % of 60 is how many times $\frac{2}{3}$ of 24?

27. Mr. Hawthorne bought wheat at 56 cts. a bushel and sold it at an advance of $12\frac{1}{2}$ %. How much did he make on 1,000 bu.? On 250 bu.?

28. What per cent was made on butter that cost 20 cts. a pound and sold for 22 cts. a pound?

29. What per cent was lost on cloth that cost \$2.40 a yard and sold for \$2 a yard?

30. Find the value of x in each of the following proportions:

(1) $\$16 : \$x = \$10 : \$5$.

(2) $40 \text{ ft.} : 10 \text{ ft.} = x \text{ ft.} : 5 \text{ ft.}$

(3) $100 \text{ yd.} : 33\frac{1}{3} \text{ yd.} = 27 \text{ yd.} : x \text{ yd.}$

31. Find the value of 45 lb. of butter at 25 cts. a pound. Of 36 lb. of cheese at $12\frac{1}{2}$ cts. Of 75 yd. of cloth at $33\frac{1}{3}$ cts.

32. What is the ratio of 2,000 lb. to 500 lb.? To 250 lb.? To 3,000 lb.?

33. Mr. Barlow sold 3,500 lb. of hay at \$24 a ton, and 4,500 lb. of straw at \$16. How much did he receive for each?

34. How many pounds are there in .7 of a ton? In $12\frac{1}{2}$ % of a ton? In $\frac{2}{3}$ of a ton?

35. What is the value of 325 lb. of butter at 20 cts. a pound?

SUGGESTION. 20 cts. per pound = \$20 per hundredweight. 325 lb. = $3\frac{1}{4}$ cwt.

36. Find the value of 450 lb. of butter at 22 cts. a pound.

37. At 16 cts. a pound, what are 225 lb. of cheese worth? $150\frac{3}{4}$ lb.? 75 lb. 8 oz.?

38. What is the ratio of 1 ft. 3 in. to 2 ft. 6 in.? Of 4 ft. to 1 ft. 4 in.?

39. In $62\frac{1}{2}\%$ of a mile there are 1,100 yd. How many yards are there in a mile?

40. How many square inches in 2 sq. ft.? In 10 sq. ft.?

41. Jerome has \$40. $\frac{3}{4}$ of his money equals $\frac{2}{3}$ of Theodore's money. How much money has Theodore?

42. If 12 men can build a barn in 20 da., how long will it take 16 men to build it?

43. Mr. Fisk employed 30 men 12 da. to dig a ditch. How many men would it have taken to dig the ditch in 4 da.?

44. If $\frac{5}{8}$ of Susan's money equals $\frac{3}{4}$ of Emily's money, and Emily has \$80, how much money has Susan?

45. Mr. Seton has a square field containing 121 sq. rd. How many yards in the perimeter of the field?

46. Florence has a square flower bed whose perimeter is 4.8 yd. What is its area in square yards? In square feet?

47. Mr. Anthony has a rectangular beet bed. Its width is 4 ft., and its length $2\frac{1}{2}$ times its width. How many square feet in its area?

48. He has a bed of carrots also, whose area is 160 sq. ft. and width 8 ft. What is its perimeter?

49. The base of a right triangle is 18 ft. 6 in.; the altitude is 20 ft. What is its area?

50. What is the average length of two ropes, one 16 ft. 8 in. long, the other $14\frac{1}{3}$ ft.?

51. Frank rode his bicycle 80 mi. Monday, 100 mi. Tuesday, 60 mi. Wednesday, and 56 mi. Thursday. How many miles did he average daily?

52. How many cubic feet in 5 loads of stone, each 8 ft. long, 4 ft. wide, and 2 ft. high?

53. A cubic foot equals approximately .8 of a bushel of grain. About how many bushels will a bin 30 ft. long, 20 ft. wide, and 10 ft. deep hold?

54. A cubic foot of liquid contains approximately $7\frac{1}{2}$ gal. What is the approximate number of gallons that a cistern 5 ft. square and 10 ft. deep will contain?

55. The water tank in Mr. Stull's house is 10 ft. long, 6 ft. wide, and 5 ft. deep. What is the approximate number of gallons that it will hold?

56. Allowing 22 bricks to a cubic foot, how many bricks will it take to build a wall 20 ft. long, 5 ft. thick, and 4 ft. high?

57. Allowing to a square foot, $4\frac{1}{2}$ bricks laid flatwise, how many bricks will it take to pave a yard 20 ft. square?

58. Allowing 9 bricks laid edgewise, how many bricks will it take to pave the same yard?

59. What is the area of an isosceles triangle whose base is 22 in. and altitude 71 in.?

60. What is the circumference of a circle whose diameter is 49 in.?

61. What is the circumference of a circle whose radius is 14 in.?

62. What is the diameter of a circle whose circumference is 440 in.?

63. Find the circumferences of circles having the following diameters:

(1) 14 in.

(3) 56 in.

(2) 1 ft. 9 in.

(4) 2 ft. 4 in.

64. Find the diameters of circles having the following circumferences:

(1) 44 in.

(3) 88 ft.

(2) 7 yd. 1 ft. ?

(4) 9 ft. 2 in.

65. The radius of a circle is 14 yd. What is its circumference in yards? In feet?

66. The inner circumference of a bicycle track is 88 rd. Henry stands in the center of the inclosure. How far is he from the track?

67. What is the altitude of a right triangle containing 675 sq. in., whose base measures 30 in.?

68. According to the latest census, Louisiana had 115,969 farms. Of these, cane for making syrup or sugar was raised on 13,881. On what per cent of the farms was cane grown?

69. 3,421,751 acres were "farmed." On 277,903 acres cane was grown. What per cent of all the farmed acres was devoted to cane? (Carry to two places.)

70. On 1,376,254 acres cotton was raised. What per cent of all the farmed acres was this?

71. Mr. Duncan bought a sugar plantation of 400 acres in Louisiana, paying \$70 an acre, including buildings and equipment. The first year the land produced 18 tons of cane to the acre. How many tons of cane were produced?

72. The cane, when ground, produced 80% of juice. How much juice did it yield?

73. The juice boiled produced 14% of sugar. How much sugar was produced?

74. It cost Mr. Duncan \$15 an acre for labor, \$16 an acre for seed, and \$1.55 a ton of cane for making the sugar. He sold the sugar for $3\frac{1}{2}$ cts. a pound. How much did he receive? What was his profit on the crop?

75. There were 79,486 cotton farms in Louisiana, covering 1,376,254 acres. What was the average acreage per farm? What is it worth at \$25 per acre?

76. The value of all the farm lands was \$39,829,180, with buildings worth \$11,233,850, implements and machinery worth \$2,950,730, and live stock worth \$13,491,383. What was the total farm valuation?

77. The cotton crop for the year was valued at \$36,823,212. The expenditures were: for fertilizers, \$182,460; for labor, \$1,513,610. What was the net value of the cotton crop?

78. What per cent of the value of the farm property was the value of the cotton crop?

79. Mr. Dunn of South Carolina had 400 acres in Sea Island cotton. He raised .3 of a bale (500 lb.) per acre. It cost him \$10.50 a bale for ginning. Labor and fertilizers cost \$4.00 an acre. What was Mr. Dunn's net income from his farm, with cotton at 20¢ a pound?

80. The "Grizzly Giant," one of the "big trees" of the Mariposa Grove in California, is 93 ft. in circumference. What is its diameter?

81. One of the largest trees in the world is a Mexican cypress, 112 ft. in circumference. What is its diameter?

82. Find the interest on \$890 for 6 yr. 7 mo. 10 da., at 5%.

83. Find the interest on \$650 from April 13, 1910, to July 19, 1910, at 4%.

84. Find the interest on \$861 for 3 yr. 2 mo. 12 da., at 6%.

85. Find the interest on \$380.40 for 4 yr. 5 mo., at 5%; at 4%.

86. Find the interest on \$267.33 for 3 yr. 3 mo., at 3%; at 7%.

87. On May 15, 1910, Mr. French borrowed \$200.75. He paid it on Aug. 4, 1910, with interest at 5%. How much did he pay?

88. How many bricks, each 8 in. $\times$ 4 in., will it take to pave a walk 60 ft. long, 3 ft. 6 in. wide?

89. How many granite blocks will it take to pave a street 920 ft. long and 50 ft. wide, if it takes 6 blocks for each square foot?

90. Mr. Hopkins, an Illinois farmer, experimented with different kinds of hay on 5 ten-acre fields as nearly alike in quality of soil as possible.

He obtained tons of hay from each field as follows:

Field 1.	Wild Prairie Grass	. . .	11 tons
Field 2.	Millet	. . .	18½ tons
Field 3.	Alfalfa	. . .	9.3 tons
Field 4.	Clover	. . .	18.3 tons
Field 5.	Timothy and Red Top	. . .	11.6 tons

(1) What was the average yield per acre for each kind of grass?

(2) What per cent larger was the yield of millet than that of each other kind? The yield of clover than the yield of Timothy and Red Top?

91. Mr. Hyland, a Colorado farmer, tried a similar experiment on his farm with the following results:

Yield from 10 acres:

Wild Grass	. . .	8.2 tons
Millet	. . .	10 tons
Alfalfa	. . .	34 tons
Clover	. . .	22.7 tons
Timothy and Red Top	. . .	15.7 tons

(1) What was the average yield per acre for each kind of grass?

(2) By what per cent was the yield of alfalfa greater than that of each other kind? The yield of clover than the yield of wild grass?

92. On a scale of 1 inch to 40 feet, draw a diagram to represent a house lot 60×120 ft. Find the cost at \$80 a front foot. On your diagram draw the house, walks, and yard as described in Problems 93 and 94.

93. The house is 40 feet wide and 60 feet deep. It stands 40 feet back from the front of the lot and at equal distances from the two sides. How many square feet are covered by the house? How many remain in the lot? A cement walk 5 feet wide is laid from the front of the lot to the house. Another walk is laid from the rear of the lot to the house. At \$.20 a square foot, what did the walks cost?

94. The remainder of the front yard was sodded at a cost of 27 cts. per square yard. The other part of the lot was seeded at 3 cents a square yard. What did it cost to sod the front yard, and to seed the remainder?

95. The sixth grade made in their manual training class a case of book shelves for the schoolroom. The case was 4 ft. high and 3 ft. wide and contained 4 shelves. Draw a diagram of the case on a scale of $\frac{1}{2}$ in. to 1 ft. and find out how many feet of timber it will take to make it.

The sides of the case are 8 in. wide and the shelves are 6 in. deep. Make estimates, as follows:

Number of square feet of boards in sides.

Number of square feet of boards in top.

Number of square feet of boards in shelves.

If boards 12 in. wide are bought, how much is wasted?

96. At 4 cts. a square foot what will the wood used cost? What is the cost of the waste?

97. Can you suggest a more economical plan of purchasing boards? How much would be saved by your plan?

98. How many books averaging $1\frac{1}{2}$ in. thick can be placed in the shelves?

99. Avery rode on his bicycle from Norton to Penrose, $15\frac{1}{4}$ miles, in $1\frac{1}{2}$ hours. At what rate did he ride?

100. A ten-dollar gold piece weighs 258 grains. $\frac{9}{10}$ of its weight is pure gold. How many grains of gold are there in a single coin? How many grains of alloy?

101. A bill was \$127.50 if paid in three months, or 6% less if paid at once. I borrowed the money at 5% and paid the bill at once. How much did I gain?

102. A room is 15 ft. 8 in. long, 12 ft. wide, and 9 ft. 6 in. high. How many square feet are there in its walls and ceiling?

103. It has 3 windows 6 ft. $\times$ $3\frac{1}{2}$ ft. What per cent of the wall space is occupied by windows?

104. Divide 83.75 by .0125 and multiply the quotient by 3.007.

105. Add $2\frac{1}{3}$; $7\frac{4}{5}$; $8\frac{5}{12}$; $10\frac{7}{8}$; $9\frac{3}{16}$.

106. Subtract $8\frac{3}{4}$ from $12\frac{5}{8}$ and multiply the difference by $2\frac{6}{7}$.

107. How many days, hours, minutes, in $\frac{3}{5}$ of a week? In $\frac{5}{7}$ of a month? Allow 30 da. to a month.

108. Make a bill for the following and find the total cost:

25 lb. sugar at $5\frac{1}{2}$ cts. per pound.

$2\frac{1}{2}$ lb. tea at 75 cts. per pound.

5 boxes of crackers at $12\frac{1}{2}$ cts. per box.

3 cans of vegetables at $17\frac{1}{2}$ cts. per can.

4 lb. of rice at $9\frac{1}{2}$ cts. per pound.

3 doz. oranges at $\$ \frac{2}{5}$ per dozen.

$1\frac{1}{2}$ doz. lemons at $\$ \frac{3}{10}$ per dozen.

109. A grocer bought 3 barrels of apples, each containing 2 bu., 3 pk., at $\$ 2\frac{1}{2}$ per barrel. He sold them at 40 cts. per peck. How much did he gain?

110. Alfred's mother gave him a pie for himself and 3 boy friends. He divided it equally. What was the angle at the center of each piece?

111. What part of a circle was each piece? How many degrees were there in each piece?

112. Elsie's mother made her a birthday cake and put 10 candles on it. Elsie cut it into 10 equal parts, 1 for each candle. What part of a circle was each part? How many degrees did it contain?

113. At 9 o'clock what kind of an angle is made by the clock hands? At 3 o'clock? At 4 o'clock?

114. How many degrees are there in the angle made by the clock hands at 9 o'clock? At 3 o'clock?

115. Miss Jewett asked to have a new circle painted on the floor of the kindergarten, large enough for 25 children. Allowing 1 yard for each child, what must the diameter of the circle be?

116. The minute marks on the dial of the big tower clock are 1 foot apart. What is the diameter of the dial?

117. Workmen who receive $\$2\frac{1}{4}$ per day when working $7\frac{1}{2}$ hr. per day are asked to work 9 hr. per day during a busy season. What increase in wages should they receive?

118. At how much must cloth that was bought for 88 cts. per yard be sold to gain 20 %? To gain 25 %?

119. A merchant sold 625 yd. of silk that was damaged in a fire at a loss of 5 %. If he sold it for 57 cts. a yard, how much did it cost him?

120. Find the interest and amount of \$ 250 for 2 yr. 6 mo. at 3 %. At 4 %. At 5 %.

121. Fred's father deposited \$25 for him in a savings bank on his tenth birthday and added \$25 on each following birthday until Fred was 21 years old. If the bank paid 4 %

interest and added the interest to the principal each year, how much money had Fred in the bank after the last deposit was made?

122. A man bought a house and lot for \$5500. He paid \$400 for repairs, \$62 for taxes, \$24 a year for water, and \$14 a year for insurance. If he estimated the money invested at 5%, what had the house cost him at the end of the first year?

123. The house was rented for \$40 per month. Did the man gain or lose on his investment, and how much?

124. Prove the following proportions by comparing the products of the extremes and the means :

(1) $12 : 24 = 20 : 40.$

(4) $9 : 63 = 23 : 161.$

(2) $18 : 54 = 24 : 72.$

(5) $13 : 52 = 19 : 76.$

(3) $16 : 80 = 18 : 90.$

(6) $35 : 105 = 45 : 135.$

THE GILBERT ARITHMETICS

BY

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BOOK TWO

PART TWO

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PART II

CHAPTER I

THE FUNDAMENTAL PROCESSES

NOTATION AND NUMERATION

1. Write in as many ways as you can the number of the present year.

Read 26,057 ; MDXCVII.

Writing numbers is called **notation**.

Reading numbers is called **numeration**.

A regular plan of writing or reading numbers is called a **system** of notation or of numeration.

There are two systems of notation and numeration in use in this country : the **Arabic** and the **Roman**.

The Arabic system was invented by the ancient Hindus, the people of India. It was introduced into Europe by the Arabs, hence its name.

The Roman system was used by the ancient Romans.

The Arabic system is the one used in all common business transactions. The Roman system is used chiefly in the numbering of certain chapters and pages of books, and on the faces of clocks and watches.

Read 349; CCCXLIX. Write eighty-eight in both Arabic and Roman notation. Which do you think the better system? Why?

THE ARABIC SYSTEM

2. The characters, or figures, of the Arabic system are 1, 2, 3, 4, 5, 6, 7, 8, 9, called **digits**, and 0, called **zero**, or **cipher**.

Ten ones, or **units** = 1 ten; $10 \times 1 = 10$.

Ten **tens** = 1 hundred; $10 \times 10 = 100$.

Ten **hundreds** = 1 thousand; $10 \times 100 = 1,000$.

1,000 **thousands** = 1 million; $1,000 \times 1,000 = 1,000,000$.

1,000 **millions** = 1 billion; $1,000 \times 1,000,000$
= 1,000,000,000.

The use of numbers higher than millions is rare, though the following numbers are found by using 1,000 as a multiplier: billions, trillions, quadrillions, quintillions, sextillions, septillions, octillions, nonillions, and decillions. 1 decillion would contain 33 ciphers.

The Arabic system is called a **decimal** system (from the Latin word **decem**, meaning "ten") because it is a system of tens, each place having a value ten times greater or ten times less than the one next to it.

The word *digit* means "finger." It indicates that the decimal system grew out of the custom of counting on the fingers.

Numbers as written by the Arabic system are divided into **periods** of three figures each. According to its position each figure stands for units, tens, or hundreds of its period.

In 3,126,374,201 the value of each figure is as follows:

	MILLIONS			THOUSANDS			UNITS		
	Hundred millions	Ten millions	Millions (units)	Hundred thousands	Ten thousands	Thousands (units)	Hundreds	Tens	Units
3,	1	2	6,	3	7	4,	2	0	1

3. Each period is read as a separate number followed by the name of the period, excepting that the name of the unit's period is not read, but is "understood."

The number above is read: "Three billion, one hundred twenty-six million, three hundred seventy-four thousand, two hundred one."

The cipher is used merely to fill vacant spaces and is not read.

The places of the figures in a number are called **orders**.

In reading numbers from left to right, a figure in unit's place of any period is called a figure of the **first** order. A figure in ten's place is called a figure of the **second** order. A figure in hundred's place is called a figure of the **third** order, and so on.

4. WRITTEN EXERCISES

1. Copy, point off, and read :

1002356.	37027017.
467392.	2080104346.
100247016.	19405007084.
403006040.	700600050200.
830042.	14308012.

2. Express with figures :

7 billion, 80 million, 2 hundred thousand, 50.

20 billion, 3 million, 15 thousand, 8.

215 billion, 0 million, 140 thousand, 4 hundred.

305 million, 1 thousand, 11.

5 million, 5 thousand, 5 hundred, 5.

3. Write in words :

206,937.	60,215,840,075.
92,010,005.	40,003,090.
209,387,016.	1,002,003,040.
587,009,307.	26,807,243,201.

THE ROMAN SYSTEM

5. The Roman notation uses seven letters, namely :

I = 1,	V = 5,	X = 10,	L = 50,
C = 100,	D = 500,	M = 1000.	

These letters are combined to represent other numbers according to the following principles :

(1) When a letter is followed by the same letter or by a letter of less value, it indicates that the values of the letters are added.

Thus, II represents 2; VI, 6; XXX, 30; XV, 15; LX, 60; CX, 110; DC, 600.

(2) When a letter is followed by a letter of greater value, its value is to be taken from that of the greater.

Thus, IV represents 4; IX, 9; XL, 40; XC, 90; CD, 400; CM, 900.

(3) Placing a dash over a letter multiplies its value by 1,000.

Thus, $\overline{V}$ represents 5,000; $\overline{X}$, 10,000; $\overline{IV}$, 4,000.

6. ORAL AND WRITTEN EXERCISES

1. Read the following :

$\overline{M}$.	$\overline{LXXIX}$.	$\overline{CCV}$.	$\overline{MDXC}$.
XCIX.	CXVIII.	DXIX.	MDCCC.
DLXXV.	CCXXVI.	DCXL.	MMDC.
CXXXII.	CDLXIII.	DCCL.	IVCCXL.
DCLIX.	MDCC.	XCVI.	CCCXL.

2. Express the following in the Roman notation :

55.	61.	101.	496.	1,607.
66.	58.	114.	509.	5,000.
79.	99.	325.	1,900.	10,200.
94.	89.	423.	2,821.	16,306.

ADDITION

7. What is the sum of $123 + 456$?

What is the process of finding the sum called?

Addition is the process of combining two or more numbers into a single equivalent number, called the **sum**.

The numbers so combined are called **addends**.

1. Add 9 to each of the following: 12, 23, 32, 44, 55, 67.

2. How do you add 9 to numbers larger than 10?

3. Add 8 to each of the following: 13, 24, 35, 46, 57, 66, 83, 87, 94.

4. How do you add 8 to numbers larger than 10?

5. Add 7 to each of the following numbers: 23, 34, 45, 56, 68, 78, 82, 94.

6. How do you add 7 to numbers larger than 10?

7. Add: 6,845

5,385

8. Explain each figure of the sum.

ORAL EXERCISES

8. In the following numbers the dollars are separated from the cents by a line instead of a decimal point. This method is used in books of accounts. Add, while some one times you:

1. $\begin{array}{r} 96|48 \\ 34|73 \\ 65|48 \\ 84|56 \\ 45|68 \\ 7|15 \\ 5|71 \\ 28|39 \\ \hline 93|82 \end{array}$

2. $\begin{array}{r} 465|73 \\ 376|45 \\ 283|91 \\ 75|60 \\ 29 \\ 25|98 \\ 9|17 \\ 724|82 \\ \hline 239|56 \end{array}$

3. $\begin{array}{r} 875|23 \\ 3,468|71 \\ 51|98 \\ 918|46 \\ 7,326|93 \\ 3,962|58 \\ 5,806|73 \\ 3,785|64 \\ \hline 97|08 \end{array}$

4. $\begin{array}{r} 28,675|09 \\ 86,750|92 \\ 67,509|86 \\ 75,092|28 \\ 50,928|67 \\ 92,764|92 \\ 89,674|85 \\ 76,658|37 \\ \hline 98,747|61 \end{array}$

TESTING ADDITION

$$\begin{array}{r}
 9. \text{ Add and test:} \quad 3,184 \\
 \quad \quad \quad 5,732 \\
 \quad \quad \quad 6,813 \\
 \quad \quad \quad 5,940 \\
 \quad \quad \quad 6,028 \\
 \hline
 \quad \quad \quad 27,697
 \end{array}$$

Test the result of the above by the following rule:

Add the columns first upward, then downward. If the two results agree, the sum is *probably* correct.

NOTE. For testing addition 'by casting out the nines,' see page 262.

ORAL EXERCISES

10. Add the following and test:

1. 169	2. 1,306	3. 321	4. 72
724	271	42	777
382	82	1,675	8,888
576	3,574	4,107	9,999
444	16,999	6,010	10,101
<u>555</u>	<u>1,004</u>	<u>7,280</u>	<u>23,326</u>

SUBTRACTION

11. What is the difference of 327 — 226 ?

Subtraction is the process of finding how many units more or less one number contains than another.

The larger number is called the **minuend**; the smaller is called the **subtrahend**. The excess is called the **difference**.

$$\begin{array}{r}
 1. \text{ Subtract: } 2,601 \\
 \quad \quad \quad 1,777 \\
 \hline
 \end{array}$$

2. How do you subtract when a figure in the subtrahend is larger than a figure of the same order in the minuend ?

WRITTEN EXERCISES

12. Subtract :

- | | | | |
|----------------------------|----------------------------|-----------------------------|------------------------------|
| 1. 67,023
<u>51,569</u> | 2. 81,456
<u>29,879</u> | 3. 103,521
<u>85,762</u> | 4. 324,510
<u>167,823</u> |
| 5. 367,531 - 12,987. | 6. 120,762 - 94,876. | | |

MAKING CHANGE

13. I bought goods amounting to \$4.15 and gave a \$5 bill. Count the change by adding aloud; thus, "\$4.15, \$4.25, \$4.50, \$5.00."

What coins were used?

Make change for the following, by adding aloud :

PURCHASE	MONEY GIVEN	PURCHASE	MONEY GIVEN
1. \$1.25	\$ 2.00	6. \$ 0.38	\$ 1.00
2. 7.23	10.00	7. 0.26	0.50
3. 8.16	10.00	8. 14.30	15.00
4. 4.10	5.00	9. 40.16	100.00
5. 6.21	10.00	10. 67.20	100.00

TESTING SUBTRACTION

14. To test the correctness of subtraction :

Add the difference to the subtrahend; if the sum equals the minuend, the difference is correct. Explain why.

WRITTEN EXERCISES

15. Subtract the following and test :

- | | | |
|----------------------------|----------------------------------|------------------------------------|
| 1. 67,584
<u>56,784</u> | 2. \$9,230.64
<u>1,873.79</u> | 3. 44,445,555
<u>34,446,666</u> |
| 4. 94,387
<u>29,699</u> | 5. 3,061.51
<u>1,982.62</u> | 6. 68,901,332
<u>27,072,459</u> |

NOTE. For testing subtraction by casting out the nines, see page 262.

FINDING NUMBERS FROM THEIR SUM AND DIFFERENCE

16. One half the sum of the sum and difference of two numbers equals the larger number.

One half the difference of the sum and difference of two numbers equals the smaller number.

$$7 + 5 = 12 \text{ (sum).}$$

$$7 - 5 = 2 \text{ (difference).}$$

$$12 + 2 = 14 \text{ (sum of sum and difference).}$$

$$14 \div 2 = 7 \text{ (the larger number).}$$

$$12 - 2 = 10 \text{ (difference of sum and difference).}$$

$$10 \div 2 = 5 \text{ (the smaller number).}$$

ORAL AND WRITTEN EXERCISES

17. Solve:

1. The sum of two numbers is 80; their difference is 20. What are the numbers?

2. The number of votes cast for two candidates was 3,200. The successful candidate had 200 more than the other. How many votes did each receive?

3. Luther and Richard together have 120 marbles. Luther has 10 marbles more than Richard. How many has each?

4. John and Henry in a relay race ran 7 miles. John ran 2 miles farther than Henry. How far did each run?

5. Mr. Allison gave his sons, Alfred and Charles, \$28, giving each \$1 for every year of his age. He gave Charles \$6 more than Alfred. How old was each boy?

6. Mary and Lucy together picked 66 qt. of strawberries. Mary picked 6 qt. more than Lucy. How many did each pick?

7. John and James have 80 cents. John has 20 cents more than James. How many cents has each?

ADDITION AND SUBTRACTION

WRITTEN EXERCISES

18. The fifteen universities in the United States having the largest number of students in the year 1909 were:

Harvard	5,558	Cornell	5,028
Columbia	6,132	* Wisconsin	4,245
* Chicago	5,487	Yale	3,276
* Northwestern	3,197	Pennsylvania	4,857
* Michigan	5,259	Syracuse	3,284
* California	4,084	* Nebraska	3,402
* Illinois	4,502	New York	3,843
* Minnesota	4,351		

1. How many students were enrolled in Harvard, Yale, and Columbia? How many in Columbia, Cornell, and Syracuse?

2. How many students were enrolled in the eastern universities? How many in the western universities?

(Stars show western universities.)

3. How many were enrolled in the fifteen universities?

4. Which of the western universities had the largest enrollment?

5. How much larger was it than that of each of the other western universities?

6. How many more were enrolled in Columbia than in each of the others?

7. Which had the more students, the eastern or western universities? How many more?

8. January 1, Mr. Brown had in the bank \$1263.75. He made the following deposits during the month; \$180, \$67.73, \$180, \$24.30, \$635.02, \$873.60, \$765, \$937.50.

(1) How much did he deposit?

(2) How much did he have in the bank during the month?

9. February 1, he called for a statement of his account with the bank, and together with the paid checks he received a slip of paper on which were written the amounts of the checks drawn.

(1) Add these amounts to find how much money he had drawn from the bank.

(2) How may the amount which Mr. Brown has in the bank February 1 be found?

(3) Find the amount of his balance in the bank on that date.

\$101.50
9.00
88.79
17.00
10.00
25.00
25.00
84.35
3.14
108.15
60.00
10.00
124.20
21.23
2.80
92.75
7.25
212.75
18.93
211.45
1.40
21.00
<u>2.52</u>

10. The country's iron production in tons during three years was as follows:

	1908-9	1907-8	1906-7
July	2,101,579	1,218,129	2,259,682
June	1,929,884	1,088,634	2,231,675
May	1,883,330	1,149,688	2,294,005
April	1,738,877	1,149,602	2,216,558
March	1,836,194	1,228,204	2,226,457
February	1,707,340	1,079,721	2,045,068
January	1,797,560	1,045,250	2,205,607
December	1,740,912	1,234,279	2,236,153
November	1,577,854	1,828,125	2,187,665
October	1,567,198	2,336,972	2,196,808
September	1,418,998	2,163,437	1,970,962
August	1,359,831	2,250,419	1,926,736

(1) What was the total production of iron for each year?

(2) How much larger was the production of 1906-7 than that of each of the other years?

(3) How much greater was the production of 1908-9 than that of 1907-8?

MULTIPLICATION

19. How many days are there in 4 years?

$$365 + 365 + 365 + 365 = 365 \times 4 = 1,460.$$

Multiplication is a short method of finding the sum of equal numbers.

Define **multiplier**, **multiplicand**, **product**.

The multiplier and the multiplicand are **factors** of the product.

The multiplicand (one of the equal addends) may be a **concrete** number, as 7 days, 34 years. The multiplier is always an **abstract** number and tells simply how many times the multiplicand is taken, or how many addends are to be added.

For example: To find out how many days there are in four weeks, we take 7 days 4 times, not 4 weeks times.

If the number to be multiplied is smaller than the multiplier, it may be simpler to perform the multiplication as if both were abstract numbers and to multiply the larger by the smaller, adding the name afterwards.

Thus: How many days are 365 times 17 days?

$$\begin{array}{r} \text{SOLUTION:} \qquad 365 \\ \qquad \qquad \qquad 17 \\ \hline \qquad \qquad \qquad 2555 \\ \qquad \qquad 365 \\ \hline \qquad \qquad 6205 \end{array}$$

There are 6,205 days in 365 times 17 days.

20. Multiply \$8,042.30 by 2,060.

$$\begin{array}{r} \text{SOLUTION:} \qquad \$8042.30 \\ \qquad \qquad \qquad 2060 \\ \hline \qquad \qquad \qquad 482538 \\ \qquad \qquad 160846 \\ \hline \qquad \$16567138.00 \end{array}$$

1. Explain what you do with each cipher in the multiplicand and in the multiplier.
2. Why is 6, the right-hand figure of the second partial product, placed under the 5 of the first?

TESTING MULTIPLICATION

21. To test a product, divide it by either the multiplier or the multiplicand. If it is correct, the quotient should equal the other term.

NOTE. For testing multiplication by casting out the nines, see page 268.

Multiply and test:

$$\begin{array}{r} 1. \quad 67,845 \\ \quad \quad 372 \\ \hline \end{array}$$

$$\begin{array}{r} 2. \quad 189,763 \\ \quad \quad 4,279 \\ \hline \end{array}$$

$$\begin{array}{r} 3. \quad 54,321 \\ \quad \quad 12,345 \\ \hline \end{array}$$

$$\begin{array}{r} 4. \quad 43,768 \\ \quad \quad 9,999 \\ \hline \end{array}$$

$$\begin{array}{r} 5. \quad 157,865 \\ \quad \quad 8,769 \\ \hline \end{array}$$

SHORT METHODS OF MULTIPLICATION

22. To MULTIPLY BY 5, 50, 500, OR 5,000.
 1. How is an integer multiplied by 10? By 100? By 1,000?
 2. What is the ratio of 10 to 5? Of 100 to 50? Of 1,000 to 500?
 3. Multiply 87,659 by 5.

$$87,659 \times 10 = 876,590.$$

$$876,590 \div 2 = 438,295.$$
 4. Multiply 8,482 by 50.

$$8,482 \times 100 = 843,200.$$

$$843,200 \div 2 = 421,600.$$
 5. Multiply 5,763 by 50. By 500.
 6. Make a rule for a short method of multiplying by 5. By 50. By 500.

23. TO MULTIPLY BY 25 AND 125.

1. What is the ratio of 100 to 25? Of 1,000 to 125?
2. Make a rule for a short way of multiplying by 25.
By 125.

WRITTEN EXERCISES**24. Solve:**

1. Multiply 46,753 by 25. By 125.
2. Multiply 436,854 by 125.
3. How many days in 5 years? In 50 years? In 500 years? In 125 years?
4. At \$50 apiece, how much will 125 bicycles cost? At \$65? At \$70.50?
5. Find the cost of:
 - (1) 37 pencils, at 5 cts. each.
 - (2) 45 Arithmetics, at 50 cts. each.
 - (3) 77 Spelling Books, at 25 cts. each.
 - (4) 3 doz. golf balls, at 50 cts. each.
 - (5) 45 tennis rackets, at \$1.25 each.
 - (6) 75 chairs, at \$5 each.
 - (7) 3 doz. brooms at 25 cts. each.
 - (8) 14 hats at \$2.50 each.
 - (9) 18 pairs of shoes at \$3.50 per pair.
 - (10) 2 doz. pairs of gloves at \$1.25 per pair.

25. TO MULTIPLY BY 9; by 99; by 999.

$$9 = 10 - 1; \quad 99 = 100 - 1; \quad 999 = 1,000 - 1.$$

Multiply 45 by 9

$$9 \times 45 = 10 \times 45 - 45 = 450 - 45 = 405.$$

Make a rule for multiplying by 9.

1. Multiply 37 by 9. By 99. By 999.
2. How many ciphers are annexed in multiplying by 99?
By 999?

WRITTEN EXERCISES

26. Find the cost of:

1. 16 pencils, at 9 cts. each.
2. 432 packages of breakfast food, at 9 cts. each.
3. 165 straw hats, at 99 cts. each.
4. 37 boys' suits, at \$9.99. each.
5. 43 two-pound boxes of chocolates, at 99 cts. each.

27. TO MULTIPLY BY $12\frac{1}{2}$.

1. Multiply 16 by $12\frac{1}{2}$.
2. What is the ratio of $100 : 12\frac{1}{2}$?
3. Make a rule for multiplying by $12\frac{1}{2}$.

WRITTEN EXERCISES

28. Solve:

1. Multiply each of the following numbers by 25 and by $12\frac{1}{2}$: 32, 64, 80, 20, 35, 66, 26, 37, 38, 42, 56, 50, 120, 124, 240, 160, 260, 168.

2. Mr. Wilmot purchased an acre of land at $\$12\frac{1}{2}$ per square rod. Find the cost.

3. Mr. Roberts bought two wood lots. One contained 85 acres, for which he paid \$25 per acre; the other contained 112 acres, for which he paid $\$12\frac{1}{2}$ per acre. Find the cost of each lot.

29. TO MULTIPLY BY $33\frac{1}{3}$ AND BY $16\frac{2}{3}$.

1. Multiply 27 by $33\frac{1}{3}$.
2. What is the ratio of $100 : 33\frac{1}{3}$?
3. Make a rule for multiplying by $33\frac{1}{3}$.
4. Multiply 36 by $16\frac{2}{3}$.
5. What is the ratio of $100 : 16\frac{2}{3}$?
6. Make a rule for multiplying by $16\frac{2}{3}$.

WRITTEN EXERCISES

30. Solve:

1. Multiply by $33\frac{1}{3}$ the number of months in a year. Of days in June. Of hours in a day. Of minutes in an hour. Of gills in a gallon. Of quarts in a bushel. Of ounces in a pound. Of rods in a mile.

2. A merchant employs two clerks. To one he pays $\$33\frac{1}{3}$ per week, and to the other $\$16\frac{2}{3}$ per week. What is the yearly salary of each?

3. Mr. Lindsley has a farm of which $33\frac{1}{3}$ acres are sown with oats, 25 acres planted with corn, and $12\frac{1}{2}$ with potatoes. If the oats yield 32 bushels per acre, the corn 41 bushels, and the potatoes 240 bushels, how many bushels of each will he have?

4. If a man earns $\$33\frac{1}{3}$ a week and spends $\$16\frac{2}{3}$, how many dollars will he save in 36 weeks? In one year, if he works every week?

ACCOUNTS

31. A record of business transactions is called an **account**.

A person who owes a debt is called a **debtor**, and a person to whom the debt is owed is called a **creditor**.

An amount due to another is called a **debit**.

An amount due from another is called a **credit**.

The difference between the sum of the debits and the sum of the credits in an account is called the **balance**.

Entering the balance on the smaller side and ruling the account is called closing or **balancing** the account.

In business the closing of an account is usually done at stated times, generally at the end of the month.

When the account is closed for the stated time, the balance is carried over to the following account of the next stated time.

Some abbreviations in common use are :

Debit or debtor . . .	Dr.	Account	Acct.
Credit or creditor . .	Cr.	Merchandise	Mdse

The following form illustrates a closed account of S. T. Delmar, a farmer, with his grocer, R. L. Falmouth. The value of farm products that Mr. Delmar sells Mr. Falmouth and any cash that he pays are put in the debtor column. They show what Mr. Falmouth owes Mr. Delmar.

The cost of the groceries that Mr. Delmar buys of Mr. Falmouth is put in the creditor column. This column shows what is due Mr. Falmouth from Mr. Delmar.

1906.		R. L. FALMOUTH		Dr.	Cr.
July	1	By 7 lb. Sugar			48
"	"	" 1 bbl. Flour			5 50
"	5	To 3 bbl. Potatoes	\$3.80	11 40	
"	"	By 5 lb. Prunes	12¢		60
"	"	" 2 lb. Crackers	10¢		20
"	8	To 8 doz. Eggs	42¢	3 36	
"	"	" 12 lb. Butter	35¢	4 20	
"	12	By 4 lb. Rice	9¢		36
"	"	" 2 doz. Oranges	40¢		80
"	16	To 3 bbl. Apples	\$3.85	11 55	
"	"	By 1 bbl. Sugar			18 75
"	20	To 8 bbl. Kindling Wood	18¢	1 44	
"	"	By 6 lb. Lard	15¢		90
"	23	" 2½ lb. Cheese	16¢		40
"	"	" 3 lb. Coffee	35¢		1 05
"	30	" 2 pkg. Cereals	15¢		30
"	31	" 3 cans Peaches	25¢		75
		Balance			1 86
				31 95	31 95
		Balance Carried Forward,		1 86	

What is the sum of the debits in the account ?

What is the sum of the credits ?

How much is the balance that Mr. Falmouth owes Mr. Delmar ?

On which side of the account is it entered ? Why ?

With what word are the debit items introduced ? The credit items ?

The same account would appear in R. L. Falmouth's book as follows :

1906.		S. T. DELMAR	Dr.	Cr.
July	1	To 7 lb. Sugar	48	
"	"	" 1 bbl. Flour	5 50	
"	.5	By 3 bbl. Potatoes \$ 3.80		11 40
"	"	To 5 lb. Prunes 12¢	60	
"	"	" 2 lb. Crackers 10¢	20	
"	8	By 8 doz. Eggs 42¢		3 36
"	"	" 12 lb. Butter 35¢		4 20
"	12	To 4 lb. Rice 9¢	36	
"	"	" 2 doz. Oranges 40¢	80	
"	16	By 3 bbl. Apples \$ 3.85		11 55
"	"	To 1 bbl. Sugar	18 75	
"	20	By 8 bbl. Wood 18¢		1 44
"	"	To 6 lb. Lard 15¢	90	
"	23	" 2½ lb. Cheese 16¢	40	
"	"	" 3 lb. Coffee 35¢	1 05	
"	30	" 2 pkg. Cereals 15¢	30	
"	31	" 3 cans Peaches 25¢	75	
		Balance	1 86	
			31 95	31 95
		Balance		1 86

What do the items in the debtor column show ? Those in the creditor column ?

WRITTEN EXERCISES

32. Write the accounts of the different persons mentioned in the following transactions, as they would appear in the books:

1. B. H. Graham, a farmer, bought of Joralemon & Co., grocers:

Feb. 6, 1906.	3 gallons molasses	.	.	@	\$.75
" 10, "	4 bags flour	.	.	"	1.25
" 15, "	2 cans salmon	.	.	"	.18
" 18, "	6 pounds butter	.	.	"	.35
" 21, "	3 pounds cheese	.	.	"	.16
" 23, "	10 pounds sugar	.	.	"	.06
" 27, "	2 dozen oranges	.	.	"	.40

Sold to Joralemon & Co.:

Feb. 8, 1906.	2 cords wood	.	.	@	\$ 5.25
" 18, "	40 pounds chickens	.	.	"	.20
" 23, "	10 dozen eggs	.	.	"	.32
" 27, "	5 gallons maple syrup	.	.	"	.95
" 28, "	12 bushels potatoes	.	.	"	.85

2. R. L. Martin sold Charles Miller:

July 1, 1906.	10 pounds rice	.	.	@	\$.10
" 5, "	4 dozen bananas	.	.	"	.15
" 12, "	2 boxes soap	.	.	"	2.75
" 15, "	1 pound pepper	.	.	"	.15
" 21, "	2 pounds raisins	.	.	"	.16
" 30, "	3 cans peaches	.	.	"	.25

Bought of Charles Miller:

July 3, 1906.	5 bushels potatoes	.	.	@	\$.95
" 10, "	3 crates strawberries				
	(96 quarts)	.	.	"	.08
" 20, "	12 pounds butter	.	.	"	.25
" 28, "	5 bushels onions	.	.	"	.80
" 28, "	15 dozen eggs	.	.	"	.25

3. Kearny & Clark sold F. H. Baldwin :

Sept. 2, 1906.	2 gross matches	. . @	\$3.75
" 9, "	28 pounds sugar	. . "	.06
" 14, "	2 pounds coffee	. . "	.28
" 20, "	10 pounds crackers	. . "	.16
" 23, "	5 pounds tea	. . "	.60
" 28, "	3 dozen oranges	. . "	.35
" 28, "	2 dozen lemons	. . "	.20
" 30, "	2 cans spice	. . "	.10
" 30, "	6 boxes cereal	. . "	.15

Bought of F. H. Baldwin :

Sept. 10, 1906.	2 barrels apples	. . @	\$2.00
" 18, "	16 dozen ears corn	. . "	.18
" 21, "	2 bags oats (5 bushels)	. "	.40 per bu.
" 27, "	1 barrel vinegar (40 gallons)	"	.28 per gal.
" 28, "	20 pounds butter	. . "	.28
" 28, "	15 dozen eggs	. . "	.24
" 30, "	8 bushels potatoes	. . "	.90

DIVISION**33. Into how many scores of years can you divide a century?**

How many times can you take 20 from 100?

Division is the opposite or *inverse* of multiplication. Division is a short method of finding how many times a given subtrahend may be taken from a number; or how many times a number contains a given number as a factor.

Define *divisor*, *dividend*, *quotient*, *remainder*.

The divisor and the quotient are factors of the dividend.

Express in three ways 6 divided by 2.

Divide 7,101 by 263.

Explain each step and name each trial divisor and trial dividend.

Is the division exact? How do you write the remainder?

34. Divide \$10,000 by 66.

$$\begin{array}{r}
 \$151.51 + \\
 66 \overline{) \$10000} \\
 \underline{66} \\
 340 \\
 \underline{330} \\
 100 \\
 \underline{66} \\
 340 \\
 \underline{330} \\
 100 \\
 \underline{66}
 \end{array}$$

In dividing cents, if the division is not exact, it is customary to omit the fractional remainder and place the sign + after the cents in the quotient.

TESTING DIVISION

35. To test the accuracy of division, multiply the quotient by the divisor, and to the product add the remainder, if any. What must the result equal? Why?

NOTE. For testing division by casting out the nines, see p. 263.

WRITTEN EXERCISES

36. Solve :

1. Divide and test :

$$(1) \quad 46 \overline{) 6175} \quad (3) \quad 321 \overline{) 19874} \quad (5) \quad 414 \overline{) 76853}$$

$$(2) \quad 195 \overline{) 5273} \quad (4) \quad 6789 \overline{) 66778899} \quad (6) \quad 1234 \overline{) 56789}$$

2. The highest mountain on the earth is Mt. Everest in India, 29,002 ft. above sea level. The greatest ocean depth known is 31,614 ft. in the Pacific near Guam. State these measurements in miles.

3. The average daily attendance in all the public schools in New York City in 1909 was 656,619 pupils. The appropriation for schools (exclusive of land and buildings) was \$27,470,736.80. How much was the cost per pupil?

4. The average daily attendance in the day schools was 545,218. If the pupils were equally divided among 15,695 teachers, how many pupils did each teacher have?

5. On a test run of 2 automobiles from New York to Boston, a distance of 250 mi., one car consumed $24\frac{3}{4}$ gal. of gasoline, and the other $33\frac{3}{4}$ gal. of kerosene. If the gasoline cost 15 cts. per gallon, and the kerosene cost 20 cts. per gallon, what was the cost of each kind of fuel per mile?

6. Following are the amounts of money of different kinds that were in circulation in the United States on August 1, 1909:

Gold coin,	\$ 596,806,435	Subsidiary silver,	\$132,857,008
Gold certificates,	806,284,359	Treasury notes of 1890,	4,156,121
Silver dollars,	71,887,688	U. S. notes,	338,928,434
Silver certificates,	477,213,767	National bank notes,	667,947,187

(1) Estimating the population at 80,000,000, what was the amount of each kind of money *per capita* (per head, or per person)?

(2) What was the amount of gold and silver coins to each person?

(3) What was the amount of all kinds of paper money per capita? How much more paper money per capita than coin was there?

(4) If all the money were divided equally among the inhabitants of the United States, how much would each person have?

(5) The yield of tobacco in Virginia in a recent year was 73,000,000 lbs. This was grown on 110,000 acres of land. Find the average number of pounds produced on each acre.

FACTORS AND MULTIPLES

DIVISIBILITY

37. A number is **divisible** by any number that will divide it without a remainder.

An **even** number is one divisible by 2.

An **odd** number is one not divisible by 2.

A number is divisible by 2, if its last figure is an even number or 0.

1. Which of the following numbers are divisible by 2 :

246, 347, 890, 2,743, 27,692, 98,310, 98,762?

A number is divisible by 3, if the sum of its digits is divisible by 3.

2. Which of the following numbers are divisible by 3 :

250, 251, 365, 368, 1,011, 258, 41,681?

A number is divisible by 4, if the last two digits together are divisible by 4.

3. Which of the following numbers are divisible by 4 :

3,133, 3,135, 3,124, 6,782, 2,528, 9,192, 6,788?

A number is divisible by 5, if its last figure is 0 or 5.

4. Write 6 numbers of 3 or more figures that are divisible by 5.

A number is divisible by 6, if it is divisible by the factors of 6 (3 and 2).

5. Which of the following numbers are divisible by 6 :

67,830, 9,183, 108, 9,768, 5,454, 123,456, 5988, 76,512?

A number is divisible by 8, if the number made by its last three figures is divisible by 8.

6. Which of the following numbers are divisible by 8 :

12,504, 769,432, 467,216, 141,414, 151,528?

A number is divisible by 9, if the sum of its digits is divisible by 9.

7. Which of the following numbers are divisible by 9 :

333, 4,788, 67,230, 578,645, 171,819, 654,321, 450,000?

A number is divisible by 10, if its last figure is 0.

8. By what other numbers is a number having 0 for its last figure divisible?

A number is divisible by 12, if it is divisible by 3 and 4.

9. Which of the following numbers are divisible by 12 :

16,848, 378,492, 38,495, 167,859, 314,152?

10. By which of the numbers from 2 to 12 is each of the following divisible :

128, 133, 500, 62, 85, 324, 252, 144, 5,278, 365, 1492, 1,908?

11. In how many different ways can a gardener set in equal rows 240 rose bushes? 2,013 asters? 124 dahlias? 1,500 pansies?

A **prime number** is a whole number that is divisible only by itself and 1.

12. Name the prime numbers from 1 to 100.

All whole numbers not prime are **composite**.

Any **exact divisor** of a number is a **factor** of the number.

A **prime factor** is a factor that is a prime number.

13. Give all the factors and the prime factors of each composite number from 1 to 20. From 90 to 100.

14. What is a multiple?

15. What are the factors of 6?

16. What are the products of 6 by 2, 3, 4, 5, 6, 7, 8, 9, and 10?

17. Are 2 and 3 factors of each of these products? Why?

A number that will divide another number without a remainder will divide any product of that number; for example, 6 will divide 12; it will also divide 24, 36, 60, 72.

ORAL EXERCISES

38. Name:

1. 5 products of 8 that contain 4 without a remainder.
2. 5 " " 10 " " 5 " " "
3. 5 " " 14 " " 7 " " "
4. 5 " " 16 " " 8 " " "
5. 5 " " 18 " " 9 " " "

WRITTEN EXERCISES

39. Solve:

1. A certain garden contains 360 sq. ft. Its length is 20 ft. What is its breadth?

SUGGESTION. 360 is a multiple of two factors, of which 20 is one. What is the other?

2. There are 900 sq. ft. in a lawn 30 ft. long. How wide is the lawn?

3. A floor 32 ft. long contains 576 sq. ft. How wide is it?

4. One side of a barn is 23 ft. high from the foundation to the roof, and contains 1035 sq. ft. How long is it?

GREATEST COMMON DIVISOR

40. A common factor or common divisor of two or more numbers is one that will divide each of them without a remainder.

What is a greatest common divisor (g. c. d.)? Find by inspection the g. c. d. of:

- | | |
|-----------------|---------------------|
| 1. 15, 25, 50. | 5. 12, 18, 36, 60. |
| 2. 14, 21, 28. | 6. 42, 84, 96. |
| 3. 16, 32, 42. | 7. 24, 96, 120. |
| 4. 36, 96, 180. | 8. 30, 60, 90, 120. |

To find the greatest common divisor of numbers not readily factored at sight, the following method may be used.

Find the greatest common divisor of 18, 36, 27, and 45.

SOLUTION:

$$\begin{array}{r} 3 \overline{)18\ 36\ 27\ 45} \\ 3 \overline{)6\ 12\ 9\ 15} \\ \underline{2\ 4\ 3\ 5} \end{array}$$

$3 \times 3 = 9$, the greatest common divisor.

The numbers are arranged in a horizontal line and divided by any number, 3, that will divide each of them without a remainder. The quotients are written below and divided by another number, 3, that will divide each of them without a remainder. This is repeated until no number will so divide all the quotients. The product of the divisors, $3 \times 3 = 9$, is the greatest common divisor. Explain.

WRITTEN EXERCISES

41. Solve:

1. Find the g. c. d. of:

(1) 50, 175, 125, 400.

(5) 21, 210, 315.

(2) 72, 48, 96, 24.

(6) 16, 48, 96, 32.

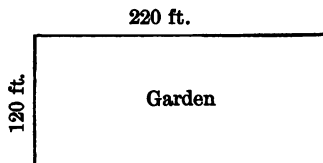
(3) 70, 490, 476.

(7) 144, 1728, 84.

(4) 180, 195, 990.

(8) 80, 40, 60, 120.

2. The posts of the fence around the four sides of a garden are set equal distances apart. What is the greatest number of feet possible between the posts (g. c. d.)? Test your answer by a diagram drawn to scale.



3. Three men purchased sheep at the same price per head. One spent \$81; another, \$36; and the third, \$72. What was the greatest price per head that they could pay (g. c. d.)? At that price, how many sheep could all buy?

LEAST COMMON MULTIPLE

42. A number that is divisible by two or more numbers is called a **common multiple** of those numbers. For example, 16 is a common multiple of 2, 4, and 8; 20 is a common multiple of 2, 4, 5, and 10.

1. What is a least common multiple (l. c. m.)?

2. Find the l. c. m. of 24, 48, 36.

SOLUTION: The prime factors of 24 are 2, 2, 2, 3.

“ “ “ “ 48 “ 2, 2, 2, 2, 3.

“ “ “ “ 36 “ 2, 2, 3, 3.

2 occurs four times in 48. Hence, 2 must be taken four times in finding the l. c. m.

3 occurs twice in 36. Hence, 3 must be taken twice in finding the l. c. m.

$$2 \times 2 \times 2 \times 2 \times 3 \times 3 = 144 = \text{the l. c. m.}$$

The product of the prime factors is the l. c. m. Explain.

To find the least common multiple of two or more numbers, multiply together all their prime factors, taking each the greatest number of times that it occurs in any one of the numbers.

ORAL OR WRITTEN EXERCISES

43. Find by inspection the l. c. m. of:

1. 12, 72, 84.

7. 24, 48, 72.

2. 42, 98, 120.

8. 18, 45, 90.

3. 16, 64, 96.

9. 5, 15, 20.

4. 28, 56, 112.

10. 6, 18, 4.

5. 4, 8, 12, 16.

11. 13, 39, 78.

6. 14, 20, 28.

12. 35, 14, 140.

44. To find the least common multiple of numbers that cannot be readily factored at sight, the following plan may be used:

Find the least common multiple of 6, 9, 12, 15, and 18.

SOLUTION:

$$\begin{array}{r}
 2)6 \quad 9 \quad 12 \quad 15 \quad 18 \\
 \hline
 3)3 \quad 9 \quad 6 \quad 15 \quad 9 \\
 \hline
 3)1 \quad 3 \quad 2 \quad 5 \quad 3 \\
 \hline
 1 \quad 1 \quad 2 \quad 5 \quad 1
 \end{array}$$

$2 \times 3 \times 3 \times 2 \times 5 = 180$, the least common multiple.

The numbers are arranged in a horizontal line and divided by the least number, 2, that will divide two or more of them without remainders. The quotients and undivided numbers are written below and divided by the least number, 3, that will divide two or more of them without remainders. This operation is continued until there is no number greater than 1 that will divide two or more of the quotients and undivided numbers without remainders.

The prime factors are the divisors and undivided numbers. Their product is the least common multiple.

When all the numbers are prime numbers, their product is their least common multiple.

WRITTEN EXERCISES

45. Factor and find the l. c. m. of:

- | | |
|------------------|------------------------|
| 1. 4, 6, 8, 12. | 4. 2, 3, 4, 8, 6. |
| 2. 5, 8, 10, 16. | 5. 12, 15, 8, 10, 30. |
| 3. 6, 9, 27, 18. | 6. 20, 45, 30, 15, 60. |

46. Find the g. c. d. and the l. c. m. of:

- | | |
|--------------------|------------------------|
| 1. 6, 12, 24, 36. | 5. 12, 24, 16, 32, 8. |
| 2. 21, 35, 42, 14. | 6. 18, 36, 12, 60, 20. |
| 3. 16, 40, 24, 56. | 7. 30, 25, 10, 40, 80. |
| 4. 9, 15, 18, 27. | 8. 12, 16, 32, 64, 8. |

REVIEW

47. Questions on subject-matter.

1. Why is the Arabic system called a decimal system?
2. In what table of denominate numbers is the decimal system used? Would it be better if the decimal system were used in all tables?
3. What is the name of the answer in each of the four different arithmetical processes?
4. Name and give examples of two different classes of numbers.
5. Work a problem in addition, beginning with the left-hand column.
6. Name and make all of the signs used thus far in arithmetic.
7. How would you test the result in a problem in subtraction?
8. How would you test the result in a problem in division?
9. What is a simple number? A compound number?
10. What is an abstract number? A concrete number?
11. In multiplication, which term is always abstract?
12. Why will 5 divide any number that 10 will divide?
13. Why is it that 10 will not divide all numbers that 5 will divide?
14. What is a factor? A prime factor?
15. In what class of problems is a knowledge of factoring important?
16. How many multiples may 2 and 3 have?
17. Can you give a reason for using the least common multiple?

WRITTEN EXERCISES

48. Solve:

1. Express in words the following numbers:

3,001,003, 11,111,101, 4,324,101, 2,002,002.

2. Express in figures the following numbers:

One million, forty thousand, ten.

Twenty-seven million, twenty-seven thousand, twenty-seven.

3. Express in Roman notation the number 5555.

4. Add once only the following numbers and test the result: 30,647, 26,004, 10,118, 96, 386, 92,896, 4379.

5. Multiply once only the following numbers and test the result: 8,604,975 by 64,893.

6. Multiply in two different ways the following numbers: 4680 by 50; 640 by 25.

7. Divide the following numbers by the ordinary method of division, and also by dividing by the factors of the divisor: 8736 by 24.

8. Multiply in two ways the following numbers: 249 by $33\frac{1}{3}$.

9. Make a bill of the following transactions, in which you show the debit and credit items and how the account stands:

March 1, Mr. Gray bought of Mr. Smith 10 lb. of sugar at \$.06 a pound and $\frac{1}{2}$ lb. of tea at \$.90 a pound. He sold Mr. Smith 10 bu. of potatoes at \$.60 a bushel and 2 doz. eggs at \$.30 a dozen.

March 10, Mr. Gray bought of Mr. Smith 5 lb. of coffee at \$.40 a pound, 9 cans of sweet corn at \$.11 a can. He sold him 20 chickens at \$.45 each.

March 20, Mr. Gray bought of Mr. Smith 25 lb. of sugar at \$.05 a pound, 2 lb. of raisins at \$.12 a pound, $1\frac{1}{2}$ lb. of pepper at \$.30 a pound. He sold Mr. Smith 4 doz. eggs at \$.25 a dozen.

10. Copy the following names of various things, and place after each the price as nearly as you can :

Apples, oranges, eggs, tea, sugar, steak, ice, horse, sheep, pair of your shoes, handkerchief, this book.

11. What numbers will divide the following number : 1728 ? Give a reason for each one named.

12. Find by inspection three multiples of the following numbers : 2, 3, 4, 6, 12.

13. Find by inspection two common divisors of the following numbers : 40, 64, 16, 56, 32.

14. A circular track is 3 miles in circumference. Three men start from the same point to travel around it until they all come together again at the place of starting. A travels 2 miles an hour, B 3 miles an hour, and C 6 miles an hour. How many hours did they travel ? How many miles does each travel ?

15. A farmer bought three farms which contained 360 acres, 450 acres, and 240 acres. He gave these to his two sons and two daughters, giving to each son twice as much as to each daughter. How many acres did each receive ?

16. A man had \$6000. He gave to James $\frac{1}{2}$ of this, to Peter $\frac{1}{2}$ of the remainder, to Susan $\frac{1}{2}$ of this remainder, and the rest to Clara. How many dollars did each receive ?

SUMMARY OF CHAPTER I

THE FUNDAMENTAL PROCESSES :

Addition, subtraction, multiplication, division, with tests and applications.

FACTORS AND MULTIPLES :

Greatest common divisor and least common multiple.

CHAPTER II

COMMON AND DECIMAL FRACTIONS

FRACTIONS DEFINED

49. Any quantity considered as a single thing, or used as a measure of other quantities, is called a **unit**.

1, 5, 20, a pound, an acre, a foot, are units.

50. A **fraction** is one, or more than one, of the equal parts of a unit, as $\frac{1}{2}$, $\frac{2}{3}$ of a pound, $\frac{3}{4}$ of an acre.

Write a fraction.

Name its numerator and its denominator.

The numerator and the denominator of a fraction are called the **terms** of the fraction.

A fraction whose value is less than 1 is called a **proper fraction**, as $\frac{1}{2}$.

A fraction whose value is 1 or greater than 1 is called an **improper fraction**, as $\frac{5}{3}$.

A number made up of an integer and a fraction written together as one number is called a **mixed number**, as $1\frac{1}{2}$.

51. There are four facts about all fractions that it is well to remember.

(1) Every fraction represents one or more of the equal parts of some unit.

(2) The unit of which the fraction is a part may be any quantity used as a single thing, and not merely the number 1.

For example, $\frac{5}{8}$ may be a fraction of 1 mile or of 30 yards or of an abstract number, as 1 or 24. If the unit is not mentioned, it is commonly understood to be 1.

(3) A fraction may itself be treated as a unit. So we may add, subtract, multiply, and divide fractions as we do whole numbers.

When we say $\frac{1}{2}$ of $\frac{1}{4}$, $\frac{1}{4}$ is treated as a unit.

(4) Every fraction shows a ratio. Its value, considered as a unit, is the quotient of its numerator divided by its denominator.

The value of $\frac{3}{4}$ as a unit is the quotient 2.

REDUCTION

52. REDUCING TO HIGHER OR LOWER TERMS.

Multiply both terms of $\frac{3}{4}$ by 2.

Divide both terms of $\frac{4}{8}$ by 2.

Is the value of the fraction changed in either case? Why?

Multiplying or dividing both terms of a fraction by the same number does not change its value.

Changing the form or the terms of a fraction without changing its value is called **reduction**.

Reduce to their lowest terms:

$$\frac{4}{16}, \frac{15}{20}, \frac{125}{1000}, \frac{1600}{8200}, \frac{3}{33}, \frac{111}{444}.$$

Give a rule for this process.

WRITTEN EXERCISES

53. Reduce to lowest terms:

1. $\frac{125}{800}, \frac{98}{343}, \frac{81}{162}.$

3. $\frac{216}{288}, \frac{567}{891}, \frac{144}{1728}.$

2. $\frac{56}{128}, \frac{77}{182}, \frac{105}{165}.$

4. $\frac{875}{1000}, \frac{256}{768}, \frac{162}{405}.$

54. REDUCING TO A COMMON DENOMINATOR.

How do you reduce $\frac{1}{3}$ to 12ths?

By what factor must you multiply the terms of $\frac{2}{3}$ to reduce the fraction to 12ths?

Reduce $\frac{3}{4}$ to 8ths; $\frac{2}{10}$ to 20ths.

Give a rule for this process.

Reduce to fractions having a common denominator:

$$\frac{1}{3}, \frac{2}{5}, \frac{3}{4}, \text{ and } \frac{6}{12}.$$

Give a rule for this process.

WRITTEN EXERCISES

55. Find a common denominator of:

1. $\frac{1}{5}, \frac{2}{3}, \frac{5}{6}, \frac{2}{15}.$

4. $\frac{1}{7}, \frac{2}{3}, \frac{1}{6}, \frac{5}{21}.$

2. $\frac{2}{7}, \frac{3}{4}, \frac{1}{2}, \frac{4}{5}.$

5. $\frac{5}{6}, \frac{2}{5}, \frac{1}{10}, \frac{7}{15}.$

3. $\frac{2}{10}, \frac{3}{5}, \frac{5}{6}, \frac{1}{5}.$

6. $\frac{2}{3}, \frac{5}{6}, \frac{2}{6}, \frac{4}{27}.$

To reduce fractions having unlike denominators to equal fractions with like denominators, find a common multiple of the denominators and change each fraction to an equal fraction having this common multiple for a denominator.

56. REDUCING TO A LEAST COMMON DENOMINATOR.

How do you find the least common multiple of several numbers?

Find the l. c. m. of 9, 5, 15, 3; 7, 3, 2, 9.

The least common multiple of several denominators is the least common denominator.

Find the least common denominator of:

$$\frac{2}{160}, \frac{1}{80}, \frac{5}{320}, \frac{3}{40}; \frac{2}{9}, \frac{4}{5}, \frac{7}{15}; \frac{5}{7}, \frac{2}{3}, \frac{8}{21}, \frac{7}{5}.$$

Give a rule for this process.

WRITTEN EXERCISES

57. Reduce to fractions having the least common denominator:

1. $\frac{12}{18}, \frac{4}{39}, \frac{7}{65}, \frac{8}{78}, \frac{8}{52}.$

4. $\frac{12}{1000}, \frac{4}{5000}, \frac{7}{2000}, \frac{9}{1500}.$

2. $\frac{4}{15}, \frac{3}{25}, \frac{17}{40}, \frac{13}{60}, \frac{6}{35}.$

5. $\frac{2}{20}, \frac{1}{30}, \frac{4}{15}, \frac{7}{60}, \frac{9}{40}.$

3. $\frac{5}{24}, \frac{3}{8}, \frac{16}{32}, \frac{4}{12}, \frac{7}{60}.$

6. $\frac{6}{15}, \frac{8}{25}, \frac{10}{75}, \frac{3}{5}, \frac{2}{3}.$

58. REDUCING IMPROPER FRACTIONS TO MIXED NUMBERS.

Reduce $\frac{11}{3}$ to a mixed number. Give a rule for this process.

ORAL EXERCISES**59. Reduce to whole or mixed numbers :**

1. $\frac{15}{2}$, $\frac{25}{8}$, $\frac{35}{5}$, $\frac{41}{7}$, $\frac{47}{9}$.
2. $\frac{51}{8}$, $\frac{59}{9}$, $\frac{62}{12}$, $\frac{65}{18}$, $\frac{71}{11}$.
3. $\frac{120}{4}$, $\frac{115}{8}$, $\frac{201}{2}$, $\frac{213}{3}$, $\frac{204}{7}$.

WRITTEN EXERCISES**60. Reduce to whole or mixed numbers :**

1. $\frac{61}{12}$, $\frac{7}{2}$, $\frac{125}{40}$, $\frac{875}{37}$, $\frac{14}{8}$, $\frac{135}{45}$.
2. $\frac{22}{30}$, $\frac{764}{184}$, $\frac{15}{14}$, $\frac{26}{12}$, $\frac{1472}{1400}$.
3. $\frac{176}{9}$, $\frac{185}{12}$, $\frac{217}{8}$, $\frac{291}{6}$, $\frac{379}{9}$.

61. REDUCING MIXED NUMBERS TO IMPROPER FRACTIONS.

Reduce $3\frac{1}{2}$ to an improper fraction.

Give a rule for this process.

ORAL EXERCISES**62. Reduce to improper fractions :**

1. $4\frac{1}{3}$, $5\frac{1}{6}$, $6\frac{2}{3}$, $8\frac{2}{5}$, $12\frac{2}{7}$, $11\frac{2}{3}$.
2. $6\frac{2}{5}$, $7\frac{5}{9}$, $10\frac{2}{7}$, $9\frac{5}{12}$, $13\frac{2}{3}$, $15\frac{1}{8}$.

WRITTEN EXERCISES**63. Reduce to improper fractions :**

1. $16\frac{1}{2}$, $7\frac{2}{3}$, $85\frac{1}{15}$, $14\frac{1}{14}$, $7\frac{1}{7}$, $18\frac{1}{3}$.
2. $125\frac{1}{4}$, $16\frac{1}{4}$, $73\frac{1}{2}$, $88\frac{1}{8}$, $91\frac{2}{5}$, $195\frac{1}{4}$.
3. $402\frac{2}{3}$, $425\frac{1}{4}$, $501\frac{1}{7}$, $475\frac{5}{8}$, $650\frac{2}{3}$.

ADDITION

64. Add $\frac{1}{3} + \frac{2}{5} + \frac{3}{7} + \frac{4}{9}$.

How do you add fractions having *like* denominators?

Add $\frac{1}{3} + \frac{2}{3} + \frac{3}{3} + \frac{5}{3}$.

In adding fractions having unlike denominators, it is sometimes easier to take the product of the denominators of the fractions for a common denominator than to find their least common denominator.

$$\begin{aligned}\text{Example: } \frac{2}{3} + \frac{4}{5} + \frac{3}{7} + \frac{1}{2} &= \frac{80+84+90+60}{120} = \frac{314}{120} \\ &= 2\frac{86}{60} = 2\frac{43}{30}.\end{aligned}$$

The product of the denominators equals 120. This is a common denominator, but not the least common denominator.

The first numerator is $2 \times 5 \times 4 \times 2 = 80$

“ second “ “ $4 \times 3 \times 4 \times 2 = 96$

“ third “ “ $3 \times 3 \times 5 \times 2 = 90$

“ fourth “ “ $1 \times 3 \times 5 \times 4 = 60$

To add fractions having unlike denominators, reduce to like fractions having a common denominator; add the numerators and place the sum over the denominator.

ORAL EXERCISES

65. Add, finding either the least common denominator or any other common denominator:

1. $\frac{4}{5} + \frac{3}{15} + \frac{7}{30} + \frac{9}{60}$.

2. $\frac{1}{120} + \frac{3}{20} + \frac{2}{5} + \frac{7}{15}$.

3. $\frac{4}{7} + \frac{1}{35} + \frac{3}{10}$.

4. $\frac{5}{12} + \frac{7}{8} + \frac{9}{24} + \frac{2}{3} + \frac{1}{4}$.

5. $\frac{1}{2} + \frac{3}{5} + \frac{6}{8} + \frac{4}{10}$.

6. $\frac{2}{3} + \frac{3}{9} + \frac{2}{6} + \frac{7}{15}$.

7. $\frac{3}{4} + \frac{6}{7} + \frac{5}{14} + \frac{9}{28}$.

SUBTRACTION

66. Subtract $\frac{2}{3}$ from $\frac{7}{7}$.

How do you subtract fractions having like denominators?

67. Subtract $\frac{2}{4}$ from $\frac{5}{5}$.

When fractions have small denominators it is often easier to multiply these together for a common denominator than to find their l. c. d.

68. Subtract $\frac{3}{8}$ from $\frac{5}{8}$.

SOLUTION: $6 \times 8 = 48$.

$$3 \times 6 = 18. \quad \frac{40 - 18}{48} = \frac{22}{48} = \frac{11}{24}.$$

$$5 \times 8 = 40.$$

If fractions have unlike denominators, reduce them to fractions having a common denominator before subtracting the numerators.

ORAL EXERCISES

69. Subtract:

1. $\frac{2}{3} - \frac{1}{4}$.

6. $9 - \frac{3}{4}$.

11. $\frac{5}{8} - \frac{1}{2}$.

16. $11 - \frac{7}{8}$.

2. $\frac{1}{2} - \frac{1}{3}$.

7. $5 - \frac{5}{8}$.

12. $\frac{7}{12} - \frac{1}{3}$.

17. $4 - \frac{5}{12}$.

3. $\frac{1}{3} - \frac{1}{6}$.

8. $10 - \frac{3}{5}$.

13. $\frac{2}{3} - \frac{7}{12}$.

18. $3 - \frac{8}{11}$.

4. $\frac{1}{2} - \frac{1}{4}$.

9. $12 - \frac{2}{3}$.

14. $7 - \frac{8}{11}$.

19. $\frac{13}{25} - \frac{2}{5}$.

5. $6 - \frac{4}{5}$.

10. $\frac{4}{5} - \frac{3}{4}$.

15. $8 - \frac{9}{10}$.

20. $\frac{140}{225} - \frac{1}{3}$.

WRITTEN EXERCISES

70. Solve:

1. $\frac{3}{4} - \frac{1}{2} + \frac{2}{3}$.

5. $\frac{16}{21} - \frac{1}{3} + \frac{3}{24}$.

SUGGESTION: $\frac{9 - 6 + 8}{12}$.

6. $\frac{5}{12} + \frac{3}{18} - \frac{1}{9} - \frac{1}{3}$.

7. $\frac{9}{16} - \frac{3}{8} + \frac{30}{32} - \frac{3}{4}$.

2. $\frac{15}{20} + \frac{2}{5} - \frac{3}{4} - \frac{3}{10}$.

8. $\frac{4}{17} + \frac{5}{7} - \frac{1}{3}$.

3. $\frac{5}{6} - \frac{3}{24} - \frac{2}{12} + \frac{1}{2} - \frac{1}{3}$.

9. $\frac{9}{25} + \frac{6}{35} + \frac{2}{5} + \frac{7}{40}$.

4. $\frac{5}{17} - \frac{3}{20}$.

10. $\frac{2}{7} + \frac{3}{14} + \frac{15}{49} - \frac{1}{2}$.

11. Mr. Jones owns $\frac{1}{3}$ of the business known as Jones & Co. Of his partners, Mr. Alfred owns $\frac{1}{4}$, Mr. Swift $\frac{1}{8}$, Mr. Allen $\frac{1}{16}$. How much more does Mr. Jones own than each of his partners?

12. How much more does he own than his three partners together?

13. $\frac{3}{8}$ of the road from Smithville to Jonesboro is good, $\frac{1}{8}$ is fair, and the remainder is bad. What part of the road is bad?

14. How much greater is the fractional part of the road that is good than the part that is fair? Than the part that is bad? Than both together?

15. As the result of a drought last year, Mr. Knapp's crops were less than the average, as follows: hay, $\frac{1}{2}$; oats, $\frac{1}{3}$; apples, $\frac{2}{3}$; potatoes, $\frac{3}{8}$; berries, $\frac{2}{3}$. Which crop suffered the greatest loss?

16. How much larger was the fractional loss on this crop than that on each of the others?

17. A field measures $\frac{7}{8}$ of a mile on one side, $\frac{5}{8}$ of a mile on another side, $\frac{3}{11}$ of a mile on the third side, and $\frac{4}{5}$ of a mile on the fourth side. How far is it around the farm?

18. Mary earned \$ $\frac{5}{8}$ on Monday, \$ $\frac{2}{11}$ on Tuesday, \$ $\frac{1}{12}$ on Wednesday, and \$ $\frac{7}{10}$ on Thursday. How much did she earn in all?

19. John weeded the garden in four hours. The first hour he did $\frac{2}{11}$ of it, the second hour $\frac{3}{10}$, the third hour $\frac{1}{6}$. How much was left to be done the fourth hour?

20. Fred had $\frac{9}{10}$ of a dollar. He spent $\frac{5}{6}$ of it for a necktie. What part of the dollar did he have left?

21. Clara made three bows for her dress. For one she used $\frac{1}{3}$ of a yard of ribbon, for the second $\frac{1}{6}$ of a yard, and for the third $\frac{1}{8}$ of a yard. How many yards did she use in all?

MULTIPLICATION AND DIVISION

71. $\frac{4}{2} = 2$. If we multiply the numerator by 4, we have $\frac{4 \times 4}{2} = 8$. 8 = the ratio, 2, multiplied by 4.

How is the value of the fraction affected by multiplying its numerator by 4?

$\frac{8}{2} = 3$. Divide the numerator by 3. We have $\frac{6 \div 3}{2} = \frac{2}{2} = 1$. 1 = the ratio, 3, divided by 3.

How is the value of the fraction affected by dividing its numerator by 3?

Multiplying or dividing the numerator of a fraction by any number multiplies or divides the value of the fraction of that number. Explain.

Multiply $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{8}$, $\frac{9}{10}$, $\frac{11}{12}$ by 2, 3, 4, 5, 6, 7, 8, 9 by multiplying the numerators.

Divide $\frac{4}{6}$, $\frac{8}{12}$, $\frac{10}{15}$, $\frac{12}{18}$, $\frac{14}{21}$, $\frac{16}{24}$ by 2 by dividing the numerators.

72. $\frac{4}{2} = 1$. Divide the denominator by 2.

$\frac{4}{4 \div 2} = \frac{4}{2} = 2$. 2 = the ratio, 1, multiplied by 2. Explain.

How is the value of the fraction affected?

Dividing the denominator of a fraction by a number multiplies the value of the fraction by that number. Explain.

$\frac{4}{2} = 2$. Multiply the denominator by 2.

$\frac{4}{2 \times 2} = \frac{4}{4} = 1$. 1 = the ratio, 2, divided by 2.

How is the value of the fraction affected?

Multiplying the denominator of a fraction by a number divides the value of the fraction by that number. Explain.

Multiply and then divide the denominators of the following fractions, and state the effects:

$\frac{1}{6}$ by 2; $\frac{3}{4}$ by 4; $\frac{5}{12}$ by 6; $\frac{7}{100}$ by 10.

WRITTEN EXERCISES

73. Multiply each of the following fractions,

(1) By multiplying the numerators by the given number,

(2) By dividing the denominators:

1. $\frac{3}{4}, \frac{5}{6}, \frac{9}{10}, \frac{11}{12} \times 2.$

5. $\frac{5}{6}, \frac{8}{12}, \frac{7}{24}, \frac{9}{36}, \frac{5}{18} \times 6.$

2. $\frac{3}{8}, \frac{5}{6}, \frac{7}{9}, \frac{5}{12}, \frac{11}{15}, \frac{7}{18} \times 3.$

6. $\frac{6}{7}, \frac{8}{14}, \frac{9}{21}, \frac{20}{28}, \frac{15}{35} \times 7.$

3. $\frac{3}{4}, \frac{5}{6}, \frac{7}{18}, \frac{21}{24}, \frac{9}{36} \times 4.$

7. $\frac{7}{8}, \frac{15}{16}, \frac{13}{24}, \frac{7}{32}, \frac{25}{40} \times 8.$

4. $\frac{4}{5}, \frac{3}{10}, \frac{7}{20}, \frac{13}{30}, \frac{14}{45} \times 5.$

8. $\frac{5}{9}, \frac{5}{18}, \frac{12}{27}, \frac{19}{36}, \frac{14}{45} \times 9.$

Divide each of the following fractions,

(1) By dividing the numerators,

(2) By multiplying the denominators by the given number:

1. $\frac{4}{5}, \frac{6}{7}, \frac{2}{3}, \frac{8}{9}, \frac{4}{11}, \frac{10}{12} \div 2.$

4. $\frac{6}{7}, \frac{12}{14}, \frac{13}{21}, \frac{24}{25}, \frac{30}{33}, \frac{12}{22} \div 6.$

2. $\frac{3}{4}, \frac{5}{7}, \frac{9}{10}, \frac{1}{11}, \frac{8}{13}, \frac{12}{15} \div 3.$

5. $\frac{8}{9}, \frac{16}{18}, \frac{22}{21}, \frac{40}{42}, \frac{16}{15}, \frac{16}{20} \div 8.$

3. $\frac{4}{5}, \frac{8}{16}, \frac{12}{18}, \frac{16}{21}, \frac{24}{30}, \frac{16}{36} \div 4.$

6. $\frac{5}{8}, \frac{10}{12}, \frac{5}{9}, \frac{15}{24}, \frac{25}{45}, \frac{10}{21} \div 5.$

MULTIPLICATION

74. Multiply $\frac{3}{4}$ by 2.

$$\frac{3}{4} \times 2 = \frac{3 \times 2}{4} = \frac{6}{4} = 1\frac{1}{2}, \text{ or } \frac{3}{4 \div 2} = \frac{3}{2} = 1\frac{1}{2}.$$

To multiply a fraction by a whole number, multiply the numerator or divide the denominator of the fraction by the whole number.

Multiply $\frac{1}{6}$ by 5; $\frac{2}{3} \times 3$; $4 \times \frac{1}{16}.$

Mr. Avery has 5 gardens containing each $\frac{1}{16}$ of an acre. How many acres in them all?

What is $\frac{1}{2}$ of $\frac{1}{4}$? Show this by a drawing.

In what other way can you obtain the result?

Multiply $\frac{3}{4} \times \frac{2}{4} \times \frac{6}{7}.$

$$\begin{aligned} \text{SOLUTION: } \frac{3}{4} \times \frac{2}{4} \times \frac{6}{7} &= \frac{3}{\cancel{4}} \times \frac{\cancel{2}}{\cancel{4}} \times \frac{6}{7} \\ &= \frac{3 \times 3}{2 \times 2 \times 7} = \frac{9}{28}. \end{aligned}$$

In the second example we cancel 2, a factor of 4, in the denominator of $\frac{3}{4}$ and in the numerator of $\frac{2}{4}$. Then we cancel 2, a factor of 4, in the denominator of $\frac{2}{4}$, and also 2, a factor of 6, in the numerator of $\frac{6}{7}$.

To multiply a fraction by a fraction, cancel all factors common to both numerators and denominators, and multiply the remaining factors of the numerators together for the new numerator, and those of the denominators for the new denominator.

Since to multiply a fraction by a fraction is to take a fraction of a fraction, it follows that when two proper fractions are multiplied together the value of the product is always less than that of either factor.

$$\frac{4}{5} \times \frac{1}{2} = \frac{\overset{2}{\cancel{4}}}{5} \times \frac{1}{\cancel{2}} = \frac{2}{5}. \quad \frac{2}{5} \text{ is less than either } \frac{4}{5} \text{ or } \frac{1}{2}.$$

WRITTEN EXERCISES

75. Multiply :

1. $\frac{3}{8} \times \frac{2}{5} \times \frac{10}{8} \times \frac{4}{5} =$

3. $\frac{1}{5} \times \frac{10}{8} \times \frac{36}{30} =$

2. $\frac{1}{2} \times \frac{6}{7} \times \frac{14}{15} \times \frac{5}{8} =$

4. $\frac{2}{3} \times \frac{9}{10} \times \frac{5}{8} \times \frac{7}{8} \times \frac{16}{25} =$

5. Mr. Thomas divided $\frac{1}{2}$ of a dollar equally among 5 children; what part of a dollar did he give each?

6. Mr. Dalrymple sowed $\frac{1}{2}$ ton of wheat on five fields: he sowed $\frac{1}{8}$ of it on field A; $\frac{1}{8}$ on field B; $\frac{1}{8}$ on field C; $\frac{2}{15}$ on field D; and the remainder on field E. What part of a ton did he sow on each?

7. In Mr. Whitman's berry patch there are $\frac{11}{8}$ of an acre. Of this $\frac{1}{2}$ was in blackberries, $\frac{2}{3}$ in raspberries, and the remainder in strawberries. What part of an acre was used for each kind of berries?

8. Our athletic field is $\frac{5}{20}$ of a mile long and $\frac{4}{15}$ of a mile wide. Its area is what part of a square mile? How many acres does it contain?

DIVISION

76. Divide :

$$\frac{4}{5} \div 2 = \frac{2}{5}; \text{ or } \frac{1}{2} \text{ of } \frac{4}{5} = \frac{4}{10} = \frac{2}{5}.$$

To divide a fraction by a whole number, divide the numerator or multiply the denominator by the number. (See p. 39.)

Divide: $\frac{4}{5} \div \frac{2}{2} = 4 \div 2 = 2.$

Test: $2 \times \frac{2}{5} = \frac{4}{5}.$

To divide a fraction by another fraction having the same denominator, divide the numerator of the dividend by the numerator of the divisor.

Divide $\frac{1}{3}$ by $\frac{1}{12}$.

$$\frac{1}{3} \div \frac{1}{12} = \frac{4}{12} \div \frac{1}{12} = 4 \div 1 = 4.$$

To divide a fraction by a fraction having an unlike denominator, reduce the fractions to a common denominator and divide the numerator of the dividend by that of the divisor.

Divide $\frac{4}{5}$ by $\frac{3}{4}$.

$$\frac{4}{5} \div \frac{3}{4} = \frac{16}{20} \div \frac{15}{20} = \frac{16}{15} = 1\frac{1}{15}.$$

ORAL EXERCISES

77. Divide :

$$\frac{6}{7} \text{ by } 3; \frac{2}{3} \text{ by } 4; \frac{5}{6} \text{ by } \frac{1}{6}; \frac{15}{16} \text{ by } \frac{3}{16}; \frac{1}{2} \text{ by } \frac{1}{5}; \frac{22}{100} \text{ by } \frac{2}{10}.$$

A SHORT METHOD OF DIVISION

78. The product of the denominators is always a common denominator, though it may not be the least common denominator. Explain why.

The new numerator in each case is the product of the given numerator by the denominator of the other fraction. Explain.

What will be the new numerator of each of the fractions in the following divisions?

$$\frac{2}{3} \div \frac{1}{2}; \frac{5}{6} \div \frac{2}{7}; \frac{4}{7} \div \frac{3}{8}; \frac{6}{7} \div \frac{3}{4}.$$

After having found the new numerators, the denominator is of no further use. We merely divide the numerators.

Hence, as we can find the new numerators by multiplying each numerator by the denominator of the other fraction, we may express their division without naming the common denominator.

$$\begin{aligned}\text{Thus:} \quad \frac{2}{3} \div \frac{1}{2} &= \frac{2 \times 2}{3 \times 2} + \frac{1 \times 3}{2 \times 3} \\ &= \frac{2 \times 2}{1 \times 3} = \frac{4}{3} = 1\frac{1}{3}.\end{aligned}$$

For convenience, we may invert the divisor and then multiply as in multiplication of fractions.

$$\text{Thus:} \quad \frac{2}{3} \div \frac{1}{2} = \frac{2}{3} \times \frac{2}{1} = \frac{4}{3} = 1\frac{1}{3}.$$

From this we have the following rule:

To divide a fraction by a fraction, invert the divisor and proceed as in multiplication of fractions.

WRITTEN EXERCISES

79. Divide:

- | | | |
|---------------------------------------|---------------------------------------|---------------------------------------|
| 1. $\frac{7}{8} \div \frac{3}{4}$. | 5. $\frac{9}{10} \div \frac{3}{20}$. | 9. $\frac{4}{7} \div \frac{5}{10}$. |
| 2. $\frac{1}{3} \div \frac{1}{4}$. | 6. $\frac{1}{2} \div \frac{1}{3}$. | 10. $\frac{7}{8} \div \frac{9}{10}$. |
| 3. $2\frac{1}{2} \div \frac{3}{5}$. | 7. $\frac{5}{8} \div 3\frac{1}{3}$. | 11. $\frac{4}{9} \div \frac{5}{6}$. |
| 4. $\frac{9}{20} \div \frac{7}{15}$. | 8. $\frac{7}{10} \div \frac{2}{7}$. | 12. $\frac{3}{4} \div \frac{7}{12}$. |

80. DIVIDING A WHOLE NUMBER BY A FRACTION OR THE REVERSE.

$$6 \div \frac{3}{4} = \frac{24}{4} \div \frac{3}{4} = 8; \text{ or } \frac{6}{1} \div \frac{3}{4} = \frac{6}{1} \times \frac{4}{3} = 8.$$

To divide a whole number by a fraction, or a fraction by a whole number, reduce the whole number to a fraction by writing 1 for its denominator and divide.

ORAL EXERCISES

81. Read, supplying the quotients:

- | | | |
|-----------------------------|----------------------------|----------------------------|
| 1. $5 \div \frac{2}{3} =$ | 4. $\frac{2}{3} \div 5 =$ | 7. $6 \div 1\frac{3}{4} =$ |
| 2. $\frac{5}{6} \div 7 =$ | 5. $9 \div \frac{3}{10} =$ | 8. $16 \div \frac{3}{8} =$ |
| 3. $14 \div \frac{5}{12} =$ | 6. $8 \div \frac{3}{4} =$ | 9. $\frac{3}{4} \div 12 =$ |

82. The division of one integer by another can be expressed in various ways; as,

$$(a) 3 \div 2, \quad (b) 3 : 2, \quad (c) \frac{3}{2}.$$

The division of one fraction by another may be expressed in the same ways; as,

$$(a) \frac{1}{2} \div \frac{1}{4}, \quad (b) \frac{1}{2} : \frac{1}{4}, \quad (c) \frac{\frac{1}{2}}{\frac{1}{4}}.$$

Form (c) is sometimes called a **complex fraction**, which is defined as a fraction in whose numerator or denominator, or both, a fraction occurs.

Complex fractions should be rewritten in the common form for division before dividing.

$$\text{Thus:} \quad \frac{\frac{1}{2}}{\frac{1}{4}} = \frac{1}{2} \div \frac{1}{4} = \frac{1}{2} \times \frac{4}{1} = \frac{4}{2} = 2.$$

$$\frac{2}{\frac{1}{3}} = 2 \div \frac{1}{3} = 2 \times \frac{3}{1} = 6.$$

The same result will be produced by multiplying the outside terms together for the new numerator and the inside terms for the new denominator.

$$\text{Thus:} \quad \frac{\frac{1}{2}}{\frac{1}{4}} = \frac{4 \times 1}{2 \times 1} = \frac{4}{2} = 2.$$

83. Change to the common form and divide the following:

$$\frac{3}{4}; \quad \frac{5}{6}; \quad \frac{2}{3}; \quad \frac{8}{9}; \quad \frac{7}{8}.$$

If mixed numbers occur in either term of a complex fraction, they should be reduced to improper fractions before dividing.

NOTE. The term *complex fraction* has no value to children studying arithmetic; it is an algebraic form. While it may be well for them to understand the meaning of the term, the simple method of treatment here given is to be preferred to the more difficult and confusing ones sometimes used.

WRITTEN EXERCISES

84. Divide:

1. $\frac{1}{15} \div \frac{2}{3}$.

3. $\frac{8}{11} \div \frac{2}{3}$.

5. $\frac{5}{8} \div \frac{2}{3}$.

2. $\frac{6}{7} \div \frac{7}{8}$.

4. $\frac{4}{7} \div \frac{9}{10}$.

6. $\frac{2}{3} \div \frac{5}{12}$.

7. $\frac{\frac{18}{21}}{\frac{8}{7}}$.

8. $\frac{\frac{15}{16}}{\frac{1}{4}}$.

9. $\frac{\frac{7}{12}}{\frac{5}{16}}$.

10. $\frac{\frac{21}{25}}{\frac{21}{4}}$.

11. $\frac{\frac{32}{6}}{\frac{61}{7}}$.

12. $\frac{2}{3}$ of the stock of a railroad is owned by 5 men in equal portions. What part of the stock does each man own?

13. How many boards $\frac{1}{6}$ of a foot wide will be required for a floor 15 ft. wide?

14. How many strips $\frac{1}{15}$ of a foot wide will be required for a mat $\frac{5}{6}$ of a foot wide?

15. Mr. Avery willed $\frac{2}{3}$ of his estate to certain charities, $\frac{1}{10}$ of the estate to each. How many charities profited?

16. A boy rowed $1\frac{1}{2}$ of a mile in an hour. How many hours did it take him to row 6 miles?

17. If Frank can do $\frac{3}{10}$ of a piece of work in an hour, how many hours will it take him to do $1\frac{1}{3}$ of it?

18. Dorothy is making silk bags from ribbon. For each bag she uses $\frac{5}{8}$ of a yard of ribbon. How many bags can she make from 10 yards?

19. John had $\$ \frac{4}{5}$ which he divided among some boys, giving $\$ \frac{1}{10}$ to each. To how many boys did he give the money?

20. Mr. Ewing used $\frac{3}{17}$ of a pound of grass seed on each square yard of his lawn. The lawn contained 27 sq. yd. Find the number of pounds that he used on the entire lawn.

21. At a certain school $\frac{7}{8}$ of an acre of land was given to the children for school gardens. $\frac{1}{10}$ of an acre was given to each child. How many children were there?

ADDITION OF MIXED NUMBERS

85. Mixed numbers may be added, either by adding the integers and fractions separately, or by first reducing the mixed numbers to improper fractions.

(1) By adding the integers and the fractions separately.

$$2\frac{1}{5} + 4\frac{1}{5} + 5\frac{2}{5}.$$

SOLUTION:

$$\begin{array}{r} 2 + 4 + 5 = 11 \\ \frac{1}{5} + \frac{1}{5} + \frac{2}{5} = \frac{4}{5} \\ 11 + \frac{4}{5} = 11\frac{4}{5} \end{array}$$

$$\begin{array}{r} 2\frac{1}{5} \\ 4\frac{1}{5} \\ 5\frac{2}{5} \\ \hline 11\frac{4}{5} \end{array}$$

(2) By first reducing to improper fractions.

$$2\frac{1}{5} + 4\frac{1}{5} + 5\frac{2}{5}.$$

SOLUTION:

$$\begin{aligned} 2\frac{1}{5} + 4\frac{1}{5} + 5\frac{2}{5} &= \frac{2}{1} + \frac{4}{1} + \frac{12}{5} = \\ \frac{27}{5} + \frac{28}{5} + \frac{78}{5} &= \frac{133}{5} = 11\frac{3}{5}. \end{aligned}$$

WRITTEN EXERCISES

86. Solve:

1. $9\frac{3}{8} + 7\frac{3}{10} + 8\frac{1}{4}.$

4. $21\frac{5}{8} + 17\frac{3}{8} + 13\frac{1}{4}.$

2. $10\frac{3}{8} + 12\frac{1}{2} + 11\frac{3}{4}.$

5. $16\frac{3}{8} + 14\frac{5}{12} + 35\frac{7}{24}.$

3. $25\frac{1}{8} + 30\frac{3}{4} + 42\frac{5}{16}.$

6. $72\frac{5}{9} + 20\frac{2}{3} + 18\frac{1}{6}.$

7. Mr. Leonard sold $125\frac{3}{4}$ pounds of sugar on Monday; $76\frac{5}{8}$ pounds on Tuesday, and $96\frac{2}{16}$ pounds on Wednesday. How many pounds did he sell during the 3 days?

8. Mr. Durand sold $46\frac{1}{12}$ dozen eggs to Mrs. Bates; $18\frac{3}{4}$ dozen to Mrs. Luke, and $35\frac{5}{8}$ dozen to Mrs. Gray. How many dozen eggs did he sell?

9. Mr. Bowers sold 4 plots of land. The first contained $17\frac{1}{2}$ acres; the second contained $25\frac{3}{4}$ acres; the third contained $35\frac{7}{8}$ acres; and the fourth contained $45\frac{7}{16}$ acres. How many acres did he sell in all?

SUBTRACTION OF MIXED NUMBERS

87. TO SUBTRACT A MIXED NUMBER FROM A WHOLE NUMBER.

From 6 take $4\frac{1}{3}$.

$$\begin{aligned}\text{SOLUTION:} \quad 6 - 4\frac{1}{3} &= 5\frac{3}{3} - \frac{1}{3} - 4 \\ 5\frac{3}{3} - \frac{1}{3} &= 5\frac{2}{3} \\ 5\frac{2}{3} - 4 &= 1\frac{2}{3}\end{aligned}$$

In subtracting a mixed number from a whole number, we subtract first the fraction, then the whole number.

From the minuend 6, we take 1 unit and change it to $\frac{3}{3}$. We then have $5\frac{3}{3} - 4\frac{1}{3} = 1\frac{2}{3}$.

88. TO SUBTRACT A MIXED NUMBER FROM A MIXED NUMBER.

(1) When the fraction of the minuend is larger than that of the subtrahend.

From $15\frac{5}{8}$ take $9\frac{3}{8}$.

$$\begin{array}{r}\text{SOLUTION:} \quad 15\frac{5}{8} = 15\frac{2\frac{1}{4}}{8} \\ 9\frac{3}{8} = 9\frac{2\frac{3}{4}}{8} \\ \hline 6\frac{1\frac{1}{4}}{8}\end{array}$$

(2) When the fraction of the subtrahend is larger than that of the minuend.

From $12\frac{3}{4}$ take $6\frac{5}{8}$.

$$\begin{array}{r}\text{SOLUTION:} \quad 12\frac{3}{4} = 12\frac{6}{8} = 11\frac{14}{8} \\ 6\frac{5}{8} = 6\frac{5}{8} \\ \hline 5\frac{9}{8}\end{array}$$

We cannot take $\frac{5}{8}$ from $\frac{6}{8}$, so we "borrow" 1 from 12, the integer of the minuend, and change it to 12ths. $1 = \frac{8}{8}$. $\frac{14}{8} + \frac{8}{8} = \frac{22}{8}$. $\frac{22}{8} - \frac{5}{8} = \frac{17}{8}$. The difference of the integers, $11 - 6 = 5$. The answer is $5\frac{17}{8}$.

To subtract a mixed number from a mixed number reduce the mixed numbers to improper fractions and subtract; or, subtract the fractions and the integers separately.

WRITTEN EXERCISES

89. Solve:

1. $12\frac{3}{4} - 8\frac{5}{8}$.

6. $42\frac{9}{14} - 25\frac{3}{7}$.

11. $20\frac{3}{8} - 16\frac{5}{8}$.

2. $16\frac{5}{8} - 9\frac{3}{8}$.

7. $19\frac{1}{2} - 12\frac{1}{4}$.

12. $25\frac{3}{4} - 18\frac{1}{2}$.

3. $20\frac{9}{12} - 12\frac{4}{5}$.

8. $60\frac{4}{5} - 40\frac{3}{4}$.

13. $32\frac{5}{12} - 18\frac{3}{4}$.

4. $25\frac{9}{16} - 17\frac{1}{4}$.

9. $75\frac{3}{4} - 54\frac{2}{3}$.

14. $40\frac{1}{2} - 20\frac{5}{8}$.

5. $30\frac{11}{12} - 20\frac{5}{8}$.

10. $16\frac{1}{16} - 8\frac{3}{8}$.

15. $48\frac{1}{20} - 36\frac{7}{10}$.

16. The prices of wheat for a week were quoted in cents as given below. Find the difference between the highest price and each of the others.

Monday $101\frac{1}{2}$

Thursday $103\frac{1}{5}$

Tuesday $99\frac{3}{4}$

Friday $104\frac{1}{4}$

Wednesday $100\frac{1}{8}$

Saturday $98\frac{7}{8}$

MULTIPLICATION OF MIXED NUMBERS

90. To MULTIPLY AN INTEGER AND A MIXED NUMBER.

How many square feet do 460 square rods equal?

Multiply: $460 \times 30\frac{1}{4}$.

$$460 \times 30 = 13,800$$

$$460 \times \frac{1}{4} = 115$$

$$\begin{array}{r} 4)460 \\ \underline{30\frac{1}{4}} \\ 13,800 \\ \underline{115} \\ 13,915 \end{array}$$

To find the product of an integer and a mixed number, multiply the integer by the integral and fractional parts of the mixed number separately and add the products.

91. To MULTIPLY A MIXED NUMBER BY A MIXED NUMBER.

Multiply $5\frac{1}{3}$ by $3\frac{1}{4}$.

SOLUTION:

$$5\frac{1}{3} = \frac{16}{3}.$$

$$3\frac{1}{4} = \frac{13}{4}.$$

$$\frac{16}{3} \times \frac{13}{4} = \frac{208}{12} = 17\frac{1}{3}.$$

To multiply mixed numbers, change the mixed numbers to improper fractions and multiply.

Or, multiply separately, thus:

$$\begin{array}{r}
 5\frac{1}{3} \times 3\frac{1}{4} \\
 \quad \quad 5\frac{1}{3} \\
 \quad \quad 3\frac{1}{4} \\
 \hline
 \quad \quad \frac{5}{4} + \frac{1}{12} \\
 15 + \frac{3}{4} \\
 \hline
 15 + \frac{3}{4} + \frac{5}{4} + \frac{1}{12} = 15 + 1 + \frac{15}{12} + \frac{1}{12} \\
 = 16 + 1\frac{4}{12} = 17\frac{1}{3}.
 \end{array}
 \qquad
 \begin{array}{l}
 \frac{1}{4} \times \frac{1}{3} = \frac{1}{12} \\
 \frac{1}{4} \times 5 = \frac{5}{4} \\
 3 \times \frac{1}{3} = \frac{3}{3} \\
 3 \times 5 = 15.
 \end{array}$$

Treat the fractions as separate units and multiply as with whole numbers; then combine the results by addition.

WRITTEN EXERCISES

92. Multiply:

1. $6\frac{1}{3} \times 3\frac{2}{3}$.

3. $16\frac{1}{2} \times 16\frac{1}{2}$.

5. $375\frac{1}{6} \times 5\frac{1}{6}$.

2. $25\frac{3}{4} \times 4\frac{1}{8}$.

4. $127\frac{3}{8} \times 2\frac{3}{4}$.

6. $347 \times \frac{1}{2} \times 3\frac{1}{8}$.

7. How many yards are there in $16\frac{1}{2}$ rods?

8. How many feet in $25\frac{1}{4}$ rods?

9. How many square yards in $10\frac{1}{2}$ square rods?

10. How many square inches in a page of a reader that is $5\frac{1}{8}$ in. by $7\frac{3}{8}$ in.?

11. Measure a page of your arithmetic and find the number of square inches in it.

12. Measure the top of your desk and find the number of square inches in it.

13. John walks $3\frac{3}{4}$ mi. in 1 hour. How far does he walk in $11\frac{1}{2}$ hours?

14. If one acre of land costs \$215 $\frac{3}{8}$, what will $8\frac{3}{8}$ acres cost?

15. How many square feet in your school room floor?

DIVISION OF MIXED NUMBERS

93. TO DIVIDE A MIXED NUMBER BY AN INTEGER OR THE REVERSE.

Divide: $105 \div 5\frac{1}{4}$.

$$5\frac{1}{4} = \frac{21}{4}; 105 \div \frac{21}{4} = 105 \times \frac{4}{21} = \frac{420}{21} = 20.$$

To divide a mixed number by an integer, or an integer by a mixed number, change the mixed number to an improper fraction and divide.

94. TO DIVIDE A MIXED NUMBER BY A MIXED NUMBER.

$$4\frac{1}{2} \div 3\frac{1}{3} = \frac{9}{2} \div \frac{4}{3} = \frac{27}{6} \div \frac{4}{6} = \frac{27}{4} = 6\frac{3}{4}.$$

To divide mixed numbers, reduce to improper fractions and divide.

WRITTEN EXERCISES

95. Divide:

1. $8\frac{1}{4} \div 2\frac{1}{2}$.

4. $6 \div 2\frac{1}{2}$.

7. $7\frac{1}{2} \div 2\frac{1}{5}$.

2. $4\frac{3}{8} \div 3\frac{3}{4}$.

5. $5 \div 3\frac{1}{4}$.

8. $10\frac{5}{8} \div 5\frac{1}{8}$.

3. $6\frac{3}{8} \div 1\frac{1}{8}$.

6. $5\frac{1}{2} \div 3$.

9. $4 \div 2\frac{1}{5}$.

96. Solve, multiplying or dividing as necessary:

1. The conductors on the street cars of a certain city receive $22\frac{1}{2}$ cents an hour.

Alfred Harris worked as follows for one week:

Sunday $8\frac{3}{4}$ hours.

Thursday $6\frac{3}{4}$ hours.

Monday $10\frac{1}{2}$ hours.

Friday $11\frac{1}{2}$ hours.

Tuesday $9\frac{1}{2}$ hours.

Saturday $13\frac{1}{2}$ hours.

Wednesday $12\frac{1}{2}$ hours.

How much did he earn during the week?

2. How many square yards in $186\frac{3}{4}$ square rods?

3. My garden is $25\frac{3}{4}$ yards long and $16\frac{1}{4}$ yards wide. How many square rods does it contain?

4. If a block of granite weighs $4\frac{1}{16}$ tons, what is the weight of a pedestal containing 96 blocks?

DECIMAL FRACTIONS

97. A decimal fraction is a fraction whose denominator is 10 or some power of 10. Usually the denominator is not written but is indicated by a decimal point placed before the numerator. .06, .004, are decimal fractions.

Decimal fractions written with a decimal point are called **decimals**.

A decimal of less value than 1 is called a **pure decimal**, as .01, .25.

A decimal containing a whole number is called a **mixed decimal**, as 2.56, 3.02.

What is the first place to the right of the decimal point called?

ORAL AND WRITTEN EXERCISES

98. Read or write as directed:

1. Read: .025; .0225; .00225; 2.25; 22.5; 2.6; 26; .260; .026; .0026; .00026; 26.000; 26:26; 262.6; 2.626.

2. Write as decimals: three thousandths; one millionth; two hundred sixty-five ten-thousandths; one, and three thousand, five hundred seventy-four hundred-thousandths; six hundred, and six hundred-thousandths.

3. Is there any difference in the values of .6 and of .6000? Tell why. Write them both as common fractions.

4. Write as decimals:

$$\frac{23}{100}$$

$$\frac{6}{100}$$

$$\frac{175}{1000}$$

$$\frac{2}{1000}$$

$$\frac{57}{10}$$

$$\frac{186}{1000}$$

$$\frac{145}{10}$$

$$\frac{287}{1000000}$$

$$\frac{28}{1000}$$

$$\frac{205}{1000}$$

5. Write in decimal form:

One dollar, two cents, three mills.

One mill.

Ten dollars, one cent, one mill.

One thousand dollars, one mill.

REDUCTION OF DECIMALS

99. REDUCING DECIMALS TO COMMON FRACTIONS.

Reduce .16 to a common fraction.

$$.16 = \frac{16}{100} = \frac{4}{25}$$

To reduce a decimal to a common fraction, supply the denominator indicated and reduce the fraction to its lowest terms.

WRITTEN EXERCISES

100. Reduce to common fractions or mixed numbers :

- | | | |
|------------|--------------|--------------|
| 1. .125. | 8. 14.22. | 15. .00144. |
| 2. .064. | 9. 37.0666. | 16. 10.001. |
| 3. .0018. | 10. 24.24. | 17. 304.516. |
| 4. .750. | 11. 708.096. | 18. .00302. |
| 5. .00018. | 12. 367.009. | 19. 20.525. |
| 6. .0002. | 13. 2.002. | 20. 3.004. |
| 7. 3.16. | 14. .0375. | 21. 256.175. |

101. REDUCING COMMON FRACTIONS TO DECIMALS.

Reduce $\frac{4}{5}$ to a decimal. Reduce $\frac{1}{25}$ to a decimal.

$$\begin{array}{r} 5 \overline{)4.0} \\ \underline{.8} \end{array}$$

$$\begin{array}{r} .04 \\ 25 \overline{)1.00} \\ \underline{1.00} \end{array}$$

To reduce a common fraction to a decimal, add as many ciphers as may be necessary to the numerator and divide by the denominator. Point off as many places as the number of ciphers added, prefixing ciphers to the quotient, if required, to make the proper number of places.

Reduce $\frac{1}{8}$ to the decimal form :

$$\begin{array}{r} .333\frac{1}{8} \\ 3 \overline{)1.000} \end{array}$$

If after three divisions a remainder is still left, it is usually better to write it over the divisor as a common fraction.

WRITTEN EXERCISES

102. Reduce to the decimal form :

1. $\frac{1}{6}$.

5. $\frac{1}{12}$.

9. $\frac{9}{10}$.

13. $18\frac{1}{2}$.

2. $\frac{1}{8}$.

6. $\frac{4}{7}$.

10. $\frac{5}{8}$.

14. $20\frac{1}{4}$.

3. $\frac{5}{7}$.

7. $\frac{18}{20}$.

11. $1\frac{2}{3}$.

15. $36\frac{1}{8}$.

4. $\frac{3}{4}$.

8. $\frac{4}{16}$.

12. $17\frac{3}{4}$.

16. $5\frac{1}{10}$.

17. $7\frac{3}{12}$.

ADDITION AND SUBTRACTION OF DECIMALS

103. How do you write decimals for addition ?

1. Add :

$$32.654 + 18.275 + 3.06; \quad 76.405 + 1.83769 + .2793.$$

How do you write decimals for subtraction ?

2. Subtract :

(1) 16.075

(2) 7.306

4.285

$.042$

(3) $16.04 - 1.604.$

(4) $.023 - .0023.$

3. One million dollars ten cents, from one million, one thousand dollars, ten cents, one mill.

WRITTEN EXERCISE

104. Solve :

1. A room is 21.5 ft. long, 16.2 ft. wide. What length of baseboard will it require ?

2. A man owns 3 city plots containing 1.413 acres, 2.04 acres, and 6.125 acres. He sold $5\frac{1}{2}$ acres. How many acres had he left ?

3. A corn plant grew .1 in. on Monday, .003 in. on Tuesday, $\frac{1}{2}$ in. on Wednesday, and on Thursday as much as on the three preceding days. How much did it grow in the 4 days ?

4. For each of the following commodities find how much the highest price exceeded each of its other prices:

	Jan. 7, 1908	July 7, 1908	Aug. 7, 1908
Wheat, No. 2 red	1.13 $\frac{1}{2}$	1.45	1.03 $\frac{1}{2}$
Corn, No. 2 mixed	.80	.78	.88
Oats, No. 2 white	.50 $\frac{1}{2}$	.57 $\frac{1}{2}$	.59
Flour, Minn. Patent	6.25	6.60	6.00
Lard, choice	11.90	12.15	9.85
Pork, mess	22.00	21.00	17.50
Beef, family	14.00	13.74	17.25
Coffee, No. 7 Rio	.06 $\frac{3}{8}$	.07 $\frac{3}{8}$	.06 $\frac{1}{2}$
Sugar, granulated	4.85	4.75	5.20
Steel billets	24.00	23.00	25.00
Lead	4.27 $\frac{1}{2}$	4.37 $\frac{1}{2}$	4.60
Tin	29.27 $\frac{1}{2}$	28.85	30.80

MULTIPLICATION OF DECIMALS

105. Multiply $1 \times \frac{1}{10}$; $\frac{1}{10} \times \frac{1}{10}$; $\frac{1}{10} \times \frac{1}{100}$.

How do the ciphers in each product compare in number with those in the two factors together? Explain.

Write each of the above multipliers, multiplicands, and products as a decimal.

1. $.1 \times 1 = .1$ ($1 \times \frac{1}{10}$). One-tenth taken once = .1.

2. $.1 \times .1 = .01$ ($\frac{1}{10}$ of $\frac{1}{10}$). One-tenth taken one-tenth times = .01.

3. $.01 \times 1 = .01$ ($\frac{1}{100}$ of 1). One-hundredth taken once = .01.

4. $.01 \times .1 = .001$ ($\frac{1}{100} \times \frac{1}{10}$). One-hundredth taken one-tenth times = .001.

In each of the above problems how many decimal places are there in the multiplier? In the multiplicand? In both? In the product?

106. The product of two numbers containing decimals contains as many decimal places as both of the numbers. Explain why.

$$46.385 \times 2.56$$

SOLUTION: 46.385

$$\begin{array}{r} 2.56 \\ \underline{278310} \\ 231925 \\ 92770 \\ \hline 118.74560 \end{array}$$

To multiply decimals, multiply as with whole numbers. In the product, point off as many places at the right as there are decimal places in both multiplicand and multiplier.

$$\begin{array}{r} .0256 \\ .047 \\ \hline 1792 \\ 1024 \\ \hline .0012032 \end{array}$$

In this example the number of figures in the product is two less than the number of decimal places of both multiplicand and multiplier; therefore, two ciphers are *prefixed* to the product. Why are they not placed *after* the product.

WRITTEN EXERCISES

107. Find the product of the following:

- | | | |
|---------------------------|----------------------------------|----------------------------------|
| 1. $2.25 \times 5.$ | 6. $.0019 \times .91.$ | 11. $.375 \times 23\frac{3}{4}.$ |
| 2. $35.04 \times 7.3.$ | 7. $475 \times .475.$ | 12. $62\frac{1}{2} \times .15.$ |
| 3. $.038 \times 46.$ | 8. $.0214 \times .078.$ | 13. $43\frac{1}{2} \times 6.25.$ |
| 4. $150.85 \times 4.013.$ | 9. $2200 \times .0022.$ | 14. $.875 \times \frac{3}{4}.$ |
| 5. $8.04 \times 408.$ | 10. $35\frac{3}{4} \times .125.$ | 15. $12.435 \times \frac{7}{8}.$ |

16. What will .016 of an acre of land cost at \$400 per acre?

17. If lumber is worth \$40 a thousand, how much will 125 feet cost?

18. A train goes 20.25 mi. in one hour. How far will it go in $3\frac{3}{4}$ hours? In 4.05 hours? In 5.001 hours? In $6\frac{1}{2}$ hours?

BILLS

108. The following is a bill receipted :

MARSHALL FIELD & COMPANY.							
Importers, Retailers, Manufacturers, State, Washington, Randolph, & Wabash Sts., CHICAGO, ILL., July 8, 1909.							
Sold to MR. JOHN DOE, 1017 State St.							
July	6	5 yd. silk,	.95	4	75		
		4 spools thread,	.04		16		
	13	2 pair rubbers,	.55	1	10		
		6 cakes soap,	.08		48		
	24	1 umbrella,		1	48	7	97
		Received payment, MARSHALL FIELD & Co.					

Make out and receipt 3 bills, selecting articles and using prices named in the following lists, and supplying the names of the parties to the transactions :

Sugar	@	\$.06 $\frac{1}{2}$	Eggs	@	\$.23
Coffee	"	.32	Milk	"	.06
Slippers	"	1.50	Lace	"	2.25
Raisins	"	.12 $\frac{1}{2}$	Muslin	"	.07
Silk	"	1.75	Crackers	"	.08
Calico	"	.10	Molasses	"	.80
Hose	"	.25	Oil	"	.14
Ribbon	"	.35	Codfish	"	.09
Gloves	"	1.25	Tea	"	.60
Collars	"	.15	Rice	"	.08
Shoes	"	3.50	Starch	"	.05

DIVISION OF DECIMALS

109. DIVIDING DECIMALS OF THE SAME DENOMINATION.

$\frac{25}{100} \div \frac{5}{100} = 5$. The denominators being the same, how do we divide common fractions?

$$.25 \div .05 = \frac{25}{100} \div \frac{5}{100} = 5.$$

$$\frac{4}{10} \div \frac{4}{10} = (.4 \div .4) = 1.$$

$$.075 \div .025 = \frac{75}{1000} \div \frac{25}{1000} = 3.$$

If a decimal is divided by a decimal of the same denomination no larger than itself, the quotient will be a whole or a mixed number. Explain why.

110. DIVIDING DECIMALS WHEN THERE ARE MORE PLACES IN THE DIVISOR THAN IN THE DIVIDEND.

Divide:

$$25 \div .5 = 50.$$

SOLUTION: $25 = \frac{250}{10}$; $.5 = \frac{5}{10}$.

$$\frac{250}{10} \div \frac{5}{10} = 250 \div 5 = 50. \quad .5 \overline{)25.0}$$

Divide:

$$2.5 \div .005 = 500.$$

SOLUTION: $\frac{2500}{1000} \div \frac{5}{1000} = 2500 \div 5 = 500.$

$$\begin{array}{r} 500 \\ .005 \overline{)2.500} \end{array}$$

Divide:

$$.2 \div .08.$$

SOLUTION: $\frac{20}{100} \div \frac{8}{100} = \frac{20}{8} = 2\frac{1}{2} = 2.5.$

$$\begin{array}{r} 2.5 \\ .08 \overline{)2.00} \\ \underline{16} \\ 40 \end{array}$$

In dividing decimals, when the divisor contains more decimal places than the dividend, annex enough ciphers to the dividend to make up the difference.

Why are the ciphers not prefixed? What will be the nature of the quotient?

111. DIVIDING DECIMALS WHEN THERE ARE MORE PLACES IN THE DIVIDEND THAN IN THE DIVISOR.

In dividing common fractions having different denominators, how do you proceed?

(Remember that inverting the divisor is a short method of reducing to a common denominator; see page 42.)

Reduce to a common denominator, $\frac{5}{10}$, $\frac{2}{100}$, $\frac{3}{1000}$, 4.

Reduce to the same denomination, .5, .02, .003, 4.

In changing several decimal fractions to the same denomination, which of the fractions will determine the new denomination? Why?

Reduce the fractions in each of the following problems to a common denominator and divide the numerators.

Divide $\frac{1}{100}$ by $\frac{1}{10}$; $\frac{1}{10}$ by 1; $\frac{1}{1000}$ by $\frac{1}{100}$; $\frac{1}{1000}$ by $\frac{1}{10}$.

Can you by observing the number of ciphers in the denominators of the dividend and the divisor tell how many that of the quotient will have? Explain how.

112. Divide .01 by .1; .1 by 1; .001 by .01; .001 by .1.

In the above problems can you tell by observing the dividend and the divisor how many decimal places the quotient will have? Explain how.

Divide: $2.5 \div 5 = .5$.

What is the common denominator?

$$2.5 = 2\frac{5}{10} = \frac{25}{10}; \quad 5 = \frac{50}{10};$$

$$\frac{25}{10} \div \frac{50}{10} = \frac{25}{50} = \frac{1}{2} = \frac{5}{10} = .5.$$

Divide: $2.5 \div .5 = 5$.

What is the common denominator?

$$\frac{25}{10} \div \frac{5}{10} = 5. \quad \begin{array}{r} 5 \\ .5 \overline{)2.5} \end{array}$$

Divide: $.25 \div .5 = .5$.

$$\frac{25}{100} \div \frac{50}{100} = \frac{25}{50} = \frac{5}{10} = .5 \quad \begin{array}{r} .5 \\ .5 \overline{).25} \end{array}$$

Divide: $.16 \div 8$.

SOLUTION: $\frac{16}{100} \div \frac{800}{100} = \frac{16}{800} = \frac{2}{100} = .02$. $\begin{array}{r} .02 \\ 8 \overline{) 16} \end{array}$

Divide: $.004 \div .2$; $6.25 \div 2.5$; $.0012 \div .03$.

When dividing decimals, if there are more decimal places in the dividend than in the divisor, the number of decimal places in the quotient will equal their difference.

If more places must be pointed off than the number of figures in the quotient, *prefix* enough ciphers to equal the difference.

113. A SHORT METHOD OF DIVIDING DECIMALS:

$$62.5 \times 10 = 625. \quad 6.25 \times 100 = 625.$$

How do you multiply a decimal by 10? By 100? By 1000?

Does multiplying both numerator and denominator of a fraction alter its value?

$$\frac{3.25}{.25} \times \frac{100}{100} = \frac{325}{25}.$$

$$\frac{47.5}{.25} \times \frac{100}{100} = \frac{4750}{25}.$$

The method given above may be used to advantage in all cases of long division, involving decimals. For example,

Divide 3.08654 by 2.15.

SOLUTION: $\frac{3.08654}{2.15} \times \frac{100}{100} = \frac{308.654}{215}$. $\begin{array}{r} 1.435 \\ 215 \overline{) 308.654} \\ \underline{215} \\ 936 \\ \underline{860} \\ 765 \\ \underline{645} \\ 1204 \\ \underline{1075} \\ 129 \end{array}$

114. In each of the following problems, by what do you multiply the divisor and the dividend to make a whole number of the divisor?

$$4368 \div .104; 34.28 \div 11.6; .3872 \div .236; 441.375 \div 26.05.$$

Before dividing move the decimal point of both dividend and divisor to the right enough places to make the divisor a whole number.

Place the decimal point of the quotient over that of the dividend and divide, prefixing ciphers to the quotient if necessary.

If there are more decimal places in the divisor than in the dividend, add enough ciphers to the dividend to equal the difference. If, after adding several ciphers and dividing, there is still a remainder, place it over the divisor, making a common fraction as in dividing whole numbers.

WRITTEN EXERCISES

115. Solve :

1. $327.56 \div .233.$

5. $6000 \div .25.$

2. $417 \div 25.$

6. $\$487.60 \div .15.$

3. $16.893 \div .16893.$

7. $24 \div .012.$

4. $2.727 \div 272.7.$

8. A lot containing 5,000 sq. ft. is $233.33\frac{1}{3}$ ft. long. How wide is it?

9. A room is $26.66\frac{1}{3}$ ft. long; its area is 780 sq. ft. What is its width?

10. The area of a circle $= 3.1416 \times$ the square of the radius (πr^2). A certain sundial has an area of 12.5664 ft. What is the length of its radius?

11. Sound travels through the air at the rate of 1134 ft. a second. If a shot is fired 234.25 ft. distant from you, in what part of a second will the sound reach you?

12. In one square rod there are 30.25 square yards. How many square rods are there in 3586.375 square yards?

13. Divide \$405.25 equally among 5 girls. How much does each receive?

REVIEW

116. QUESTIONS ON SUBJECT MATTER

1. What is a simple fraction? Give an example.
2. When is a fraction in its lowest terms?
3. What does the numerator of a fraction show? What does the denominator show?
4. What is the general meaning of the word *reduction*?
5. What does reduction mean as used in arithmetic?
6. Why must fractions be reduced to a common denominator before they can be added?
7. In how many ways can the division of 3 by 4 be indicated?
8. Which of the following fractions are decimal fractions: $\frac{4}{10}$, $\frac{3}{4}$, .04, $\frac{5}{200}$, $\frac{7}{1000}$, .033, 2.8, $4\frac{3}{4}$?
9. What is the denominator of a decimal?
10. What is the numerator of a decimal?
11. What is meant by a unit?
12. What is meant by the unit of a fraction?
13. How may a complex fraction be reduced to a simple one?
14. How do you reduce a decimal to a common fraction?
15. Show that the process is a real case of reduction.
16. How would you multiply $\frac{3}{4}$ by $\frac{5}{7}$? What two things are required to be done?
17. Since division is the converse of multiplication, how could you divide $\frac{1}{2}\frac{5}{8}$ by $\frac{3}{4}$? By $\frac{5}{7}$?
18. Show that the decimal form is preferable to the fraction form in the following problem:

$$.008 \times 2.04 = \text{what?}$$

$$\frac{8}{1000} \times 2\frac{4}{100} = \text{what?}$$

WRITTEN EXERCISES

117. Solve:

1. Reduce to lowest terms:

$$\frac{15}{75}, \frac{48}{80}, \frac{24}{30}, \frac{40}{60}, \frac{42}{56}, \frac{64}{144}$$

2. Find the sum of:

$$\frac{6}{11}, \frac{7}{12}, \frac{9}{14}, \frac{5}{16}$$

3. Find the difference of $\frac{1}{12} - \frac{5}{8}$.

4. Fred picked half a bushel of peas. He sold them at \$.11 $\frac{1}{2}$ a quart. How much did he get for them?

5. What will be the cost of 3 $\frac{3}{4}$ doz. oranges at \$ $\frac{9}{20}$ a dozen?

6. $\frac{3}{4}$ of the value of a house was \$1248. What was the whole value of the house?

7. A man bought a cow for \$48.50. The cow cost him $\frac{2}{5}$ as much as he paid for a horse. How much did he pay for the horse?

8. A room is 24 $\frac{3}{4}$ ft. wide. How many strips of carpeting $\frac{3}{4}$ yd. wide will be required to cover it?

9. A sugar dealer had 15 $\frac{3}{8}$ gal. of maple syrup which he put up in bottles, each holding $\frac{3}{8}$ of a gallon. How many bottles did he have?

10. He sold the syrup at \$ $\frac{5}{8}$ a bottle. He had paid \$6 $\frac{1}{2}$ for it. How much did he gain?

11. The circumference of a circle is 3.1416 times the diameter. Find the circumference of a circle whose radius is 42.15 ft.

12. A real estate dealer bought 1013.88 acres of land which he divided into lots of equal size. There were 426 in all. How many acres were there in each lot?

13. A man left $\frac{2}{7}$, $\frac{1}{5}$, and $\frac{2}{15}$ of his money to three different

charity organizations. The total amount of property that he left was \$21,105. How much did he leave to each?

14. A tank of oil is $\frac{4}{5}$ full. If $\frac{3}{5}$ of the contents of the tank are drawn off and then $\frac{1}{6}$ of the remainder, what part is left?

15. From a pile of apples containing $368\frac{1}{2}$ bu., $72\frac{1}{2}$ bu. were sold. How many remained?

16. Divide 6.26 by .00025; 3430 by .049.

17. A rectangular field is $33\frac{3}{4}$ rods long and $15\frac{1}{2}$ rods wide. How many square rods does it contain?

18. A farmer had 650 bushels of corn. He sold $\frac{5}{18}$ of them. How many did he sell? How many had he left?

19. From his income a man saves \$250 a year. He spends $\frac{1}{5}$ of his income for household expenses, $\frac{1}{4}$ of it for clothing, and $\frac{1}{12}$ of it for his vacation expenses. What is his income?

20. An automobile went 275.16 miles in 10 hours. How far did it travel in one hour?

21. The area of a rectangle is 3798.75 sq. ft. One side of the rectangle measures 100 feet. What are the lengths of the other sides?

22. A dealer bought 100 pictures for which he paid \$965. What was the cost of one picture?

SUMMARY OF CHAPTER II

COMMON FRACTIONS.

Reduction.

Fundamental processes.

Simple Fractions.

Mixed Numbers.

DECIMAL FRACTIONS.

Reduction.

Fundamental processes.

CHAPTER III

DENOMINATE NUMBERS—APPLICATIONS OF DENOMINATE NUMBERS

SIMPLE AND COMPOUND

118. What is an abstract number? A concrete number? A unit?

Name some unit of measurement.

Concrete numbers naming units of measurement fixed by law are called **denominate numbers**.

Denominate numbers of a single denomination, as 2 pounds, are called **simple denominate numbers**; those of more than one denomination, as 2 lb. 3 oz., are called **compound denominate numbers**.

For the tables of denominate numbers, see p. 264.

REDUCTION

119. REDUCING TO LOWER DENOMINATIONS.

How many pint bottles can be filled from a can of milk containing 3 gal. 3 qt. 1 pt.?

SUGGESTION: Gal. 3×4 (qt. in gal.) = 12 qt.

Qt. $3 + 12 = 15$; 15×2 (pt. in qt.) = 30 pt.

Pt. $1 + 30 = 31$ pt.

Reducing a denominate number to lower denominations is called **reduction descending**.

To reduce a compound denominate number to its lowest denomination, multiply the number of units of each denomination, beginning with the highest, by the number representing the ratio of its unit to that of the next lower denomination, and add to the product in each case the units, if any, of the lower denomination.

120. Reduce to lower denominations:

1. \$1, 5 dimes 6 cts.
2. 16 gal. 3 qt. 1 pt. 1 gi.
3. 1 T. 3 cwt. 17 lb. 9 oz.
4. 12 cwt. 3 gr.
5. 1 lb. 10 $\frac{3}{4}$ 7 $\frac{3}{4}$ 2 $\frac{1}{2}$ 15 gr.
6. 1 mi. 150 rd. 3 yd.
7. 4 yd. 2 ft. 10 in.
8. 2 mi. 75 ch. 80 li.
9. 3 sq. mi. 500 A. 15 sq. rd. 26 sq. yd. 8 sq. ft. 120 sq. in.
10. 5 sq. mi. 320 A. 5 sq. ch. 12 sq. rd. 315 sq. li.
11. 10 cu. yd. 25 cu. ft. 1600 cu. in.
12. 3 bundles of paper, 1 ream, 15 quires, 20 sheets.
13. 10 gr. gross, 8 gross, 9 doz.
14. How many years are 3 score and 10?
15. 1 yr. 4 mo. 2 wk. 6 da.

121. REDUCING TO HIGHER DENOMINATIONS.

In 24 pt. how many gallons?

$$\begin{array}{r} 2 \overline{)24} \text{ pt.} \\ 4 \overline{)12} \text{ qt.} \\ 3 \text{ gal.} \end{array}$$

Reduce 6500 lb. to higher denominations.

$$\begin{array}{r} \text{SOLUTION:} \quad 100 \overline{)6550} \text{ lb.} \\ \quad \quad \quad 20 \overline{)65} \text{ cwt. 50 lb.} \\ \quad \quad \quad \quad \quad 3 \text{ T. 5 cwt.} \end{array}$$

Ans. 3 T. 5 cwt. 50 lb.

Changing denominate numbers to higher denominations is called *reduction ascending*.

To reduce a denominate number to one of higher denominations, divide the number of units given by the ratio of a unit of the next higher denomination to one of the given denomination. The remainder, if any, will be of the lower denomination. Divide each succeeding quotient in like manner.

WRITTEN EXERCISES

122. Reduce :

- 400 pt. (dry) to bushels.
- 850 lb. to hundredweight.
- 672 cwt. to tons.
- 5267 sec. to hours.
- 139,968 cu. in. to cubic yards.
- 1467 in. to rods.
- 800 rd. to miles.
- 245 hr. to days.
- 122 sq. ft. to square yards.
- 520 sq. rd. to acres.

ORAL EXERCISES

123. How many inches in 4 yd. 2 ft. 8 in.?

How many pounds in 64 ounces?

How many pints in 2 bu. 3 pk. 1 qt.?

How many sq. in. in 3 sq. yd.?

How many ounces in 5 cwt.?

WRITTEN EXERCISES

124. Solve:

1. Reduce to lower denominations :

- (1) 30 sq. yd. 8 sq. ft. 56 sq. in.
- (2) 25 " " 6 " " 100 " "
- (3) 18 " " 5 " " 75 " "

2. Mr. Morse sold 5 bu. 3 pk. 6 qt. of chestnuts for 15 ct. a quart. How much did he receive for them?

3. Mr. Dunlap bought $12\frac{1}{2}$ bu. of potatoes for \$12.50 and sold them for \$.35 a peck. How much did he make?

4. Mr. Harris sold $31\frac{1}{2}$ gal. of molasses at \$0.12 $\frac{1}{2}$ a quart. How much did he receive for it?

5. A milk dealer sold to Mr. Carter 12 gal. 3 qt. of milk ; to Mr. Hammond 15 gal. 2 qt. 1 pt. ; to Mr. Francis 25 gal. The purchasers were grocers and sold the milk to their customers for \$0.04 a pint. How much did each grocer receive for his milk?

6. What is the value of 150 gal. of milk at \$0.03 a pint?

7. Change 327 pints to quarts and gallons and prove the result.

8. How many cords in a pile of wood 4 ft. wide, 8 ft. high, 120 ft. long?

9. A body immersed in water loses in weight as much as a body of water of equal size weighs. How much will a cubic foot of lead weigh in water?

1 cubic foot of water weighs 62.5 lb.

1 cubic foot of lead weighs in the air 709.5 lb.

10. A diver examined a sunken vessel $6\frac{1}{2}$ fathoms under water. How many feet under water was it?

11. He brought up a treasure chest 2 ft. $\times$ 1 ft. $\times$ 1 ft., that weighed, on land, including contents, 200 lb. How much did it weigh in the water? How many cubic feet in the chest?

12. The steel boat *Cupid* weighs 2400 lb. When empty, it displaces 880 cu. ft. of water; when fully loaded, 2100 cu. ft. How many pounds of load can be put on the boat before sinking it?

A body will float if it weighs no more than a body of water of equal size.

13. The steamship *Norma* weighs about 3800 tons; her machinery, 1310 tons; her fuel for a trip across the ocean, 1500 tons. When loaded she weighs 9550 tons. What is her load? How many cubic feet of water does she displace when loaded?

125. REDUCING TO DECIMALS OR COMMON FRACTIONS OF HIGHER DENOMINATIONS.

1. Reduce 5 bu. 2 pk. 8 qt. to mixed decimals of a bushel.

SOLUTION: 1 bu. = 32 qts.

$$8 \text{ qts.} = \frac{1}{4} = .25 \text{ of a bushel.}$$

$$1 \text{ bu.} = 4 \text{ pk.}$$

$$2 \text{ pk.} = \frac{1}{2} = .5 \text{ of a bushel.}$$

$$5 + .25 + .5 = 5.75 \text{ bu.}$$

2. Reduce 20 rd. 4 yd. 2 ft. 3 in. to mixed decimals of a rod.

SOLUTION: 3 in. = $\frac{1}{4}$ ft. = .25 ft.

$$\begin{array}{r} 3 \overline{)2.25} \text{ ft.} \\ \underline{.75} \text{ yd.} \end{array}$$

$$5.5)4.7500(.863 +$$

$$\begin{array}{r} 4 \ 40 \\ \underline{350} \\ 330 \\ \underline{200} \\ 165 \end{array}$$

NOTE. Carry to three places.

Ans. 20.863+ rd.

3. Reduce 4 gal. 2 qt. 1 pt. to gallons and decimals of a gallon. Change the result to common mixed numbers.

4. Reduce 4 da. 2 hr. 40 min. to a decimal of a week.

5. What fraction of a ton is 15 cwt. 75 lb.?

In the ordinary transactions of life the use of a common fraction, or mixed number, of a single unit, is much more common than the use of compound numbers. We say, $\frac{1}{2}$ lb. of coffee, not 8 oz.; a yard and a half of ribbon, not 1 yd. 1 ft. 6 in.

WRITTEN EXERCISES

129. Solve:

1. Mr. Alford, in taking an account of his stock, found the following remnants of ribbon:

10 yd. 1 ft. 6 in.;	5 yd. 9 in.;
7 yd. 2 ft. 3 in.;	2 yd. 1 ft. 6 in.

He sold the lot to a dealer in remnants at 20 cents a yard. What did he receive for it?

2. The purchaser sold the following pieces to girls, dressing dolls for a fair, at 25 cents a yard:

2 yd. 6 in.;	5 yd. 1 ft.;
4 yd. 1 ft. 3 in.;	3 yd. 9 in.

How much did he receive for them?

3. How much did the dealer have left after making the above sales?

4. George Washington was born Feb. 22, 1732. He died Dec. 14, 1799. How old was he at his death?

5. Exactly how long ago to-day was the Declaration of Independence signed?

6. Exactly how old are you to-day?

7. Dewey captured Manila May 1, 1898. How long ago was that?

8. Mr. Smith has 2 fields, one containing 7 A. 9 sq. rd. 3 sq. yd. 2 sq. ft. of land, and the other 5 A. 121 sq. rd. 4 sq. ft. How much larger is the first than the second?

9. A farmer sold to one man 1 T. 2 cwt. 40 lb. of hay, to another 4 T. 70 lb., to the third 2 T. 15 cwt. 10 lb., and to the fourth 5 T. 13 cwt. 23 lb. How much did he sell to all?

10. In the month of March a grocer sold 125 gal. 2 qt. 2 gi. of kerosene, but in August he sold only 52 gal. 1 pt. 3 gi. How much more did he sell in March than in August?

MULTIPLICATION

130. How much oil will three cans hold, if each holds 2 gal. 3 qt. 1 pt.?

Gal.	Qt.	Pt.
2	3	1
		3

SOLUTION: $3 \times 1 \text{ pt.} = 3 \text{ pt.} = 1 \text{ qt. } 1 \text{ pt.}$
 $3 \times 3 \text{ qt.} = 9 \text{ qt.}$
 $9 \text{ qt.} + 1 \text{ qt.} = 10 \text{ qt.} = 2 \text{ gal. } 2 \text{ qt.}$
 $3 \times 2 \text{ gal.} = 6 \text{ gal.}$
 $6 \text{ gal.} + 2 \text{ gal.} = 8 \text{ gal.}$
Ans. 8 gal. 2 qt. 1 pt.

Another method is to reduce to common fractions of the highest denomination before multiplying.

SOLUTION: $1 \text{ pt.} = \frac{1}{8} \text{ gal.}$
 $3 \text{ qt.} = \frac{3}{4} \text{ gal.}$
 $\frac{3}{4} + \frac{1}{8} = \frac{7}{8}$
 $2\frac{7}{8} \text{ gal.} \times 3 = 8\frac{5}{8} \text{ gal.}$

Still another method is to reduce to decimal fractions of the highest denomination before multiplying.

SOLUTION: $1 \text{ pt.} = .125 \text{ gal.}$
 $3 \text{ qt.} = .75 \text{ gal.}$
 $2.875 \text{ gal.} \times 3$
 $\quad \quad \quad 3$
 $\hline 8.625 \text{ gal.} = 8\frac{5}{8} \text{ gal.}$
 $\quad \quad \quad = 8 \text{ gal. } 2 \text{ qt. } 1 \text{ pt.}$

Which is the easiest way?

In some problems the reduction to a decimal or common fraction may be too complicated. In such a case it is some-

times easier to reduce the compound number to its lowest denomination; thus:

131. Multiply 1 rd. 3 yd. 2 ft. by 16.

$$\begin{array}{r} \text{SOLUTION:} \quad 1 \text{ rd.} = 16\frac{1}{2} \text{ ft.} \\ \quad \quad \quad 3 \text{ yd.} = 9 \text{ ft.} \\ \quad \quad \quad \quad \quad 2 \text{ ft.} \\ \hline \quad \quad \quad 27.5 \text{ ft.} \times 16 = 440 \text{ ft.} \end{array}$$

WRITTEN EXERCISES

132. Multiply (using the simplest method):

1. A field is 20 rd. 4 yd. 2 ft. 5 in. long. How far must a plowman walk in plowing 25 furrows the long way of the field?

2. The "mean" solar year is 365 da. 5 hr. 48 min. 46 sec. exactly. How long a time does one live in 15 "mean" solar years?

3. How many square feet in a table 4 ft. 6 in. $\times$ 8 ft. 4 in.? Which method do you employ in this example?

4. What is the volume of a box 2 ft. 6 in. $\times$ 3 ft. 2 in. $\times$ 4 ft. 8 in.?

5. An automobile tank holds 9 gal. 2 qt. 1 pt. of gasoline. How much gasoline will it take to fill it once each day for a week?

6. What will be the actual weight in pounds of 12 pound-packages of cereal, if each falls $\frac{1}{2}$ oz. short in weight?

7. A real estate dealer divided a plot of ground into 25 lots, each containing 2500 sq. ft. How many acres did the plot contain?

8. A man spent 2 hr. 25 min. 45 sec. on the train going to business each day. Find the time that he so spent in February, 1908.

DIVISION

133. The committee have 20 lb. 10 oz. of candy, with which to fill 66 bags for the Christmas tree. How much can be put into each bag?

Divide 20 lb. 10 oz. by 66.

SOLUTION: 20 lb. 10 oz. = 330 oz.
 330 oz. ÷ 66 = 5 oz.

Divide: 8 gal. 3 qt. by 5.

SOLUTION: 8 gal. 3 qt. = 8.75 qt.
 8.75 ÷ 5 = 1.75.

Divide: 14 bu. 3 pk. 7 qt. 1 pt. by 4.

	Bu.	Pk.	Qt.	Pt.
SOLUTION:	4)14	3	7	1
	3	2	7	1½

14 bu. ÷ 4 = 3 bu. and 2 remainder.

2 bu. = 8 pk.; 3 pk. + 8 pk. = 11 pk.

11 pk. ÷ 4 = 2 pk. + 3 remainder.

3 pk. = 24 qt.; 24 qt. + 7 qt. = 31 qt.

31 qt. ÷ 4 = 7 qt. + 3 remainder.

3 qt. = 6 pt.; 6 pt. + 1 pt. = 7 pt.

7 pt. ÷ 4 = 1½ pt.

Which of the methods is the simplest?

Divide, using first one and then another method:

15 yd. 2 ft. 11 in. ÷ 6.

25 mi. 75 rd. 1 ft. ÷ 8.

To divide a compound number by an abstract number, divide the number of each denomination in turn by the divisor. Reduce the remainder, if there is one, to the next lower denomination and add it to the number of that denomination before dividing. Or,

Reduce the dividend to a simple denominate number and divide. Or,

If the divisor is a compound number, reduce both terms to the same denomination, regard the numbers as abstract, and divide.

WRITTEN EXERCISES

134. Solve:

1. 25 bu. 3 pk. 6 qt. 1 pt. + 5.
2. 46 yd. 2 ft. 11 in. + 7.
3. 60 T. 1600 lb. 12 oz. + 11. •
4. 12 dollars, 6 dimes, 9 cents + 5.
5. 48 gal. 3 qt. 1 pt. + 10.
6. A man purchased land which faced on Green Street for a distance of 40 rods and 15 feet. He divided this distance into 10 equal parts. What was the width of each lot?
7. A lot 6 rd. 1 ft. $\times$ 5 rd. 10 ft. is to be laid out in tennis courts. How many square feet does it contain? How many courts 78×36 ft. can be made in it? How much land will be left on the sides? the ends?
8. It is to be fenced with 2 strips of poultry wire, each 4 ft. wide. How many linear feet of wire will it take? How many square feet?
9. The wire is supported on posts 10 ft. apart. How many posts will be required? (Draw an illustration.)
10. The poultry wire costs $\frac{1}{2}$ cent a square foot. What is the total cost?
11. The posts are 4 in. square and are set 3 ft. in the ground. At 6 cents a foot, what is their cost?
12. Into how many fields, each containing $10\frac{1}{2}$ acres, can a farmer divide his farm of $157\frac{1}{2}$ acres?
13. A grocer bought 3 barrels of apples and sold them by the basket, each basket containing 1.5 pk. How many baskets of apples did he sell?
14. A carload of coal containing 86,000 lb. was put into bins that held 10 T. 15 cwt. each. How many bins were filled from the carload?

MEASUREMENT OF PLANE SURFACES

135. To avoid misunderstanding and dishonesty all governments fix **standard measures**. To these standards all measures in use in business transactions must conform. The standard is usually a single measure of a kind, as a yard for length, a bushel and a gallon for capacity, a pound for weight. All other measures used must bear a certain ratio to the standard.

Not only do governments fix standards of measurements, but they pass laws making it a crime to use false measures, and have provided for the punishment of those doing so.

RECTANGLES AND TRAPEZOIDS

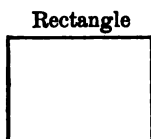


FIG. 1.

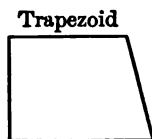


FIG. 2.



FIG. 3.

136. What is the difference between Fig. 2 and Fig. 1? Between Fig. 3 and Fig. 1? Between Fig. 2 and Fig. 3? What is Fig. 2 called?

Of what kind of lines is it made?

Lines that have the same direction are said to be **parallel**.

A quadrangle having its opposite sides parallel is called a **parallelogram**.

A **rectangle** is a plane figure bounded by four straight lines, its opposite sides being parallel, and its corners forming right angles.

A **trapezoid** is a plane figure bounded by four straight lines, only one pair of sides being parallel.

137. 1. Draw a trapezoid whose parallel sides are 4 in. and 6 in., the distance between them being 2 in. Cut out the trapezoid.

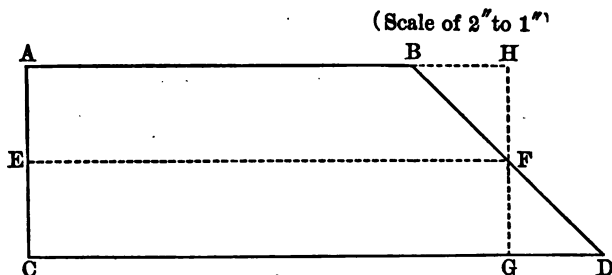
2. Bisect the vertical side AC, and draw from E, the point of bisection, a line EF parallel to the base.

3. From the point F, where this line touches the oblique line, draw a line FG, perpendicular to the base CD.

4. Fold over the triangle FGD thus formed; then cut to point of intersection F, on line FG.

5. Turn F as on a pivot, until triangle FGD takes the position FHB.

6. What kind of figure is now formed?



7. What is its length? 8. Its width? 9. Its area?

10. What is the average length of its parallel sides?

11. How does this compare with the length of the rectangle AHGC that was made from the trapezoid?

The average length of the parallel sides of a trapezoid is equal to the length of a rectangle of the same size.

The area of a trapezoid is equal to the area of a rectangle of equal altitude, whose length is the average of the two parallel sides.

To find the area of a trapezoid, find the average length of the two parallel sides and multiply this by the altitude.

ORAL EXERCISES

138. What is the area of a trapezoid whose parallel sides are:

1. 6 in. and 10 in., distance between them 4 in.?
2. 8 in. and 12 in., distance between them 6 in.?
3. 5 in. and 17 in., distance between them 10 in.?
4. 7 in. and 11 in., distance between them 7 in.?
5. 12 in. and 20 in., the distance between them 11 in.?

WRITTEN EXERCISES

139. Solve:

1. Draw a figure 2 inches by 1 inch. If it is drawn on a scale of 1 inch to 25 feet, what is the perimeter of the rectangle that it represents?

2. Find the perimeter of the rectangle represented by the figure on page 75, if drawn on a scale of $\frac{1}{2}$ inch to 25 feet.

3. Using the scale of Exercise 2, find the number of square feet the figure represents. Of square yards.

4. Measure the area of your schoolroom and draw a plan of it on the scale of $\frac{1}{4}$ in. to 1 ft.

5. Measure the surface of your desk and draw in the proper places on your plan all the desks, using the same scale.

6. Draw to scale a trapezoid whose parallel sides are 16 in. and 64 in., and the distance between them 32 in. Indicate by dotted lines the process of finding the area.

7. Find the area of the trapezoid you have drawn.

8. Mr. Scott had a farm with the parallel sides respectively 180 rods and 150 rods and the distance between them 60 rods. (Draw to scale 1 inch = 60 rods.)

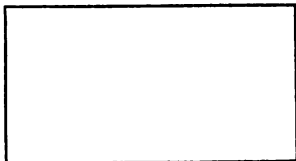
9. How many acres are there in the Scott farm?

10. Find the area of an isosceles triangle whose base is 71 in. and altitude 30 in.

WRITTEN EXERCISES

140. Solve:

1. A rectangular field is four times as long as it is wide, and contains 400 sq. rd. If it is 10 rd. wide, how long is it? What is its perimeter? How many acres does it contain?



400 sq. rd.

2. A man took a contract for making a concrete walk at \$.10 a square foot. If the walk was 10 rd. long and 5 ft. wide, how much did he receive?

3. A man built a 6-foot walk around a square plot 100 ft. on a side. What was the length of the walk inside? What was the length of the walk outside?

Make a drawing of the plot and the walk.

4. In the government survey of public lands, a square mile is called a section. The square miles are divided into 4 parts, called quarter sections. These are named from their position in the section, as, northeast quarter, or N.E. $\frac{1}{4}$, southeast quarter, or S.E. $\frac{1}{4}$, etc. Each quarter section contains 160 acres.

Draw a square to represent a square mile, and locate each of the quarter sections.

5. How many acres are there in a section? In $\frac{1}{4}$ of a quarter section? In $\frac{1}{2}$ of a quarter section?

6. A stockman purchased $10\frac{1}{2}$ sections of land for $\$3\frac{1}{2}$ an acre. What was the total cost?

7. A garden is 30 rd. long and 15 rd. wide. A man builds a fence around it 4 ft. high with posts 1 rd. apart. The posts cost \$.20 each, and the other material and labor cost \$1 per rod. How many posts will be required? What will be the total cost of the fence?

LONGITUDE AND TIME

141. The exact location of any place on the globe may be described by telling its distance north or south and east or west from other places that are known. For convenience all measurements are made to start from two lines imagined to be drawn around the earth at right angles to each other. The east and west line equally distant from the two poles is the equator.

Distance north or south of the equator is called **latitude**.

Imagine a circle drawn, through the place where you are standing, around the world through the north and south poles. Such a circle is called a **meridian**. You can imagine a meridian drawn through any place on the globe; so every place has its own meridian.

Distance east or west from a north or south line, or meridian, agreed upon as a starting place, is called **longitude**.

It has been agreed that the starting place for measuring longitude shall be the Meridian of Greenwich, England, where there is an astronomical observatory. The distance halfway around the world to the west from Greenwich is called west longitude, and halfway around the world to the east from Greenwich, east longitude.

Is your meridian in east or west longitude?

142. Every circle, large or small, is supposed to be divided into 360 parts called degrees, by which it is measured. Longitude is measured in degrees. Being a circle, the distance around the earth is divided into 360 degrees (360°); 180° east longitude and 180° west longitude. Each degree is divided into 60 minutes ($60'$), and each minute into 60 seconds ($60''$).

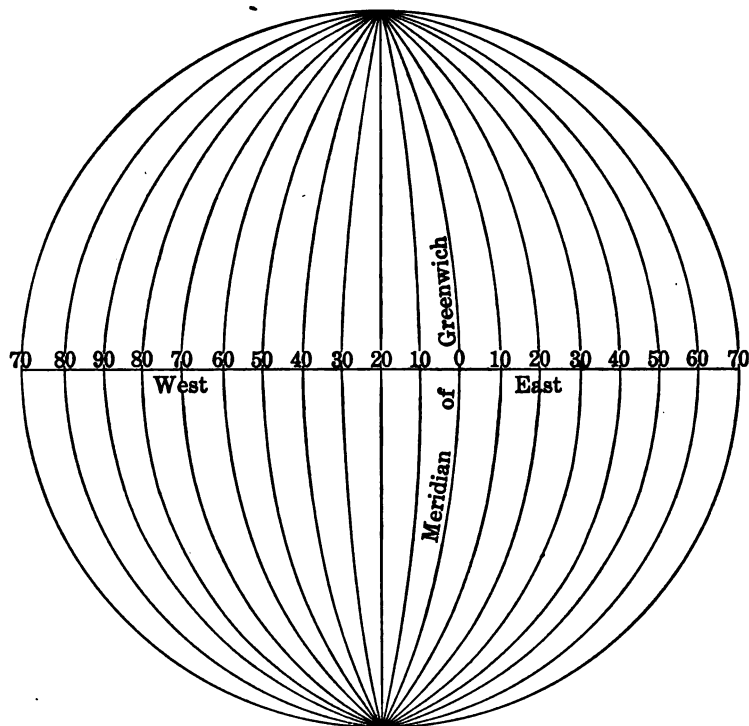
60 seconds ($60''$) make 1 minute.

60 minutes ($60'$) make 1 degree.

360 degrees (360°) make 1 circle.

TIME

143. Any meridian may be considered as 0, but the one most commonly used is the Meridian of Greenwich, which is shown on the following cut:



Time is determined by the position of the sun. When the sun is directly overhead, it is noon. As the sun appears to move from east to west, it is noon at all places on any north and south line, or meridian, at the same time.

When it is noon where you are, is it past noon or before noon at places east of you? Places west of you?

The difference in time between two places is 4 hr. 10 min. 30 sec. When it is 20 minutes past 5 in the easterly place, what time is it in the westerly place?

$$\begin{array}{r} 5 \text{ hr. } 20 \text{ min. } 00 \text{ sec.} \\ 4 \text{ hr. } 10 \text{ min. } 30 \text{ sec.} \\ \hline 1 \text{ hr. } 9 \text{ min. } 30 \text{ sec.} \end{array}$$

EXPLANATION: As we wish to find the time in the westerly place, we subtract the difference in time from the easterly time. Explain why.

The answer is 9 min. 30 sec. past 1.

The difference in time between two places is 3 hr. 50 min. 10 sec. When it is 20 minutes past 9 in the westerly place, what time is it in the easterly place?

$$\begin{array}{r} 9 \text{ hr. } 20 \text{ min. } 00 \text{ sec.} \\ 3 \text{ hr. } 50 \text{ min. } 10 \text{ sec.} \\ \hline 13 \text{ hr. } 10 \text{ min. } 10 \text{ sec.} \end{array}$$

EXPLANATION: As we wish to find the time in the easterly place, we add the difference in time to the westerly time. Tell why.

The answer is 10 min. and 10 sec. past 13 o'clock; or, 10 min. and 10 sec. past 1 P.M.

Why is 13 o'clock the same as 1 o'clock in the afternoon?

To find the time at a given place from an easterly place subtract the difference in time; or, to a westerly place add the difference in time.

It is 20 minutes after 8 P.M. in a certain place when it is 30 minutes to twelve in another. What is the difference in time? In what direction is the first place from the second?

$$\begin{array}{r} 11 \text{ hr. } 30 \text{ min.} \\ 8 \text{ hr. } 20 \text{ min.} \\ \hline 3 \text{ hr. } 10 \text{ min.} \end{array}$$

We find the difference of time by subtraction. The place of later time will be east of the other place.

A place called A is 3 hr. 40 min. east of a given place, while a place called B is 4 hr. 50 min. west. What is the difference in time between A and B?

$$\begin{array}{r} 3 \text{ hr. } 40 \text{ min. East} \\ 4 \text{ hr. } 50 \text{ min. West} \\ \hline 8 \text{ hr. } 30 \text{ min.} \end{array}$$

Inasmuch as one place is east and the other is west, the difference in time is found by adding.

LONGITUDE AND TIME

144. The sun appears to move around the earth in 24 hours. Hence, it appears to move through 360° in 24 hours, or 1440 minutes. To move through one degree it will take $\frac{1}{360}$ of 1440 minutes, or 4 minutes. Hence, the difference in time between two places 1 degree of longitude apart is 4 minutes.

The difference in time between two places is 6 hr. 10 min. What is the difference in longitude?

$$\begin{array}{r} \text{SOLUTION: } 6 \text{ hr. } 10 \text{ min.} = 370 \text{ min.} \quad \begin{array}{r} 92^\circ 30' \\ 4 \overline{)370 \text{ min.}} \end{array} \end{array}$$

EXPLANATION. Since the sun passes through 1 degree in 4 minutes, it will pass through as many degrees in 6 hr. 10 min., or 370 minutes, as 4 minutes are contained in 370 minutes = 92 times and 2 remainder = $92\frac{2}{4}^\circ$.

$$\frac{2}{4} \text{ of } 60 \text{ minutes} = 30 \text{ minutes} \quad \text{The answer is } 92^\circ 30'.$$

To find the difference in longitude when the difference in time is given, divide the number of minutes by 4.

The difference in longitude between two places is $15^\circ 12' 15''$. What is the difference in time?

$$\begin{array}{r} \text{SOLUTION:} \quad \begin{array}{r} 15^\circ 12' 15'' \\ \quad \quad \quad 4 \\ \hline 60' 49'' 0 \text{ or } 1 \text{ hr. } 49 \text{ sec.} \end{array} \end{array}$$

EXPLANATION: The earth in its rotation passes over 1° of longitude in 4 min. of time, $1'$ of longitude in 4 sec. of time.

Therefore, the number of degrees difference in longitude between two places $\times 4$ = the number of minutes difference in time.

The number of minutes difference of longitude $\times 4$ = seconds of difference in time.

(1) A is $40^\circ 45' 35''$ east longitude and B is $65^\circ 30' 40''$ east. The difference in longitude is found by subtracting the less from the greater.

(2) C is $15^\circ 20' 40''$ west longitude and D is $42^\circ 16' 18''$ west. The difference in longitude is found by subtracting the less from the greater.

(3) E is $24^{\circ} 20' 45''$ east and F is $30^{\circ} 15' 30''$ west. The difference in longitude is found by addition. Why?

From your geographies find the longitudes of Chicago, New Orleans, St. Louis, St. Paul, San Francisco, New York, Philadelphia, Boston, and of your own city or town.

1. Find the difference in longitude between your town or city and each of the above-named cities.

2. Find the longitude of Yokohama, Peking, Calcutta.

3. Find the difference in longitude between your town and each of the cities named in Problem 2.

4. To find the difference between a place in east longitude and one in west longitude do you add or subtract? Explain.

5. When it is noon in London, what time is it in New York? Do you add or subtract to find it?

6. In taking a long journey, I changed my watch every 15° of longitude. How much must I have changed it each time? I was traveling westward. In which direction did I change it?

7. If you were to travel exactly halfway around the earth without changing your watch, would it then need changing? Explain why?

TIME BELTS

145. Each meridian has its own time. No two places have the same time unless they are on the same north and south line, or meridian. As it would be very inconvenient to have our watches wrong whenever we go to a place only a short distance east or west, the governments of this continent have agreed to change time only when the difference is a whole hour, or until the earth has revolved through 15° . This is called Standard Time.



There are four divisions, or "time belts," on this continent: Eastern, Central, Mountain, and Western.

The eastern time is that of the 75th meridian, the western, that of the 120th.

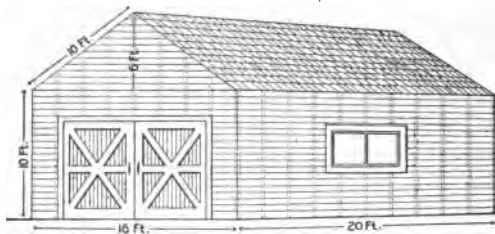
1. How many hours difference (Standard Time) between New York and San Francisco?
2. In traveling across the continent from San Francisco to New York, how many times would you have to set your watch to keep Standard Time?
3. In going west, would you move your watch backward or forward? In going east?
4. How many times was it necessary for the men in the New York to Paris motor car race, by way of San Francisco, to change their watches in their race around the world?
5. How many times was it necessary to change their watches in crossing the United States? Asia? Europe?
6. Were their watches set back or ahead in each case? Explain why.

REVIEW

146. Solve :

(See Tables, p. 264.)

1. How many loads* in a bank of sand 6 ft. high, 20 ft. long, and 15 ft. wide?
2. How many loads in the excavation of a cellar, 25 ft. long, 16 ft. wide, and 8 ft. deep?
3. A contractor received 75 cents a load for excavating a cut for a road, 116 ft. long, 20 ft. wide, and averaging 15 ft. deep. How much did he receive?



This picture is a representation of a garage. The frame is constructed of timber, 3 in. by 4 in., each stick used being 10 ft. or 16 ft. long and used with the least waste. The rafters are 2 ft. apart. The joists, represented by the dotted lines, are also 2 ft. apart all around the main part of the building.

4. How many sticks of timber, 16 ft. long, must be used in building this garage? How many sticks of timber, 10 ft. long, can be used?
5. Allowing three 16-ft. sticks of timber for the gable ends, how many feet of timber were used? What is it worth at $3\frac{1}{2}$ cts. per running foot?

* In measuring excavations the unit is 1 cubic yard, which is called a *load*.

6. Making no allowance for windows, doors, or waste, how many square feet of boards would it take to cover the main part of the building? How many square feet in the gable?

7. How many square feet in the roof? What would the boards to cover the sides and ends of the building cost at \$40 per M?

8. (Shingles are laid 4 in. to the weather and a shingle is 4 in. wide. Each shingle covers 16 sq. in., or there are 9 shingles to a square foot.) How many shingles did it take to shingle the roof? What would they cost at \$8 per M?

9. Allowing \$7.50 for extra work on the doors, and \$5 for the windows, and 12 days' carpenter work, at \$4.80 a day, what was the cost of the building?

10. The survey for a new road made the length 312 chains, 75 links. How long was it in miles, rods, and feet? In miles and decimals of a mile?

11. It is 12 miles and 3 rods from the post office of Amity to that of Coleville. How many chains and links?

12. The steamship *Adriatic* draws 22 ft. of water. How many fathoms deep must the water be for her to sail in?

13. How many D in a pound? In an Z ?

14. How many Z in $1\frac{1}{2}$ lb.?

15. Write as decimals of a pound, $3\text{Z } 4\text{Z } 50\text{D}$.

16. A gold dollar weighs 25.8 grains. What is the weight of a gold eagle? Of \$1,000,000 of gold?

17. A silver dollar weighs 412.5 grains. What is the weight of \$1,000 in silver?

18. If you are worth your weight in silver, how many dollars are you worth? (Reduce to grains. Why?)

19. The longitude of Berlin, Germany, is $13^{\circ} 23' 43.5''$ E. That of Dublin, Ireland, is $6^{\circ} 20' 30''$ W. When it is 7 A.M. at Dublin, what time is it at Berlin?

20. A man bought a plot of land in the form of a trapezoid. The length of one side was 256 ft., and that of the other side parallel to it was 328 ft. The distance between the parallel sides was 110 ft. How many acres did the plot contain?

21. Find the difference in area between a rectangle 36 ft. long and 24 ft. wide, and a trapezoid whose parallel sides measure 30 ft. and 42 ft. respectively, the distance between the parallel sides being 24 ft.

22. Classify the following units of measurement, making 3 groups of them. In Group One place all that refer to linear measurement, in Group Two all that refer to area, and in Group Three all that refer to volume.

Acre, inch, square foot, cord foot, mile, yard, section, cubic yard, rod, cubic foot, perch, square, square inch, square mile, square yard, link, chain.

23. Draw a rectangle; a square; a circle; a right triangle; a trapezoid.

24. There are two fields, each of which has a perimeter of 180 rd. One of them is a square, and the other a rectangle 8 times as long as it is wide. Which is the larger field?

25. What is the length of the rectangular field?

SUMMARY OF CHAPTER III

DENOMINATE NUMBERS

Reduction.

The Fundamental Processes.

APPLICATIONS OF DENOMINATE NUMBERS.

Measurements.

Longitude and Time.

CHAPTER IV

PERCENTAGE—APPLICATIONS OF PERCENTAGE

FINDING PERCENTAGE

147. 1 Per cent means 1 in each hundred, or $\frac{1}{100}$. It is written 1 % or .01. Per cent (centum) means by the hundred.

Of what per cent is each of the following common fractions the equivalent :

$$\frac{1}{2}, \frac{1}{4}, \frac{1}{3}, \frac{1}{6}, \frac{1}{5}, \frac{1}{8}, \frac{1}{10}, \frac{1}{12}, \frac{1}{16} ?$$

Write each as a decimal.

In decimals, one tenth is the unit of calculation ; in percentage one hundredth is the unit, but the decimal notation is used.

The terms of percentage are base, rate, percentage, amount, and difference.

The **base** is the quantity on which the percentage is reckoned.

The **rate** is the stated number of hundredths of the base. It is read "per cent."

The **percentage** is the quantity that equals the stated number of hundredths of the base. It is the product of the base by the rate.

The **amount** is the sum of the base and the percentage.

The **difference** is the base less the percentage.

The **amount** is represented by $100 \% + \text{the rate}$.

The **difference** is represented by $100 \% - \text{the rate}$.

In the following example name each term :

$$\begin{array}{r}
 \$455 \\
 .05 \\
 \hline
 \$22.75 \\
 455 \\
 \hline
 \$477.75
 \end{array}$$

148. As the relations of the quantities expressed by these terms are always the same, a clear understanding of these relations will make all problems in percentage simple of solution.

The relations may be stated in terms either of an equation or of a proportion.

I. Equation :

$$\text{Base} \times \text{Rate} = \text{Percentage.}$$

$$\text{Percentage} \div \text{Base} = \text{Rate.}$$

$$\text{Percentage} \div \text{Rate} = \text{Base.}$$

$$\text{Base} + \text{Percentage} = \text{Amount.}$$

$$\text{Base} - \text{Percentage} = \text{Difference.}$$

These formulas should be memorized.

II. Proportion :

Three terms are given and x is used to represent the fourth or missing term. Remember that 100 % always equals the whole of any quantity.

Find 6 per cent of \$600.

SOLUTION :

$$\text{By the equation:} \quad \$600 \times .06 = \$36.$$

$$\text{By proportion:} \quad 6 \% : 100 \% :: x : \$600.$$

$$x = \frac{\$600 \times 6}{100} = \$36.$$

NOTE. In finding the percentage the use of the equation is much simpler. This is true also of the greater number of problems for finding other terms. But in some the proportion may be used to advantage.

149. TO FIND THE PERCENTAGE.

What is 12 % of \$400 ?

SOLUTION: $\$400 \times .12 = \$48.$

To find the percentage, base and rate being given, multiply the base by the rate written as a decimal. .

ORAL EXERCISES

150. Find the percentage :

BASE	RATE	BASE	RATE
1,200 ft.	7 %	4,000 yd.	4 %
32	8 %	300	50 %
\$18.75	10 %	\$1,500	33 $\frac{1}{3}$ %
6	60 %	400	25 %

WRITTEN EXERCISES

151. Solve:

1. Mr. Labon, in dividing his estate of 1,600 acres, gave 15 % of it to his son, and 20 % of the remainder to his daughter. How many acres did his son receive? His daughter?

2. Mr. Murray bought 400 yards of silk. He sold to one customer 8.5 %, and to another 16 $\frac{1}{4}$ %. How many yards did he sell to each?

3. Mr. Jeffries built a house at a cost of \$6,500. He sold it at a profit of 10 $\frac{1}{2}$ %. How much did he make? For how much did he sell it?

4. He built another house at a cost of \$7,000 and sold it at a loss of 6 $\frac{1}{4}$ %. How much did he lose?

5. Mr. Adams had a field 42 rods long and 25 rods wide. He sold 33 $\frac{1}{3}$ % of it. How many acres did he sell?

6. Mr. Dunn bought 820 yards of carpet at \$.96 per yard. He sold it at a profit of 18 $\frac{1}{2}$ %. How much did he make?

7. Mr. Holden has 27,000 feet of white pine boards, and 54,000 feet of oak boards. The pine cost him \$48.50 per thousand feet. The oak cost him \$65.50 per thousand. If he sells the boards at a profit of 15%, how much will he make?

8. A merchant bought 18 yards of cloth at \$1.75 a yard. He sold 12 yards at a gain of $33\frac{1}{3}\%$ and 6 yards at a gain of 25%. What was his entire gain? What was his selling price?

FINDING AMOUNT OR DIFFERENCE

152. TO FIND THE AMOUNT OR DIFFERENCE.

By the equation:

$$\text{Amount} = \text{Base} + \text{Percentage.}$$

Mr. Sturgis had 48 cows. He increased his stock 25 percent. How many cows did he then have?

SOLUTION: $48 \times .25 = 12;$

$$48 + 12 = 60;$$

or,

$$48 = 100\%;$$

$$100\% + 25\% = 125\%,$$

$$48 \times 1.25 = 60.$$

$$\text{Difference} = \text{Base} - \text{Percentage.}$$

Mr. Reid bought a motor car for \$1,800 and after using it a year sold it for $33\frac{1}{3}\%$ less. What did he receive for it?

SOLUTION: $\$1,800 \times .33\frac{1}{3} = \$600;$

$$\$1,800 - \$600 = \$1,200 \text{ difference;}$$

or,

$$100\% - 33\frac{1}{3}\% = 66\frac{2}{3}\%;$$

$$\$1,800 \times 66\frac{2}{3}\% = \$1,200 \text{ difference.}$$

To find the amount or difference, base and rate being given, find the percentage and add it to or subtract it from the base; or,

Add the rate to or subtract it from 100% and multiply the base by the sum or difference.

WRITTEN EXERCISES

153. Solve :

1. Mr. Collinson, a dealer in heavy wire fence, bought fence at the following rates :

Fence 3 ft. high	. . .	\$.40 per ft.
3 ft. 4 in. high	. . .	.44 per ft.
4 ft. high	. . .	.48 per ft.

He sold to one man 120 ft. of the \$.40 fence, to another 80 ft. of the \$.44 fence, and to the third 75 ft. of the \$.48 fence. He made a profit of 20 %. How much did he receive for the three sales ?

2. Gates cost Mr. Collinson as follows :

SINGLE GATES		DOUBLE GATES	
3 ft. wide	\$4.00	8 ft. wide	\$8.50
4 ft. wide	4.40	9 ft. wide	9.75
5 ft. wide	5.00	10 ft. wide	10.00

Mr. Williams bought a single gate 4 ft. wide and a double gate 9 ft. wide, Mr. Gillen bought a single gate 5 ft. wide and a double gate 8 ft. wide, and Mr. Dalton bought 3 single gates each 5 ft. wide. If Mr. Collinson's selling price is an advance of 20 % on the cost, what will be the price of each one of the gates ? What will he receive for the gates sold to the persons named ?

3. Mr. Goldsmith has watches that cost him \$12 and \$25.50. He sold a \$12 watch at a profit of 22 %, and a \$25.50 watch at a profit of $18\frac{1}{5}$ %. How much did he receive for each watch ?

4. Mr. Morton has several kinds of tea that cost him \$.30, \$.45, and \$.65 per pound. Retiring from business, he sold the \$.30 tea $12\frac{1}{2}$ % less than cost, the \$.45 tea $16\frac{2}{3}$ % less, the \$.65 tea 20 % less. What did he receive per pound for each kind ?

5. In one year there were 6,578 deaths from tuberculosis in a certain state. By separating the sick from the well, the annual deaths from this disease were reduced in 5 years 46 %. How many died in the fifth year?

6. In a school having an average attendance of 1,278 pupils, the per cent of tardiness was $3\frac{1}{2}$. What was the average number of pupils present on time? The average number late?

7. There are 30 breeds of sheep known. $\frac{1}{10}$ of these are found in Asia, $\frac{1}{3}$ in Africa, $\frac{2}{15}$ in Europe, and the others in America. What per cent of the known breeds are American? How many of the known breeds are American?

8. 100 Angora goats will keep 40 acres of land cleared of brush. An Iowa farmer who owns 640 acres has a sufficient number of goats to keep 25 % of it clear. How many goats has he? How many goats would be needed to keep his land clear?

9. A goat will produce at a shearing 4 lb. of fleece. If 70 % of it is worth 30 ct. a pound, what is the value of the 30-cent fleece from the goats kept on the 640-acre farm?

10. If 30 % of the fleece is short hair (noils) worth 16 ct. a pound, what is the value of the entire product?

11. A dealer sold a cask of oil containing 32 gal. He agreed to deliver it for \$5.12. In delivering it $6\frac{1}{4}$ % of the oil was spilled. How many gallons were spilled? How much less than \$5.12 did the dealer receive?

12. Mr. Ewing had a library of 875 volumes. In a fire 44 % of them were destroyed. How many books had he left?

13. Mrs. Nelson's piano cost \$842. She was forced to sell it for 33 % less than it cost. How much did she lose on it?

FINDING BASE

154. TO FIND THE BASE.

SOLUTION:

By analysis:

25 is 25 % of what number?

$$25 \% = \frac{1}{4}.$$

If 25 is $\frac{1}{4}$, the whole number or

$$100 \% = 4 \times 25 = 100.$$

By the equation:

$$\text{Base} = \text{Percentage} \div \text{Rate}.$$

$$25 \div .25 = 100.$$

To find the base, rate and percentage being given, divide the percentage by the rate.

WRITTEN EXERCISES

155. Find the base:

RATE PER CENT	PERCENTAGE	RATE PER CENT	PERCENTAGE
1. $12\frac{1}{2} \%$	\$22.50	5. 40 %	30 yd.
2. $16\frac{2}{3} \%$	\$8.25	6. 25 %	\$16.25
3. $8\frac{1}{3} \%$	20	7. $33\frac{1}{3} \%$	6.50
4. 10 %	45.5	8. $6\frac{1}{4} \%$	200 ft.

9. A milk dealer sold 10.5 quarts of milk, which was $16\frac{2}{3} \%$ of all he had. How much milk had he? What per cent had he left?

10. Mary has \$20, which is $8\frac{1}{3} \%$ as much as Louise has. How much money has Louise? How much more has Louise than Mary?

Make five similar problems based upon the rates and percentages given above.

11. Miss Riley saves \$4.50 each week that she teaches. She teaches 40 weeks a year and saves 18 % of her salary. What salary does she receive for the year?

12. 15% of Mr. Rhodes' strawberry crop was 420 quarts. He averaged $6\frac{1}{2}$ cents a quart for his berries. What did he receive for his crop?

SOLUTION:

By the equation $420 \div .15 = 2800$;

$$2800 \times \$.065 = \$182.$$

By proportion: $100\% : 15\% = x \text{ qt.} : 420 \text{ qt.}$

$$x = \frac{420 \text{ qt.} \times 100}{15} = 2800 \text{ qt.}$$

13. Mr. Lathrop saves $8\frac{1}{2}\%$ of his salary. What is his salary if he saves \$75?

14. During a violent storm 64 bushels of apples dropped from the trees in Mr. Hendrickson's orchard. This was $12\frac{1}{2}\%$ of his entire crop. How many bushels had he in all?

15. Mr. James gave \$25,000 towards purchasing land for a public park, \$5,000 for grading, \$2,000 for a fountain, and \$6,400 for a bridge across the park. Mr. James' donation was 60% of the entire cost of the park. How much was the cost?

16. The salt water which was obtained from the bottom of a mine of rock salt contained 8% of its weight of pure salt. What weight of water was it necessary to evaporate in order to obtain 3,894 pounds of salt?

17. If the salt water contained 12% of pure salt, how many pounds of water would it be necessary to evaporate to secure 11,568 pounds of pure salt? How many pounds if it contained .05? 6%? 5%?

18. In Los Angeles, there were at a certain date 10,032 homes owned by the occupants, which is 44% of all the homes in the city. How many homes did the city contain?

19. In Philadelphia 196,248 homes were rented. This was 78% of all the homes in the city. How many homes were there?

FINDING RATE

156. To FIND THE RATE.

\$8 is what per cent of \$32?

$$\text{Percentage} \div \text{Base} = \text{Rate}.$$

SOLUTION:

$$32)8.00(.25, \text{ or } \frac{8}{32} = \frac{1}{4} = .25.$$

$$\begin{array}{r} 64 \\ \hline \end{array}$$

$$\begin{array}{r} 160 \\ \hline \end{array}$$

$$\begin{array}{r} 160 \\ \hline \end{array}$$

To find the rate, base and percentage being given, divide the percentage by the base.

WRITTEN EXERCISES

157. Solve:

1. 200,000 immigrants settled in New York in one year. If the population of New York was 4,500,000 at the beginning of the year, what was the per cent of increase from immigration during that year?

2. In Chicago there are 258,582 homes rented and 86,435 homes owned by the occupants. What per cent of all the homes are owned by the occupants?

3. The population of the United States is about 80,000,000. That of the world is estimated at 16,000,000,000. What per cent of the people of the world live in the United States?

4. A graduate of a trade school received \$3.50 a week at first. After 3 years she was receiving \$10 a week. What was the per cent of increase?

5. According to a recent estimate there are in New York 12,000 persons employed as cigar makers. Of these 9,000 are men. What per cent are women?

6. There are employed as tailors in New York approximately 50,000 persons, of whom 8,000 are women. What per cent of the tailors are men?

REVIEW

158. Solve:

1. The area of a rectangle 3 inches by 4 inches is what per cent of the area of a rectangle 6 inches by 8 inches?

2. The area of a right triangle whose base is 4 inches and perpendicular 5 inches is what per cent of the area of a right triangle whose base is 8 inches and perpendicular 10 inches?

3. The circumference of a circle whose diameter is 14 feet is what per cent of the circumference of a circle whose diameter is 28 feet?

4. Land was sold for \$8,000, which was a loss of 20%. What was the cost?

5. Land was sold for \$12,000, which was a gain of 20%. What was the cost?

6. A house that cost \$6,400 was sold at an advance of $37\frac{1}{2}\%$. What was received for the house?

7. A house that cost \$5,400 was sold at a loss of $16\frac{2}{3}\%$. What was received for the house?

8. On goods that were sold for \$600, 20% was made. If they had been sold for \$550, what would have been the gain or loss per cent?

9. Goods damaged by fire were sold for \$900, at a loss of 25%. What would have been the gain or loss had they been sold for \$1,000?

10. Goods marked at \$1,500 were sold for 20% less than the marked price. For what were they sold?

11. Goods marked at \$800 were sold for 25% less than the marked price. For what were they sold?

12. A class was given 200 words to spell. Minnie misspelled 15, Clara misspelled 19, Richard misspelled 23. What per cent did each spell correctly?

13. What number increased by 25 % of itself equals 80?

14. What number decreased by 25 % of itself equals 120?

15. A woman spent 40 % of her money and had \$80 left. How much money had she at first?

16. What number increased by $62\frac{1}{2}$ % of itself equals 5746.39?

17. What number decreased by $37\frac{1}{2}$ % of itself equals 875.15?

18. Mr. Dayton bought $4\frac{1}{2}$ tons of hay for \$18 $\frac{1}{2}$ per ton, and sold it for \$95.50. What per cent did he make, approximately?

19. A retail dealer bought 2 bunches of bananas, containing 16 dozen, for \$1.20. If $\frac{1}{4}$ of them are spoiled, and he sells the remainder at 16 cts. a dozen, what per cent will he gain?

20. Mr. Martin bought a pile of wood 24 feet long, 6 feet high, and 4 feet wide for \$25. He sold it for \$6.50 per cord. What per cent did he make?

21. Mr. Dunlap sold Mr. Smith a horse for \$322, which was 8 % less than its cost to him. Mr. Smith sold it for \$375. What per cent more than the cost to Mr. Dunlap did Mr. Smith receive?

22. A house cost \$3500. It rents for 18 % of its cost. At what price does it rent?

23. In a school of 1580 pupils, 45 % of them are boys. How many girls are there in the school? What per cent of the pupils are girls?

24. If a man works 8 hours a day and sleeps 8 hours, what per cent of the day has he for other purposes?

25. Four boys ran an 880-yard relay race, running equal distances. What per cent of a mile did they run?

26. What per cent of a mile did each boy run?

27. John, William, Henry, and David ran a 440-yard junior relay race. What per cent of a mile did they run? What per cent of a mile did each boy run?

28. In the season of 1905 the New York National Base Ball Team won the pennant, winning 105 games and losing 48 games. What per cent of the games did the team win?

29. In the season of 1906 the Chicago National Base Ball Team won the pennant, winning 116 games and losing 36 games. What per cent of the games did they win?

30. Mr. DeMott sold to the railroad enough ties to lay a mile of track, the ties being 2 feet apart and averaging 9 inches wide. He paid for them 20 cts. apiece standing, \$5 a hundred for cutting, and \$10 a hundred for carting. He sold them at a profit of 25 %. How much did he receive for them.

31. Mr. Rathburn built a house on a lot 35 feet wide, for which he paid \$60 a front foot. The mason's contract was \$2,650, the carpenter's 85 % more than the mason's, the plumber's 50 % of the mason's, and the painter's 35 % of the plumber's. He sold the house for 15 % more than the entire cost. How much did he receive for it?

32. The floor of a room 60 feet by 70 feet was laid with maple boards. 25 % was allowed for waste. The boards were bought for \$42.50 per thousand feet, with 5 % off for cash. Two carpenters were 12 days laying the floor. They received \$3.80 a day each. Allowing \$3.75 for the nails used, what did the floor cost?

33. Two newsboys, Jim and Joe, invested \$3.60 in three different evening papers, paying an equal amount for each. They bought one paper at the rate of three copies for 5 cts. and sold them for 2 cts. apiece; they bought a second paper at the rate of five for 3 cts.; and the third at the rate of five for 4 ct. The last two papers they sold for 1 ct. each. What per cent did they make on their investment?

34. Find the percentage of each of the seven players in the National Base Ball League having the highest averages for the year, 1906, as follows (add "assisted" to "put out"):

	PUT OUT	ASSISTED	ERRORS	TOTAL CHANCES
First Baseman, McGann, New York . .	1391	83	8	1482
Second Baseman, Ritchie, Pittsburg . . .	326	439	27	792
Third Baseman, Arndt, St. Louis	108	139	9	256
Short Stop, Tucker, Chicago	288	472	45	805
Outfielder, Gessler, Chicago	27	4	0	31
Catcher, Bowerman, New York .	300	80	6	386
Pitcher, Raulbach, Chicago . . .	17	74	3	94

35. A house that cost \$6,000 rents for \$45 per month. The tax rate for 1906 was \$1.25 per hundred dollars, the repairs cost $1\frac{1}{2}\%$, the insurance was \$12, and the water tax \$18. What per cent did the owner receive for the money invested in the house?

TRADE DISCOUNT

159. It is customary for wholesale merchants to issue catalogues containing lists of goods with prices. Frequently, for business reasons, they sell goods for less than the list price. To avoid changing the catalogue when prices are lowered, the merchants give their customers what is called **trade discount**; that is, a certain per cent of the list price is deducted or "taken off" the bill. For example, cloth listed at 60 cents a yard might be sold "10 off," which would mean that 10 % was to be taken from the list price of 60 cents.

What would be the selling price of goods listed at 60 cents per yard if sold 10 % off?

Sometimes there are two or more discounts. Then the price might be 60 cents, "5 and 10 off," which would mean that 5 % was to be taken from the 60 cents, and then 10 % of the remainder taken off.

What would be the real selling price in that case?

Is this more or less than "15 off"? How much?

WRITTEN EXERCISES

160. Solve:

1. Mr. Bannister, who has a wholesale hardware store, sold Mr. Atwood a bill of articles amounting to \$750 and allowed him 20 % and 10 % off. What did Mr. Atwood pay for his articles?

2. Mr. Leland bought different articles from Mr. Atwood to the amount of \$450. He was allowed 18 % and 12 % off. What did Mr. Leland pay?

3. Mr. Van Nest bought 25 doz. brass hooks at \$2.50 a dozen. He was allowed 15 % and 5 % off. What did he pay for the hooks?

4. How much less would they have cost him if he had been allowed 20 % discount?

5. Mr. Bigelow bought a bill of goods amounting to \$3,000. He was allowed 15 %, 10 %, and 5 % off. What did the goods cost him?

6. For how much less could he have bought them at a discount of 30 %?

7. The trade discounts on a piano listed at \$800 were 20 %, 15 %, and 10 % off. It was sold at a discount of 45 %. For how much less than the trade discount was it sold?

8. A wholesale silk dealer offered goods listed at \$4,000 for 20 % and $12\frac{1}{2}$ % off for cash, or a discount of 30 % and a note for 30 da. without interest. Which was the better offer, money being worth 6 % interest?

9. Mr. Pierson bought an automobile listed at \$4,000, for 20 %, 10 %, and 5 % off. What did he pay for it?

10. A plumber bought supplies amounting to \$480, at 35 % discount and 5 % for cash. What was the net price?

11. A cotton manufacturer, on account of a fall in the market, marked his cloth down 10 % from the list price, and an additional discount of 5 % for cash. What per cent of the list price did he receive for cash sales?

12. What single discount is equivalent to two successive discounts of 20 % and 10 %?

13. What single discount is equivalent to two successive discounts of 10 % and 5 %?

14. A dealer in wood and willow ware buys his goods at an average of 10 % and $5\frac{1}{2}$ % off from list prices, and sells them at list prices. What per cent does he make on his sales?

15. A hardware dealer offered a bill of goods to a customer for \$1200 with a discount of $\frac{1}{4}$ off, or a discount of 15 %, 10 % and 5 %. The customer accepted the first proposition. Was it the better one?

COMMISSION

161. The percentage, or compensation, allowed an agent for transacting business is called a **commission**.

Mr. Sweet sold for Mr. Howard merchandise amounting to \$1560 and received $2\frac{1}{2}\%$ for transacting the business. What was his commission? What did Mr. Howard receive for his merchandise?

SOLUTION: \$1560

$$\begin{array}{r} .025 \\ \hline 7800 \\ 3120 \\ \hline \$39.000 \end{array} \quad \begin{array}{l} \$1560 - \$39 = \$1521, \text{ Mr. Howard received.} \\ \\ \\ = \text{commission.} \end{array}$$

ORAL EXERCISES

162. Give the commission on each of the following :

AMOUNT OF SALES	RATE OF COMMISSION
1. \$ 800	$2\frac{1}{2}\%$
2. 1,200	$2\frac{1}{2}\%$
3. 1,600	$2\frac{1}{4}\%$
4. 9,000	5 %
5. 6,000	5 %
6. 4,000	$2\frac{1}{2}\%$
7. 2,000	$2\frac{1}{4}\%$

WRITTEN EXERCISES

163. Solve :

1. Of a tract of land containing 150 acres, 45 acres were sold at \$75.; 24 acres at \$96.50; 38 acres at \$136; and the remainder at \$150. The agent received 5% on what was sold for less than \$100 per acre, and $3\frac{1}{2}\%$ on what was sold for more than \$100 per acre. What was his commission? What did the owner receive?

2. Mr. Morris sent to his agent, A. J. Holland, 1,450 baskets of peaches, 425 barrels of potatoes, 364 barrels of apples, and 2,000 pounds of cabbage. The peaches were sold for \$1.25 per basket, the potatoes for \$2.75 a barrel, the apples for \$3.45 per barrel, and the cabbage at 3¢ per pound. What was Mr. Holland's commission at 10%?

3. If Mr. Morris paid for freight 15¢ on each basket of peaches, 25¢ on each barrel of potatoes and apples, and $\frac{3}{8}$ ¢ on each pound of cabbage, how much did he receive for his crop?

4. Jan. 1. Mr. Holbrook sent his agent \$2,000 to purchase flour. The agent bought 235 barrels at \$5.40. His commission was 5%. What was the amount of the commission?

What amount remained to the credit of Mr. Holbrook?

SOLUTION: $\$5.40 \times 235 = \$1269$ cost of the flour.
 $\$1269 \times .05 = \underline{63.45}$ commission.
 $\$1332.45$ cost, including commission.
 $\$2000 - \$1332.45 = \$667.55$ amount to the credit of Mr. Holbrook.

5. Jan. 10. The agent purchased for Mr. Holbrook 80 tubs of butter, averaging 56 pounds a tub, for 28¢ a pound. What was his commission? How much money should Mr. Holbrook send his agent to pay him in full?

6. Jan. 20. Mr. Holbrook sent his agent \$1,000 to purchase tea and coffee. The agent bought 25 chests of tea, averaging 37 pounds a chest, for 43¢ a pound, and 2,160 pounds of coffee at $16\frac{3}{8}$ ¢ a pound. What was his commission? What remained to the credit of Mr. Holbrook?

7. Jan. 31. The agent purchased for Mr. Holbrook 15 barrels of sugar, averaging 300 pounds a barrel, at $6\frac{1}{4}$ ¢ a pound. What was the commission? What amount should Mr. Holbrook send his agent to close the January account?

TAXES

TERMS DEFINED

164. During the year 1906, a certain large city of this country spent \$4,182,456.85 in maintaining the city government, the public schools, streets, police force, and other institutions for the welfare of its citizens. The money all came from **taxes** paid to the city by people who lived in it or owned property there.

A small part of this tax was a **poll tax** (poll means head), a fixed amount paid by every man over twenty-one years of age. The balance was paid by owners of property at a fixed per cent of the value of the property.

Officials known as **assessors** placed a value upon all the property owned in the city. Then the amount of money required to pay the expenses of the government was estimated. From this the amount that each property owner was required to pay for every dollar of property owned by him was determined.

For convenience, property is divided into two classes, called **real** and **personal**. Real property or real estate consists of buildings and lands, property that is fixed in place. Personal property is movable property, as money, stocks, bonds, cattle, horses, and carriages.

Taxes levied as above stated are called **direct taxes**. Town, county, and state governments are supported by money received from direct taxes.

In addition to the above sources of income, many cities receive money from **licenses**. These licenses give permission to conduct certain kinds of business within the city, and are paid for at a fixed rate. Some of the kinds of business that pay licenses are peddlers, push carts, theaters, and liquor saloons. In this city money from licenses is not estimated.

In addition to the taxes for supporting the city government, a county tax is levied for paying the cost of the county government and other expenses, of which a city situated in the county must pay its share. Most of the courts of justice, as well as the office of the sheriff, belong to the county.

165. A tax is a sum of money assessed on persons or property for public purposes.

A **property tax** is a tax assessed upon property.

A **poll tax** is a sum of money levied on male citizens more than 21 years of age.

The **assessed value** of property is taken as a basis of taxation.

An **assessor** is the officer who appraises the property.

166. The assessments for the maintenance of the several departments of the city, for 1906, were as follows:

Assessing and Collecting Taxes	\$55,000
Poor and Alms	49,000
Incidental Expenses for Public Buildings	25,000
Hospitals	136,000
Insurance	25,000
Interest on Temporary Loans	15,000
Public Schools	905,271.37
Streets and Sidewalks	390,000
Lighting Streets	320,000
Street and Water Department	49,000
Sewers	40,000
City Home	37,500
Fire Department	488,500
Public Library	54,423
Public Health Department	475,000
Police Department	520,000
Shade Trees	18,500
* Sinking Funds and Interest	536,450.91

* **NOTE.** A sinking fund is a fund set aside each year for the payment of certain of the city's debts that fall due in the future.

Sundry other expenses	<u>\$42,811.57</u>
Total amount needed by the city for carrying on the government in all its branches, to be raised by taxation in 1906, was	\$4,182,456.85
From this amount was deducted 54,000 polls, at \$1	<u>54,000</u>
Leaving to be raised by property tax	\$4,128,456.85
The county tax was	<u>1,256,422.47</u>
The total tax was	\$5,384,879.32
The real estate of the city listed for taxation was	239,801,926.00
The personal property	<u>45,101,304.00</u>
Total "ratables"	\$284,903,230.00

The amount to be raised, $\$5,384,879.32 \div 284,903,230$, the total ratables, = .0189, the tax rate on \$1, which equals \$1.89 on \$100, or \$18.90 on \$1,000.

Tax rates are often stated as so many points, thus .0189 on \$1 = \$1.89 on \$100 = 189 points.

ORAL EXERCISES

167. What is the rate on \$1, on \$100, and on \$1000, from the following numbers of points:

185	190	115	214	226
201	222	216	122	135
126	302	120	167	250

WRITTEN EXERCISES

168. Solve:

1. What was the tax, for 1906, of Mr. Ballard, whose property was in the above-mentioned city, and was assessed for \$47,500? He also paid a poll tax.

2. Mr. W's real estate was assessed for \$250,800 and his personal property \$75,000. What was his tax on each? What was his total tax?

3. Mr. Howard's store was assessed for \$60,500 and its contents for \$45,000. What was his tax?

4. The taxable property in a town is \$596,400. The rate of taxation is \$.015. What is the amount to be collected? How much is the collector's commission at $2\frac{1}{2}\%$?

NOTE. In some localities discounts are allowed for prompt payment of taxes, and penalties are exacted of delinquents, if taxes are not paid by a certain date, as from Dec. 21 at the rate of 12%.

The rate in another city was 2.26 %.

The following discounts for the year 1905 were allowed:

From Oct. 20 to Oct. 31 inclusive . . . 1%

From Nov. 1 to Nov. 20 inclusive . . . $\frac{1}{2}$ of 1%.

From Nov. 21 to Dec. 20 inclusive . . . exact amount of tax.

5. Mr. Halsey's property was assessed for \$47,500. He paid his tax Oct. 28. How much did he pay?

6. Mr. Humphrey's real estate was assessed for \$145,000, and his personal property for \$65,000. He paid his tax Nov. 10. How much discount did he receive? How much tax did he pay?

7. Mr. Smith paid his tax Dec. 12, 1905. His property was assessed for \$6800. How much was his tax?

8. Mr. Hammond's property is valued at \$18,000 and is assessed for $66\frac{2}{3}\%$ of its face value. He paid his tax March 1, 1906. What was the amount of his tax?

9. The real estate in a certain county is assessed for \$116,000,000; the personal property for \$64,000,000; and there are 28,250 polls assessed at \$1.50 each. The tax to be raised is \$3,912,375. What is the rate?

10. What is Mr. Randall's tax, if his property is assessed for \$36,875, and he is exempt from poll tax?

11. Mr. Howard's store is assessed for \$60,500 and its contents for \$85,260. He pays a poll tax. What is the amount of his tax?

12. Mr. Hill has three houses, assessed at \$8,275, \$14,207, and \$9,840 respectively. What is his tax on the houses?

INDIRECT TAXES

169. Every year the United States government makes large appropriations of money to be expended for the support of the army, the navy, and for other purposes. This money is received from taxes, but these are of a different kind from the taxes paid for the maintenance of the city government.

City taxes are paid upon polls and upon the value of property located in the city. The taxes for the support of the United States government are paid upon the value of certain articles manufactured in the United States, and upon certain articles bought in other countries and brought here. They are called **indirect taxes**.

Indirect taxes are of two classes, **internal revenue taxes** and **duties**.

Internal revenue taxes are paid chiefly on the value of liquors and tobacco manufactured in the country.

Duties are taxes paid on the value of goods brought into the country.

Duties are paid upon certain goods only.

A list of the goods with the amount of tax each is to pay is called a **tariff**.

WRITTEN EXERCISES

170. Solve :

1. During the fiscal year ended June 30, 1905, the aggregate collections of the Internal Revenue Bureau were \$234,187,976.37. It cost \$4,705,296.32 to collect it. What per cent of the amount received was the cost of collection?

2. The aggregate collections of duties during the same period were \$261,798,856.91, and the cost of collection was \$9,115,499.91. What per cent of the amount received was the cost of collection?

INTERNAL REVENUE

171. A **proof gallon** of alcoholic liquors is a gallon containing half alcohol and half water. The distilled spirits of commerce are usually 90 % proof.

WRITTEN EXERCISES

172. Solve :

1. The tax on proof spirits is \$1.10 per gallon. What is the tax on distilled spirits of commerce per gallon?

2. Estimating 50 drinks per gallon, what is the approximate governmental tax per drink?

3. Approximately 18,630,100 drinks of distilled spirits are taken in the United States each day. At 2¢ a drink, what is the revenue for a day? For a year?

4. The tax on fermented liquors, as beer, wine, ale, etc., is \$1 a barrel of $31\frac{1}{2}$ gallons. It is estimated that a barrel of beer yields 400 glasses. How much tax does the government receive from each glass of beer?

5. If the government receives from the tax on beer a daily revenue of \$137,000, how many glasses of beer are drunk each day in the United States?

6. The tax on a cigar is $\frac{3}{10}$ ¢. If the smokers of the United States consume 19,158,333 cigars each day, what is the daily revenue of the government from this source?

7. The tax on cigarettes is \$1.08 per thousand. If the average daily income from these is \$9,180, what is the number of cigarettes consumed?

8. The tax on chewing and smoking tobacco is 6¢ per pound. During the fiscal year of 1905 the consumers of chewing and smoking tobacco paid a government tax at the rate of \$54,984 per day. How many pounds of chewing and smoking tobacco were used each day in the United States?

DUTIES

173. Duties are of two kinds, **specific** and **ad valorem**.

A tax levied on goods without reference to their value is called a specific duty.

Specific duties are usually levied at a fixed sum on each pound, or yard, or other measure of quantity.

A tax levied upon goods at the cost at which they are invoiced in the country from which they were imported is called an **ad valorem** duty.

Sometimes both specific and **ad valorem** duties are levied on the same article.

WRITTEN EXERCISES

174. Solve:

1. What is the duty on 80 yards of silk invoiced at \$2.25 per yard, duty 60 %?

2. At what price per yard must the importer sell it to make $12\frac{1}{2}\%$?

3. What is the duty on 15 rolls of silk, 45 yards in a roll, invoiced at \$1.75 per yard, duty 60 %?

4. At what price per yard must the importer sell it to gain 20 %?

5. The cost of a rug in Japan was \$450. The duty was 40 %. For how much must the importer sell it to make $16\frac{2}{3}\%$?

6. A merchant imported 850 yards of flannel invoiced at $62\frac{1}{2}\phi$ per yard, duty 35 %. What did it cost him?

7. At 20 % what is the duty on 25 bales of wool, 420 pounds each, invoiced at $6\frac{3}{4}\phi$ per pound?

8. A merchant imported 800 yards of velvet, invoiced at $87\frac{1}{2}\phi$ per yard, duty $12\frac{1}{2}\phi$ per yard. He sold it for \$1.37 $\frac{1}{2}$ per yard. What per cent did he make?

9. If the specific duty on a dozen pairs of gloves is \$2.50, how much is the duty on 720 pairs?

10. What is the duty on 40 pieces of carpet, 25 yards each, invoiced at \$1.50 per yard, the specific duty being $12\frac{1}{2}\%$ per yard and the ad valorem duty 40%?

11. A merchant imported 70 rolls of woolen cloth, each containing 30 yards, invoiced at 95¢ per yard, duty 50%. He sold the cloth for \$1.75 per yard. How much did he make?

12. An importer bought cut glass in Germany, invoiced at \$45,000, when the duty was 35% ad valorem. He sold one-half of it at a profit of 25%. After his purchase the tariff was raised to 60%. He sold the other half at a profit of 20% on the invoice price, plus the new duty. How much more was his gain on the last half of his stock than on the first half?

13. A cigar dealer imported 950 pounds of tobacco, invoiced at \$4.50 per pound. The ad valorem duty was 25% and the specific duty \$2.75 a pound. How much was the duty?

14. A lady brought from Europe for her own use, 25 yards of lace worth \$1.25 per yard, upon which the duty was 50% ad valorem; 6 pairs of kid gloves upon which the duty was \$2.25 per dozen pairs; a dress pattern upon which the duty was \$27.50; and 15 yards of linen at 80¢ per yard, ad valorem duty 45%. How much more would the goods have cost her had she paid the duty?

15. Mrs. Lane brought from Germany 23 yards of lace invoiced at \$2.35 a yard, and 12 yards of cloth invoiced at \$1.90 a yard. The duty on the lace was 70% and that on the cloth 50%. What was the duty on each? What was the entire cost of both?

NOTE. Persons are allowed to bring from abroad a limited amount of goods for their own use, without paying duty upon them.

INTEREST

TERMS DEFINED

175. Interest is a sum paid for the use of money. It is reckoned as a percentage of the amount loaned for a specified time.

The terms used are principal, rate, interest, time, and amount.

The **principal** is the base upon which the interest is reckoned.

The **rate** is the stated number of hundredths of the principal.

The **interest** is the percentage of the principal for the stated time at the given rate.

The **time** is the period for which interest is reckoned.

The **amount** is the sum of the principal and interest.

METHODS OF COMPUTING INTEREST

176. There are various methods of computing interest. Some are better for short time loans, others for long time loans. Banks commonly use tables in which the interest on certain amounts for different periods is stated (see p. 134).

177. THE DIRECT OR ALIQUOT PART METHOD

This is especially simple when the time includes a number of months that are simple fractions of 1 year.

Find the interest on \$ 186 for 3 yr. 3 mo. at 5 %.

SOLUTION: \$ 186

$$\begin{array}{rcl}
 & .05 & \\
 \hline
 & \$ 9.30 & \times 3 = \$ 27.90 \quad \text{interest for 3 yr.} \\
 4) \$ 9.30 & = & \underline{2.325} \quad \text{interest for 3 mo.} \\
 & & \$ 30.225
 \end{array}$$

WRITTEN EXERCISE

178. Find the interest on:

1. \$285.50 at 5% for 2 yr. 6 mo.
2. \$378 at $4\frac{1}{2}\%$ for 4 yr. 4 mo.
3. \$720.25 at 7% for 3 yr. 3 mo.
4. \$187 at $3\frac{1}{2}\%$ for 5 yr. 2 mo.
5. \$865.45 at $6\frac{1}{2}\%$ for 1 yr. 9 mo.
6. \$236 at 6% for 6 yr. 1 mo.
7. \$240 at 5% for 8 yr. 8 mo.
8. \$350 at 5% for 4 yr. 6 mo.
9. \$975 at 4% for 5 yr. 8 mo.

179.

SIX PER CENT METHOD

This method is used largely in computing interest when the time includes time units that are not easily found fractions of larger units, as: 5 mo. 17 da.

Interest is computed upon a year of 12 months of 30 days each, using the following formula:

$$\begin{aligned} \text{Interest on \$1 for yr.} &= \$.06 \\ \text{Interest on \$1 for 1 mo.} &= \frac{1}{12} \text{ of } \$.06 = \$.005 \\ \text{Interest on \$1 for 1 da.} &= \frac{1}{30} \text{ of } \$.005 = \$.000\frac{1}{6} \end{aligned}$$

180. Solve:

1. Find the interest on \$250 for 1 yr. 5 mo. 25 da. at 6%.

WRITTEN EXERCISES

SUGGESTION

$$\begin{aligned} \text{Interest on \$1 for 1 yr.} &= \$.06 && \$.06 \\ \text{Interest on \$1 for 1 mo.} &= \frac{.06}{12} = \$.005 \\ \text{Interest on \$1 for 5 mo.} &= 5 \times \$.005 = \$.025 \\ \text{Interest on \$1 for 25 da.} &= 25 \times \$.000\frac{1}{6} = \$.004\frac{1}{6} \\ &&& \$.089\frac{1}{6} \\ \$250 \times .089\frac{1}{6} &= ? \end{aligned}$$

2. Mr. Brown borrowed \$450 of Mr. Avery, June 15, 1909. He gave his note on demand with interest at 6%. He paid the note Feb. 21, 1910. How much did he pay?

NOTE. Interest at 3% = $\frac{1}{2}$ interest at 6%.

Interest at 2% = $\frac{1}{3}$ interest at 6%.

Interest at 4% = $\frac{2}{3}$ interest at 6%.

Interest at 5% = $\frac{5}{6}$ interest at 6%.

Interest at 7% = $\frac{7}{6}$ interest at 6%.

3. Find the interest on :

PRINCIPAL	TIME	RATE
\$1,725.00	2 yr. 7 mo. 16 da.	3 %
1,650.00	5 yr. 3 mo. 10 da.	2 %
763.00	1 yr. 10 mo. 11 da.	4 %
927.50	8 mo. 17 da.	5 %
362.	1 yr. 8 mo.	6 %
215.45	7 mo.	4 %

SIXTY DAY METHOD

181. This is frequently used by banks.

182. Find the interest on \$875, at 6%, for 85 da.

EXPLANATION. 60 da. = 2 mo. = $\frac{1}{2}$ yr.

At 6% for 1 yr., the interest on any principal for 60 da. = $\frac{1}{2}$ of 6% = 1% = $\frac{1}{100}$ of itself.

85 da. = 60 da. + 20 da. + 5 da. Since any principal, at 6% draws $\frac{1}{100}$ of itself, in 60 da., interest on \$875 for 60 da. = \$8.75.

Since 20 da. = $\frac{1}{3}$ of 60 da., we find $\frac{1}{3}$ of \$8.75 = \$2.916 +, interest for 20 da.

5 da. = $\frac{1}{4}$ of 20 da. We find $\frac{1}{4}$ of \$2.916 = \$.729, interest for 5 da.

\$8.75 + \$2.916 + \$.729 = \$12.395.

SOLUTION: \$875 = principal.

3)8.75 = interest for 60 da.

4)2.916 = interest for 20 da.

.729 = interest for 5 da.

\$12.395

85 da.

ORAL EXERCISES

183. Find the interest at 6% on the following:

- | | |
|-----------------------|----------------------|
| 1. \$148 for 60 da. | 7. \$240 for 10 da. |
| 2. \$52.50 for 60 da. | 8. \$240 for 70 da. |
| 3. \$720 for 30 da. | 9. \$900 for 60 da. |
| 4. \$640 for 15 da. | 10. \$900 for 20 da. |
| 5. \$820 for 75 da. | 11. \$300 for 80 da. |
| 6. \$480 for 90 da. | 12. \$900 for 50 da. |

WRITTEN EXERCISES

184. Find the interest on the following:

1. \$270.60 for 75 da. at 6%.
2. \$280.40 for 45 da. at 6%.
3. \$250.60 for 80 da. at 6%.
4. \$175.75 for 95 da. at 6%.
5. \$860.50 for 100 da. at 6%.
6. \$420.40 for 75 da. at 7%.
7. \$560.80 for 45 da. at 5%.
8. \$360.20 for 80 da. at 4%.
9. \$180.16 for 95 da. at 5%.
10. \$300.40 for 100 da. at $3\frac{1}{2}\%$.
11. \$459.30 for 75 da. at 4%.
12. \$656.90 for 80 da. at 6%.
13. \$200.60 for 90 da. at 5%.
14. A man borrowed \$375 on May 1, 1906. He repaid it in 90 days with interest at 5%. How much did he have to pay?
15. On Feb. 2, 1905, Mr. Floyd borrowed \$670. He repaid it May 3, 1905, with interest at 6%. What amount did he have to pay?

INTEREST BY CANCELLATION

185. What is the interest on \$340.25, at 4%, for 99 da.?

The interest on \$340.25, at 4%, for 1 yr., equals \$340.25 $\times \frac{4}{100}$. The interest for 99 da. equals $\frac{99}{360}$ of the interest for 1 yr. $\$340.25 \times \frac{4}{100} \times \frac{99}{360} = \text{Ans.}$ All factors common to both numerator and denominator are cancelled.

SOLUTION:

$$\begin{array}{r} .34025 \\ \cancel{340.25} \times \frac{\cancel{4}}{100} \times \frac{99}{\cancel{360}} = \$3.74275 \end{array}$$

186. Find by cancellation the interest on the following:

1. \$840 at 5% for 75 da.
2. \$360.50 at 4% for 80 da.
3. \$240 at 5% for 90 da.
4. \$200 at $3\frac{1}{2}\%$ for 48 da.
5. \$80.84 at 4% for 18 da.
6. \$960.40 at $4\frac{1}{2}\%$ for 52 da.
7. \$176.58 at 3% for 66 da.
8. \$300 at $5\frac{1}{2}\%$ for 36 da.
9. \$980 at 4% for 72 da.
10. \$19.50 at $4\frac{1}{2}\%$ for 24 da.

REVIEW

187. Solve:

1. The motor cars in the New York-Paris race left New York on February 12, 1908. On March 12 the American car, in the lead, had reached Bitter Creek, Colorado, having traveled 2,321 miles. What was the average number of miles a day?

2. The trip to San Francisco occupied 42 days. The distance from New York by the route followed is 4,836 miles. What per cent of the distance did the American car travel in the first month? In an average day?

3. The length of the Dnieper River is 1,250 miles; the Danube is 40% longer. How long is the Danube?

4. The length of the Elbe is 800 miles; the Rhine is $12\frac{1}{2}\%$ shorter. How long is the Rhine?

5. The length of the Orinoco is 1600 miles; the St. Lawrence is $31\frac{1}{4}\%$ longer. What is the length of the St. Lawrence?

6. A gas bill reads as follows:

STATE OF METERS

May 18, 1907,	16,000
April 18, 1907,	<u>15,600</u>
Consumption	400 cu. ft., at \$1.10 per 1,000 ft.

Discount of 10 ct. per 1,000 cu. ft., if bill is paid 5 days after presentation. Bill presented June 3. Find the amount of the bill and the discount.

7. John Jacob Astor left a bequest of \$400,000 to found the Astor Library, which was opened in the winter of 1853. William B. Astor, his son, gave during his lifetime a sum equal to 75% of his father's bequest, and left a bequest equal to $62\frac{1}{2}\%$ of his father's bequest. How much did William B. Astor give to the library? What was the whole amount contributed by father and son?

8. John Jacob Astor, 2d, gave to the library during his lifetime \$250,000 and left a bequest equal to $37\frac{1}{2}\%$ of all previous contributions. How much did he give to the library?

9. William Astor gave a sum equal to $12\frac{1}{2}\%$ of the bequest of John Jacob Astor, 1st. How much did he give to the library? What was the total amount contributed?

10. The population of Newark, N.J., was 240,000 in 1890; in 1900 it was 275,000. What per cent was the increase in population?

11. A watch gained 3 min. 45 sec. from 9.30 A.M. to 3 o'clock P.M. What per cent of the time did it gain?

12. Mr. Rutan sold two watches for \$31.50 each. On one of them he lost $12\frac{1}{2}\%$, and on the other he gained $12\frac{1}{2}\%$. Did he lose or gain on the transaction?

13. In the 4-mile cross country run at Columbia University each class was represented by four runners. The class whose runners had the lowest average score won. How much less was their time than that of each of the other classes?

Seniors

B. Sanders	.	.	.	.	.	.	22 min. 45 sec.
F. W. Kennedy	.	.	.	.	.	.	23 min. 20 sec.
F. Kudlich	.	.	.	.	.	.	23 min. 25 sec.
M. Smith	.	.	.	.	.	.	24 min. 11 sec.

Juniors

J. W. Wheeler	.	.	.	.	.	.	23 min. 26 sec.
A. P. Montgomery	.	.	.	.	.	.	23 min. 54 sec.
H. N. Snevly	.	.	.	.	.	.	23 min. 55 sec.
F. Ware	.	.	.	.	.	.	25 min. 15 sec.

Sophomores

F. L. Hopkins	.	.	.	.	.	.	24 min. 44 sec.
G. Hoynes	.	.	.	.	.	.	24 min. 50 sec.
G. E. Crunsky	.	.	.	.	.	.	25 min. 05 sec.
F. S. Hetherington	.	.	.	.	.	.	23 min. 0 sec.

Freshmen

A. H. Meyer	.	.	.	.	.	.	25 min. 0 sec.
R. H. Seymour	.	.	.	.	.	.	25 min. 16 sec.
A. G. Marsh	.	.	.	.	.	.	26 min. 17 sec.
L. B. West	.	.	.	.	.	.	26 min. 55 $\frac{1}{2}$ sec.

14. Mr Grant sold 2 acres of land for \$720 each. On one of them he gained 20% and on the other he lost 20%. What did the land cost him?

15. Find the difference between $2\frac{1}{2}\%$ of \$460.50 and $3\frac{1}{4}\%$ of \$380.40.

16. Mr. Kendall pays \$25 a year for \$1,000 life insurance. What rate per cent does he pay?

17. A house valued at \$7,200 is assessed for \$5,400. What per cent of its value is the assessment? If the tax rate is \$2.32 on \$100, how much is the tax?

18. A farm valued at \$16,000 is assessed for 60 % of its value. What is the tax on the farm at the rate of \$2.14 for \$1,000?

19. Bennie bought 60 newspapers at the rate of 3 for 5 cents and sold them for 3 cents apiece. How much did he make? What per cent?

20. If Lena had taken 20 % more stitches she would have taken 720 stitches. How many stitches did she take?

21. On 4 days of the week Thomas has spelling lessons of 20 words each. On Friday all the words of the week are reviewed. If he misses an average of 5 words on Fridays, what is the average per cent of words spelled correctly? Of words missed?

SUMMARY OF CHAPTER IV

PERCENTAGE.

Definitions and applications.

To find Percentage, Amount or Difference, Base, Rate.

APPLICATIONS OF PERCENTAGE.

TRADE DISCOUNT.

COMMISSION.

TAXES.

Direct.

Indirect.

INTEREST.

Definitions.

Most-used Methods.

Direct or Aliquot.

Sixty Day.

Six Per Cent.

Cancellation.

CHAPTER V

BANKING AND BUSINESS PRACTICE

METHODS OF DEPOSITING MONEY

188. **Banks** are institutions that deal in money and take care of other people's money. Most people do not keep much money on their persons or in their houses or places of business. They put it (or **deposit** it) in a **bank**. Money so intrusted to a bank is called a **deposit**.

Business men generally deposit every day the greater part of their cash funds. Commonly depositors use a slip like this, which the bank keeps to show just how much has been deposited.

The bank clerk also enters the amount in a **bank book**, belonging to the **depositor**.

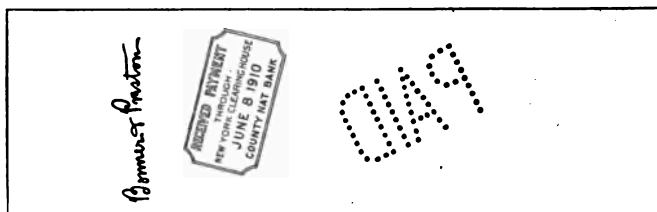
189. When depositors make payments to others, instead of paying out money they use checks. A **check** is an order on a bank to pay to some one an amount of money specified. The bank **honors** the check; that is, it pays the specified amount to the person named, after making sure that the one whose name is signed really signed it, and that he has money enough on deposit to pay it.

BY 10-25 2002X		
Deposited to the account of		
<u>Wm. Andrews</u>		
IN		
The Eighth National Bank		
New York, <u>Apr. 4</u> 19 <u>10</u>		
PLEASE ENTER EACH CHECK SEPARATELY		
	DOLLARS	CTS.
Bills	125	
Coin	3	17
Check <u>First Nat</u>	47	32
" <u>Borough</u>	13	46
" <u>Phila</u>	4	12
" <u>Chicago</u>	172	
"		
"	385	07
"		

Read this check and tell on what bank the order was drawn. Who made the check? To whose order was it made? For how much was it made?

No. <u>356</u>	New York, <u>May 7th</u> 1910
EIGHTH NATIONAL BANK	
Pay to the order of <u>Bonner & Preston</u>	
<u>Sixty five & 24/100</u>	Dollars
<u>\$65.24</u>	<u>Richard Smith</u>

If the check is made payable to a particular person, he must indorse it, that is, write his name on the back, before he can get the money.



It is not necessary for the man who receives a check to go to the bank on which it is drawn to get his money.

190. Richard Smith owes Bonner & Preston \$65.24. He has a deposit in the Eighth National Bank of New York. So he sends them a check for the amount of the bill. They indorse the check and deposit it, as if it were cash, in the Second National Bank, where they keep their money. The Second National Bank collects the amount, \$65.24, from the Eighth National Bank, for them, and places it to their credit in that bank.

These banks thus save both of these parties a great

amount of time and trouble. By such means banks provide a convenient and prompt method of collecting and paying money. To pay themselves for this, they loan on interest the money deposited with them.

CLEARING HOUSES

191. For the convenience of banks in collecting from one another money due on the checks of their depositors, there are in most cities "**clearing houses.**"

A **Clearing House** is a place where representatives from the different banks meet to exchange checks and to adjust balances.

At a certain time every day the different banks send to the clearing house all the checks on other banks which have been deposited during the day. These checks are compared, the amounts due between the various banks canceled so far as they are equal, and any balance due paid in money.

In the above example, the Second National Bank sends to the clearing house Mr. Smith's check drawn on the Eighth National Bank. The Eighth National Bank has a check drawn by Mr. Jones on the Second National Bank for \$78.40. So the messenger of the Second National Bank pays to the messenger of the Eighth National Bank, in money, the difference between the two checks, \$78.40 — \$65.24 or \$13.16. This is done through clearing house officials.

Each of the checks mentioned will be stamped on the back with the clearing house stamp which gives the date of its clearance.

192. All the checks due from each bank to all the others are taken by the officers of the clearing house, classified, and added. Then the **balances** are paid by those whose checks payable are less in amount than their checks receivable.

CERTIFICATE OF DEPOSIT

193. Banks receiving and caring for deposits do not pay interest on them, unless the depositors agree to leave the money in the bank for a considerable time. In this case the ordinary bank issues to the depositor a certificate of deposit.

Walter Emory TELLER	No 23056	New York April 3 rd 1916
	This Certifies that	
	<u>Fredrick Layton</u> has deposited in this Bank	
	<u>Four hundred</u> Dollars	
	payable <u>four</u> months after date to <u>his</u> order hereon on	
return of this certificate with interest at 2 per cent per annum		
Amount deposited \$ _____ months interest \$ _____ Due _____ 19____		
EIGHTH NATIONAL BANK		<u>Robert Goldward</u> CASHIER

Sometimes interest is paid on the average of deposits left in the bank for 60 days or more.

WRITTEN EXERCISES

194. Find the interest on the average amounts of the following deposits for the periods named, at $3\frac{1}{2}\%$:

1. \$135 from Feb. 2, 1907 to Feb. 10, 1908.
2. \$1,275 " Mar. 6, 1908 " July 16, 1908.
3. \$6,385 " Apr. 5, 1908 " May 7, 1909.
4. \$10,150 " July 16, 1908 " Sept. 20, 1908.

5. Find the interest at 4% on the average amounts of the following deposits for the periods named :

- | | |
|---------------------|---------|
| Sept. 1 to Nov. 15, | \$400. |
| Oct. 15 to Jan. 15, | \$375. |
| Nov. 1 to March 15, | \$1000. |
| Jan. 1 to March 15, | \$1200. |
| Feb. 1 to June 1, | \$1500. |

TRUST COMPANIES AND SAVINGS BANKS

195. There are two kinds of banking institutions that receive deposits not subject to check and pay interest on them. Such banks are called **Trust Companies** and **Savings Banks**.

Trust companies usually receive money in large amounts, such as entire estates, and care for it, charging for time and services, and paying interest on funds deposited.

Some trust companies also do a regular banking business and pay out money on checks of depositors. But they do not pay interest on deposits subject to check, unless a special arrangement is made.

Savings banks receive money in small amounts and pay compound interest.

COMPOUND INTEREST

196. Savings banks and some other institutions pay interest on the money deposited with them. At stated times they add to the principal the interest then due, and compute interest on the amount for the next period, thereby paying not only interest on the original principal, but interest on interest.

This is called **compound interest**.

197. Interest is computed from a certain date in each month, usually not later than the 5th. Money deposited before that date receives interest from the 1st. Money deposited after that date draws no interest until the 1st of the next month. In some institutions no interest is allowed unless the money has been on deposit 6 months if compounded semiannually, or 3 months if compounded quarterly.

Money withdrawn from a savings bank before the end of any six months' period does not draw interest for that period if interest is compounded semiannually. Money withdrawn is called a **draft**.

198. Henrietta's father deposited in her name \$600 in a savings bank. The bank pays its depositors 4 % interest, which is to be added to the principal semiannually. The money remained in the bank 3 years, when Henrietta withdrew it. How much money did she receive?

SOLUTION: \$600

First Principal.

.02

Rate for one half year.

\$12.00

Interest for 6 months.

600.00

\$612.00

Second Principal.

.02

Interest for second 6 months.

\$12.24

612.00

\$624.24

Third Principal.

.02

Interest for third 6 months.

\$12.4848

624.24

\$636.725

Fourth Principal.

.02

Interest for fourth 6 months.

\$12.73550

636.725

\$649.460

Fifth Principal.

.02

Interest for fifth 6 months.

\$12.98920

649.46

\$662.449

Sixth Principal.

.02

Interest for sixth 6 months.

\$13.24808

662.449

\$675.697

Amount received.

Find the interest for the first interest period. Add the interest thus found to the principal, and find the interest on the amount for the second interest period, and continue in this manner until the money is withdrawn.

WRITTEN EXERCISES

199. Solve:

1. What would have been the simple interest on the foregoing deposit? How much more was the compound interest?

2. Mr. Hugo has 3 sons. On the day when Edward was just 19 years 6 months old, Rudolph 18 years old, and Victor 16 years old, the father deposited \$4,000 to the credit of each son. Each was to receive his money when 21 years old. The bank in which it was deposited pays 4 % interest, adding interest semiannually. How much will each receive when he is 21 years old?

3. On his fifteenth birthday, Harold Chase deposited \$450 in a savings bank, which paid annual compound interest at 5 %. How much was due him when he was 21 years old? What would have been the amount at simple interest?

200. Most banks find interest by using tables already computed, giving the amounts on \$1, for different periods at different rates.

AMOUNT OF \$1, AT COMPOUND INTEREST, COMPOUNDED
ANNUALLY

YEARS	2%	2½%	3%	4%	5%
1	\$1.02	\$1.025	\$1.03	\$1.04	\$1.05
2	1.0404	1.05063	1.0609	1.0816	1.1025
3	1.06121	1.07689	1.09273	1.12486	1.15763
4	1.08243	1.10381	1.12551	1.16986	1.21551
5	1.10408	1.13141	1.15927	1.21665	1.27628

Find by the table the amount of \$320, at 4 %, for 3 yr.

The amount of \$1, for 3 yr, at 4 %, is \$1.12486.

$$\$1.12486 \times 320 = \$359.955.$$

WRITTEN EXERCISES

201. Find, by using the above table, the amounts of the following principals, compounded annually:

1. \$874, at 5 %, for 2 yr.

2. \$346, at 4 %, for 3 yr.

3. Mr. Glenny loaned Mr. Faulkner \$500, at 5 %, for 3 yr. 6 mo., the interest to be compounded annually. What was due when the time expired?

4. What is the amount of \$420, at 4 %, for 4 yr. 4 mo., interest compounded annually?

5. What is the interest on \$200, at 4 %, for 3 yr., compounded semiannually? Compounded annually?

202. Make five problems and compute the interest, using the following table:

SAVINGS BANK TABLE SHOWING INTEREST ON DIFFERENT AMOUNTS, ESTIMATED BY MONTHS, AT $3\frac{1}{2}\%$

	1 Mo.	2 Mo.	3 Mo.	4 Mo.	5 Mo.	6 Mo.
\$ 5	\$.01	\$.03	\$.04	\$.06	\$.07	\$.09
10	.03	.06	.09	.12	.15	.17
15	.04	.09	.13	.17	.22	.26
20	.06	.12	.17	.23	.29	.35
25	.07	.15	.22	.29	.36	.44
30	.09	.17	.26	.35	.44	.52
35	.10	.20	.31	.41	.51	.61
40	.12	.23	.35	.47	.58	.70
45	.13	.26	.39	.52	.66	.79
50	.15	.29	.44	.58	.73	.87
100	.29	.58	.87	1.17	1.46	1.75
150	.44	.87	1.31	1.75	2.19	2.62
200	.58	1.17	1.75	2.33	2.92	3.50
250	.73	1.46	2.19	2.92	3.65	4.37

203. Interest is computed and added to the principal, Nov. 1 and May 1, in the following savings bank accounts.

1907.

JOHN SMITH

	DRAFTS	DEPOSITS	INTEREST
Nov. 3		\$ 500	\$ 8.75
Dec. 3		50	.73
Jan. 3		100	1.17
Feb. 3		75	.66
Mar. 3		25	.15
Apr. 3		50	.15
			<u>\$ 11.61</u>

1907.

WM. R. LEE

	DRAFTS	DEPOSITS	INTEREST
Nov. 1		\$ 500	\$ 8.75
Nov. 28		50	.36
Dec. 24	\$ 25		
Feb. 8		100	.29
Feb. 15	50		
Mar. 3		700	1.75
Apr. 1	400		
Apr. 3		50	.15
			<u>\$ 11.30</u>

In the first account Mr. Smith made no drafts on the bank. Each deposit was made during the specified time and drew interest from its date to May 1. The amount of interest, \$ 11.61, is added to the sum of the deposits, \$ 800, and the amount, \$ 811.61, is the new principal from May 1. The same process is repeated for each succeeding six months. The rate is $3\frac{1}{2}\%$.

In the second account, Mr. Lee has made several drafts. In such an account, interest is computed on the smallest balance during the six months, which is \$500. The second deposit, \$50, was made November 28, therefore it would have drawn interest from December 1; but December 24, there is a draft of \$25, which is deducted from the last deposit, $\$50 - \$25 = \$25$, on which the interest is computed for five months. February 8, a deposit of \$100 was made, and February 15 a draft was made for \$50. $\$100 - \$50 = \$50$, which draws interest from March 1, two months. On March 3 \$700 was deposited and April 1, \$400 was withdrawn, $\$700 - \$400 = \$300$, which drew interest from March 1, two months. On April 3, \$50 was deposited, which drew interest one month. The amount of deposits, plus the interest, \$11.30, equals \$1,411.30, from which \$475, the amount of drafts, is deducted. $\$1,411.30 - \$475 = \$936.30$, balance in bank May 1. Rate $3\frac{1}{2}\%$.

WRITTEN EXERCISES

204. Solve:

1. Mr. R. S. Dunlap's account with The Citizens' Savings Bank, from May 1 to Nov. 1, 1907, was as follows:

THE CITIZENS' SAVINGS BANK

1907.

To R. S. DUNLAP, DR.

	DRAFTS	DEPOSITS	INTEREST
May 1		\$ 800.00	
June 4		75.00	
July 3		200.00	
Aug. 1		25.00	
Oct. 4		50.00	

What was due Mr. Dunlap Nov. 1? Consult the table.

2. Mr. C. H. Childs' account with the same bank, for the same period, was as follows :

THE CITIZENS' SAVINGS BANK
1907. To C. H. CHILDS, DR.

	DRAFTS	DEPOSITS	INTEREST
May 1		\$ 600.00	
May 20		150.00	
June 15	\$ 50.00		
July 7		100.00	
July 20	75.00		
Sept. 3		500.00	
Oct. 1	30.00		
Oct. 2		80.00	

What amount has Mr. Childs in bank Nov. 1, 1907?

3. The State Savings Bank had in 1908 deposits amounting approximately to \$16,500,000. It paid interest at the rate of $3\frac{1}{2}\%$ on an average amount of \$14,500,000 for the first half year and \$15,000,000 for the second half year. How much was paid out in interest?

4. The bank loaned an average amount of \$7,000,000 at 5%, and of \$7,000,000 at $4\frac{1}{2}\%$. What was the income from interest on loans? What was the excess of interest received over interest paid?

5. The expenses of the bank for rent, salaries, stationery, and other items, amounted to \$35,500. What was the total surplus for the year?

6. The trustees, in view of the surplus, decided to raise the interest paid on deposits for 1909 one half of 1%. On the basis of the business for 1908, will this leave a surplus above all expenditures? How much?

LOANS

PROMISSORY NOTES

205. Individuals and banks loan money on interest, taking from the borrower a **promissory note** like this:

\$400 ⁰⁰	Morristown N.J. Aug. 15 1907
Four months after date I promise to pay to	
the order of The National Morristown Bank	
Four hundred & 00/100	Dollars
at The National Morristown Bank	
Value received	
No. 436	Given Dec. 15, 1907. Robert Remington

206. The borrower, who signs the note, is called the **maker**.

The note is the maker's promise to repay the amount borrowed. Sometimes the maker also promises to pay interest. Banks, however, usually require that the interest be paid in advance, or they deduct it from the amount loaned.

If a note is not paid when due, the holder may sue the maker; that is, ask the officers of the law to make the borrower pay.

207. Some notes specify a time of payment, as three months. They are called **time notes**. Notes given to banks for loans are almost always time notes and are seldom for a longer period than three or four months.

208. Some notes are payable on **demand**; that is, whenever the lender wants the money. They are called **demand notes**, and are usually expected to be paid soon.

209. Frequently banks require that the note shall be **indorsed** by some other person. **Indorsing** a note is writing one's name on the back of it. The person so indorsing becomes responsible for the payment of the note and may be compelled to pay it, if the maker fails to pay.

210. Usually the borrower puts in the bank **security**; that is, some property which the bank can sell in order to get its money, in case both the maker and the indorser fail to pay it when due.

Banks, other than savings banks and trust companies, usually require that the security given with notes be in the form of stocks, or bonds, or other property for which there is always a market, so that there may be no delay in collecting the debt, if the bank needs the money quickly to pay checks or drafts.

BANK DISCOUNT

211. Banks loaning money usually require the interest to be paid in advance. The interest is deducted from the loan.

This practice of requiring that the interest be paid in advance causes the borrower to pay interest on a greater amount than that specified. Although this makes a difference that, in ordinary transactions, is small, still in the course of a year the advantage to a bank is very great. This difference can be seen in the example given below.

On Aug. 15, Mr. Remington gives his four months' note to the bank for \$400. But he receives \$400 less the interest for 4 months. When the 4 months are past, he pays the bank exactly \$400.

212. Interest paid in advance is called **Bank Discount**.

Since notes are seldom discounted for more than 4 months, the discount is most easily found by the 60-day method, computing the interest for the exact number of days between the dates.

\$400 face of note.

SOLUTION: Days of discount in August,	16
“ “ “ “ September,	30
“ “ “ “ October,	31
“ “ “ “ November,	30
“ “ “ “ December,	15
Total days of discount	<u>122</u>

\$8.00 = discount for 120 days at 6%.

.13 = “ “ 2 “ “ 6%.

\$8.13 “ “ 122 “ “ 6%.

\$400.00 — \$8.13 = \$391.87 = the amount received by Mr. Remington.

213. The difference between the face of the note and the bank discount is called the **proceeds**. In this example, \$391.87 is the proceeds.

If Mr. Remington had paid interest on the amount actually received, how much would he have paid?

How much did the bank gain by this method?

To find bank discount, compute the interest on the face of the note from the date of discount to the date of maturity. The discount deducted from the face of the note gives the proceeds.

ORAL EXERCISES

214. Find the discount, at 6 %, on the following :

- | | |
|------------------------------|----------------------------|
| 1. 60-day note for \$125. | 7. 90-day note for \$150. |
| 2. 60-day note for \$160.50. | 8. 90-day note for \$120. |
| 3. 30-day note for \$840. | 9. 20-day note for \$630. |
| 4. 30-day note for \$240.50. | 10. 80-day note for \$280. |
| 5. 15-day note for \$600. | 11. 10-day note for \$636. |
| 6. 75-day note for \$480. | 12. 70-day note for \$240. |

215. Bankers in computing discount commonly use interest tables already computed.

100 Dollars

Days	$\frac{1}{2}$ per ct.	1 per ct.	2 per ct.	3 per ct.	$3\frac{1}{2}$ per ct.	4 per ct.	$4\frac{1}{2}$ per ct.	5 per ct.	6 per ct.
1	0	0	01	01	01	01	01	01	02
2	0	01	01	02	02	02	03	03	03
3	0	01	02	03	03	03	04	04	05
4	01	01	02	03	04	04	05	06	07
5	01	01	03	04	05	06	06	07	08
6	01	02	03	05	06	07	08	08	10
7	01	02	04	06	07	08	09	10	12
8	01	02	04	07	08	09	10	11	13
9	01	03	05	08	09	10	11	13	15
10	01	03	06	08	10	11	13	14	17
11	02	03	06	09	11	12	14	15	18
12	02	03	07	10	12	13	15	17	20
13	02	04	07	11	13	14	16	18	22
14	02	04	08	12	14	16	18	19	23
15	02	04	08	13	15	17	19	21	25
16	02	04	09	13	16	18	20	22	27
17	02	05	09	14	17	19	21	24	28
18	03	05	10	15	18	20	23	25	30
19	03	05	11	16	18	21	24	26	32
20	03	06	11	17	19	22	25	28	33
21	03	06	12	18	20	23	26	29	35
22	03	06	12	18	21	24	28	31	37
23	03	06	13	19	22	26	29	32	38
24	03	07	13	20	23	27	30	33	40
25	03	07	14	21	24	28	31	35	42
26	04	07	14	22	25	29	33	36	43
27	04	08	15	23	26	30	34	38	45
28	04	08	16	23	27	31	35	39	47
29	04	08	16	24	28	32	36	40	48
30	04	08	17	25	29	33	38	42	50

WRITTEN EXERCISES

216. Using the table, find the proceeds, at 6%, on :

1. A 15-day note for \$180.50.
2. A 25-day note for \$415.75.
3. A 30-day note for \$270.50.
4. Find the proceeds of Example 1 at 7%.
5. Find the proceeds of Example 2 at 5%.
6. Find the proceeds of Example 3 at $4\frac{1}{2}\%$.

Assuming that each of the following notes was discounted for the time given, find the discount on :

7. A 2 months' note, for \$340, dated April 4, 1906, at 6%.
8. A 3 months' note, for \$160.50, dated May 12, 1906, at 6%.
9. A 4 months' note, for \$420.80, dated Oct. 5, 1905, at 6%.

In the following examples find

- (1) Date of maturity.
- (2) Time for which the note was discounted.
- (3) Discount.
- (4) Proceeds.

10. A note for \$225.50, June 8, 1905, for 3 months, was discounted July 15, 1905. Rate, 5%.

Date of maturity, Sept. 8, 1905.

Time from July 15 to Sept. 8; time for which note was discounted, 55 days.

Discount, \$1.71.

Proceeds, \$223.78.

11. Face of note, \$75.80; date, Jan. 16, 1906; time, 4 months; rate, 6%; discounted, March 12, 1906.

12. Date, Feb. 20, 1906; face of note, \$320.60; time, 4 months; discounted, April 12, 1906; rate, 6%.

13. Date, March 11, 1906; face of note, \$96.80; time, 90 days; discounted, May 14, 1906; rate, 5%.

14. Date, April 1, 1906; face of note, \$80.50; time, 100 days; discounted, June 24, 1906; rate, 6%.

INTEREST-BEARING NOTES

217. Most notes given for loans, whether *secured* by mortgages or not, except notes given to banks for short-time loans, require that interest be paid, not in advance, but when the principal is due or at stated times, as annually or semiannually. Such notes are called **interest-bearing notes**.

\$ 850 ⁵⁰ / ₁₀₀	New York, April 1, 1910
Ninety days	after date I promise to pay to
the order of The St. Paul Lumber Co.	
Eight hundred & fifty & ⁵⁰ / ₁₀₀	Dollars
with interest at 5% ⁵⁰ / ₁₀₀	
Value received	
No. 39 Due June 30, 1910	C. C. Hendricks

218. Write two interest-bearing notes for different times, one at $4\frac{1}{2}\%$ and one at 6%.

Compute the interest on each and give the amounts to be paid when the notes fall due.

219. Sometimes the holder of a note, instead of waiting until it is due, sells it to the maker or to some one else, and gets his money in advance. To do this, he must deduct the portion of the interest not yet due.

Accepting payment of a note less the interest to be earned is called **discounting** the note.

220. Sometimes the holder of a note bearing interest takes the note to a bank and has it discounted; that is, the bank pays him what the note is worth at the time. This is usually computed as the amount of the note less the interest for the time it has to run.

Mr. Gaynor gave Mr. Bannard his note for \$600 for 6 mo. without interest. After 3 mo. Mr. Bannard sold the note to his bank and received its **present worth**. This was \$600 less the interest for 3 mo. at the bank's regular rate of 5%. $\$600 - \$7.50 = \$592.50$.

221. Sometimes a borrower, having paid the interest on a note in advance, desires to repay the loan before maturity. In such cases the bank deducts from the principal due the interest paid for the unexpired portion of time.

WRITTEN EXERCISES

222. Solve:

1. The Gem Manufacturing Company in payment for goods sold took the notes of their customers, without interest, as follows:

Jan. 1, 1909,	The Hackett Hardware Co. .	\$575.50	3 mo.
Feb. 15, 1909,	The Cortlandt Company . .	\$1587.75	4 mo.
Mar. 4, 1909,	H. S. Kendall	\$5685.00	2½ mo.

On March 15 all the notes were sold to the Liberty Bank, which discounted them at 5%. What amount did the Gem Manufacturing Company receive?

SAN FRANCISCO, CAL., May 1, 1908.

2. Ninety days after date, for value received, I promise to pay G. C. Baker Six Hundred and Seventy-five Dollars (\$675), with interest at 6%.

JOHN SCAIF.

How much should Mr. Baker receive?

On June 20 Mr. Scaif paid the note, with the interest then due. How much did he pay? How much interest did he save?

NEW YORK, April 3, 1908.

3. Sixty days after date, I promise to pay to the Liberty Bank, Three Hundred Dollars, with interest at 6 %. Value received.

HENRY SCHENCK.

How much did Mr. Schenck receive for the above note?

4. On May 3 he paid the note. For how much interest did he receive credit? How much did he have to pay to take up the note?

Make and solve five similar problems.

5. Write a note, correct in form and wording, from the following data :

Maker of note, yourself.

Payee, your teacher.

Face of the note, \$1400.

Rate, the legal rate in your state.

Length of time, 1 yr. 3 mo. from date.

What will be the interest? The amount? What kind of a note is it?

223. Sometimes the holder of a note bearing interest, in order to secure payment before maturity, discounts it at a higher rate of interest than that named in the note. That is, he pays the difference between the discount and the interest so that he may get his money.

NEWARK, N.J., July 5, 1906.

Seventy-five days after date I promise to pay C. T. Shipman, or order, Ninety-six and $\frac{50}{100}$ Dollars, value received, with interest at 5 %.

F. N. DOREMUS.

The above note was discounted Aug. 21, 1906, at 6%. Find proceeds.

SOLUTION: \$96.50 face of note.

\$.965 interest for 60 days.

\$.241 interest for 15 days.

\$1.206 interest for 75 days at 6%.

\$1.206 ÷ 6 = \$.201 interest for 75 days at 1%.

\$.201 × 5 = \$1.005 interest for 75 days at 5%.

\$96.50 + \$1.005 = \$97.505, amount of note at maturity.

Date of maturity, September 18.

Time of discount, from Aug. 21 to Sept. 18, 28 days.

Discount on \$97.505, for 28 days, at 6% = \$.455.

\$97.505 - \$.455 = \$97.05, proceeds.

To find the proceeds, find the amount of the note for the given time and rate, and then find the discount on this amount from the date of discount to date of maturity. Deduct the discount from the amount of the note and the result will be the proceeds.

WRITTEN EXERCISES

224. Solve:

1. Mr. Delavan sold a house for \$8000. In payment he received a note dated Feb. 16, 1906, for 4 months, at 6%. He had the note discounted April 10, 1906, at 5%. How much did he receive?

2. Mr. Morris sold his farm for \$12,000, taking in payment a note dated January 12, 1906, for 3 months, at 5%. He had the note discounted 20 days before it was due, at 6%. What were the proceeds?

3. Find the proceeds of a 90-day note for \$180.36, bearing interest at 7%, discounted 36 days after date, at 6%.

4. Paul Brothers sold 25 tons of coal for \$6.50 a ton, taking in payment a note dated July 6, 1906, for 4 months, at 5%. They had the note discounted October 15, 1906, at 6%. What were the proceeds of the note?

MORTGAGES

225. Individuals who have money to loan, and also Savings Banks and Trust Companies, frequently take as security valuable property, like land or houses, that cannot be sold as quickly as stocks. As real estate cannot be deposited with a note, the security is put into a special form called a mortgage.

A **mortgage** is a contract giving the lender the right, in case a loan is not paid, to take possession of the property, have it sold by an officer of the law, and retain the amount due him.

WRITTEN EXERCISES

226. Solve:

1. Mr. Charles Gray bought a farm for \$7000. He paid \$2000 in cash and gave a mortgage for the balance. The mortgage was for 3 years with interest at 6%, payable semi-annually. If the mortgage is paid when due, what will be the total cost of the farm including interest? Why did the seller take a mortgage besides the note?

2. Mr. Appleby, owning a farm, insured his barn and contents for \$1,000. A fire destroyed it, including his stock, wagons, and hay. He built a new barn costing \$1,200, bought cattle for \$500, and wagons and implements costing \$300.

To pay for these, besides his insurance money, he borrowed of the savings bank the amount needed, giving a mortgage on the farm at 5% for five years. What was the amount of the mortgage? How much interest did he pay yearly?

3. Each year he saved money to pay the mortgage when due. This money he deposited in the savings bank, where it drew interest at 4% compounded semiannually. If he deposited \$300 each year, and paid the interest on the mortgage regularly, how much remained to his credit in the bank after paying off the mortgage at the end of five years?

EXCHANGE

227. Banks make it possible for persons living in different places to do business without sending sums of money from place to place.

The following will illustrate the method used :

Mr. Totten, in New York, buys oranges of the California Fruit Co. for \$1,000. To send \$1,000 in cash to California would be troublesome and expensive ; therefore he sends his check for \$1,000 on the Chemical Bank of New York to the company in California. This company indorses the check and deposits it in the First National Bank of Los Angeles.

The Angelus Hotel, of Los Angeles, has bought of McCutcheon & Co., of New York, \$1,000 worth of linen. They send their check on the First National Bank of Los Angeles for \$1,000. McCutcheon & Co. indorses the check and deposits it in the Chemical Bank.

The Chemical Bank of New York and the First National Bank of Los Angeles now each holds a check for \$1,000 against the other. The two checks cancel each other, and the two debts of \$1,000 each are paid without sending any money.

This is called **exchange**.

A plan similar to this is followed in sending money from one country to another. The banks act as agents and take charge of the transfer at very little cost to the senders. The liability of loss is small, because of the integrity of banks.

228. Exchange between places in the same country is called **domestic exchange**.

229. Exchange between places in different countries is called **foreign exchange**.

By means of exchange, very large business transactions can be carried on between distant places with the transportation of money enough to pay balances only.

FORMS OF EXCHANGE

230. BY BANK DRAFTS.

For making payments at a distance there are other means besides checks. The most common are bank drafts.

\$ 500 ⁰⁰ / ₁₀₀	New York, May 16, 1910
Eighth National Bank N.Y. pay to	
the order of John Doe on demand.	
Eight hundred & 00	Dollars
Value received and charge the same to account of	
To The Publishing Co.	J. E. Edmondson

231. Mr. Bell, living in Winona, Minn., wants to send \$320 to Mr. Hulbert, in Chicago. He goes to the First National Bank, of Winona, where he has a deposit, and asks for a draft on Chicago. The bank has an account with the First National Bank of Chicago, so Mr. Bell receives a draft. Mr. Hulbert either collects the money of the Chicago Bank, or deposits the draft in his own bank and the bank collects it for him, through the clearing house.

232. It is customary for banks to charge a small percentage to cover the various expenses of exchange. If payment is made by check, the bank finally receiving the check frequently deducts this cost from the amount paid.

Thus, if Mr. Bell had sent his check to Mr. Hulbert for \$320, the Chicago Bank would probably have paid \$320 - 15 cents, or \$319.85. The amounts charged vary. As Mr. Bell sent a draft, he probably had to pay the Winona Bank 10 or 15 cents for it.

233. Foreign Exchange usually is charged at a fixed rate per cent.

234. For vegetables sold, a commission merchant sent returns which amounted to \$2,400. If the exchange was $\frac{1}{8}\%$, what did the vegetables net the farmer?

235. BY POSTAL ORDERS.

The government provides a convenient method of sending small sums of money through the post office, by means of **postal money orders**.

Following is a list of charges:

For orders for sums not exceeding	\$ 2.50,	3 cents.
" " over \$ 2.50 and not exceeding	5.00,	5 "
" " " 5.00 " " "	10.00,	8 "
" " " 10.00 " " "	20.00,	10 "
" " " 20.00 " " "	30.00,	12 "
" " " 30.00 " " "	40.00,	15 "
" " " 40.00 " " "	50.00,	18 "
" " " 50.00 " " "	60.00,	20 "
" " " 60.00 " " "	75.00,	25 "
" " " 75.00 " " "	100.00,	30 "

236. Make and solve five problems involving the sending of money by postal orders.

237. BY EXPRESS ORDERS.

Express companies also sell money orders collectible at their own offices in different places. The rates are about the same as those of the post office.

238. BY TELEGRAPH ORDERS.

The telegraph companies furnish the quickest means of sending money. They sell orders and communicate with their agents by telegraph, so that a man in New York or London may send money to San Francisco to be paid the day

it is sent. The charges for this service are somewhat higher than for post office or express orders.

239. BY LETTERS OF CREDIT.

Persons traveling and not wanting to carry large sums of money may buy **letters of credit**.

240. Mr. Rising, of St. Paul, Minn., about to take a trip through Europe, put \$5,000 in his bank in St. Paul. He received a letter of credit for \$5,000, less 2%, the charge for exchange. This letter was addressed to banks in the cities of Europe that he planned to visit. At each of these banks he could draw money as he needed it, the amount drawn being entered on the letter. How much credit did he receive?

Make four similar problems.

241. English money is measured in pounds, shillings, pence and guineas.

12 pence (*d*) make 1 shilling.

20 shillings (*s*) make 1 pound (£).

21 shillings make 1 guinea.

The value of the pound as compared with the dollar varies according to demand, but generally $\text{£ } 1 = \$4.86$ nearly.

242. The standard French coin is the franc, which is worth approximately 20 cents. The 5-franc piece corresponds nearly to one dollar.

243. The German standard coin is the mark, worth about 24 cents; so that 4 marks equal approximately \$1.

244. The Dutch standard is the guilder, worth about 40 cents.

245. In Austria the standard is the gulden, worth nearly 48 cents.

WRITTEN EXERCISES

246. Solve:

1. Mr. Rising collected in London \$600 on his letter of credit. How much did he get in English money at \$4.86. His hotel bill was £52. How many dollars were required to pay it?

2. He spent £3 for cab hire, 12s. for flowers, 10 guineas for clothes, and 5s. 6d. for tips. What did they amount to in dollars?

3. If a dollar was worth 5.2 francs, how many francs did Mr. Rising receive when he drew \$750 in Paris?

4. Make and solve three problems covering his expenses in Paris.

5. Mr. Rising drew \$800 in Berlin. With 4 marks worth 95 cents, how many marks did he receive?

6. Make two problems for his expenses in Berlin.

7. In Amsterdam he drew \$650. With guilders worth 40.2 cents, how many did he get?

8. Make and solve five problems covering his expenses in Amsterdam.

9. Here he drew \$1,000. How many gulden did he receive?

Make and solve five problems covering his expenses in Vienna.

The French standard is used in several other countries, forming what is known as the Latin Union. Among them are Italy, Greece, Switzerland, Spain, and Belgium.

10. Antonio Capelli wants to send money to a brother in Italy to pay the expenses of his coming to America. He goes to the banking house of Morosini and Company and buys a draft on the Bank of Italy at Rome. He gives the bank \$150. After deducting 2% exchange, what is the amount of the draft in francs?

INCORPORATED UNDER THE LAWS OF THE STATE OF NEW YORK

NON-ASSESSABLE

FULL-PAID

No. 125

15 Shares

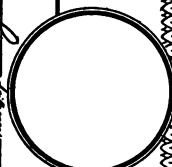


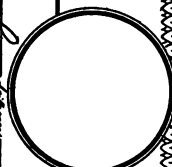
KLAUDER-WILSON DYEING MACHINE COMPANY

Capital Stock \$120,000

Shares \$100 Each

This is to Certify that Frederick Harris is the owner of Fifteen Shares of the Capital Stock of
KLAUDER-WILSON DYEING MACHINE COMPANY
transferrable only in the books of the Company by the holder hereof, person or by attorney upon surrender of this Certificate.
Witness the hand of the Company and the signatures of its authorized officers
at New York 28 June 1908


Henry Green
TREASURER


James Tuttle
PRESIDENT

STOCKS

TERMS DEFINED

247. If several people desire to do business together, they may form a **company**. All states have laws governing the formation of companies. A company formed under such laws is said to be incorporated and is called a **corporation**.

Any number of people may belong to a corporation, each owning a part of its property. Large corporations, such as those that own railroads, have often many thousand members.

Money invested in business by a person or a corporation is called **capital**.

The capital of a corporation is called **stock**. It is divided into **shares**. A share may represent any amount. The most common value of a share is \$100.

248. A person who invests money in a corporation receives one or more **certificates of stock**, indicating the number of shares he owns.

249. The amount named in the certificate is called its **face** or **par value**.

250. If the corporation makes a profit, each stockholder receives as his share of the profit a percentage of the face value of his stock. Such percentage is called a **dividend**.

251. The act of a corporation in deciding to give a dividend is called **declaring a dividend**. Sometimes when a business has paid a good dividend, the corporation does not pay it out, but uses the gains and profits to enlarge the business or to make improvements. For this reason an investor in stock may not get returns for his money for several years, and yet it may be absolutely safe.

252. The business of a corporation is managed by a **Board of Trustees** or **Directors**, elected by the stockholders, each share being entitled to one vote.

253. The value of a share of stock does not depend upon its original cost, or its face value, so much as upon the profits of the business, as shown by the dividends.

Stocks are sold sometimes for much more and sometimes for much less than their face value. The prices of stocks often change from day to day. The prices of many stocks are published daily. Such publications are called **quotations**.

254. When stocks are sold for more than the par value, they are sold "at a **premium**"; when for less than par value, "at a **discount**."

Stocks are quoted in dollars and fractions of a dollar, thus: "Erie R. R., $44\frac{5}{8}$ " means that Erie R. R. stocks sell for \$44.625 a share.

BUYING AND SELLING STOCKS

255. An agent who buys and sells stocks, bonds, etc., is called a **broker**.

256. The commission, or **brokerage**, is always reckoned on the par value of the stock.

The commission charged by brokers is $\frac{1}{8}$ of one per cent, or $12\frac{1}{2}\%$ on a share, par value \$100.

WRITTEN EXERCISES

257. Find the brokerage on the following:

1. 9500 shares Reading R. R. bought for $139\frac{1}{2}$, sold for $137\frac{3}{4}$.
2. 900 " Met. St. Ry. " " $114\frac{5}{8}$ " " $117\frac{3}{4}$.
3. 500 " N. Y. Dock " " 45 " " $47\frac{1}{4}$.
4. 600 " Amal. Copper " " $112\frac{1}{4}$ " " 110.
5. 8000 " Gt. North. Pf. " " 323 " " $327\frac{1}{2}$.

6. When a stock is quoted at 107, how much does a share cost? At 95? At $98\frac{1}{2}$? At par?

7. What was the cost of :

(1)	30,300	shares of Chicago, Milwaukee & St. Paul	at 119?
(2)	1,025	" " Chicago & Northwestern	" 147 $\frac{1}{2}$?
(3)	3,300	" " Colorado Fuel & Iron	" 207 $\frac{1}{2}$?
(4)	1,250	" " Delaware & Hudson	" 158 $\frac{1}{2}$?
(5)	2,800	" " Erie	" 15 $\frac{1}{2}$?
(6)	200	" " Illinois Central	" 128 $\frac{1}{2}$?
(7)	1,200	" " Louisville & Nashville	" 98 $\frac{1}{2}$?
(8)	300	" " Mexican Central	" 18 $\frac{1}{2}$?
(9)	6,700	" " New York Central	" 100 $\frac{1}{2}$?

Add to each of the above $\frac{1}{8}$ of 1 % brokerage.

8. Mr. Melvin bought, through his broker, 250 shares of U. S. Steel at 35 $\frac{1}{8}$, and sold it at 41 $\frac{1}{4}$. How much did he make?

SOLUTION:

$\$35.875 + \$.125$ (commission) = $\$36.00$, cost of 1 share.

$\$41.25 - \$.125$ (commission) = $\$41.125$, selling price of 1 share.

$\$41.125 - \$36.00 = \$5.125$, gain on one share.

$\$5.125 \times 250 = \$1281.50$, gain on transaction.

9. What is the annual income from 50 shares of stock, par value \$100, paying 1 $\frac{1}{2}$ % quarterly?

10. I invested \$1000 in stock, paying the par value of \$25 a share. How many shares did I buy? How much did each share cost me? (No brokerage was paid.)

11. The stock earned a dividend of 2 $\frac{1}{4}$ % semiannually. What was my annual income from it?

12. Mr. Tibbits bought 300 shares of C. M. & St. P. stock at 119 $\frac{3}{8}$. What did he pay, including brokerage?

13. The stock paid a quarterly dividend of 2 %. How much did he receive annually? What rate was this on his investment?

14. Mr. Alberts invested, through his broker, in 140 shares of U. S. Steel stock at 34. How much did it cost him?

15. If they pay $2\frac{1}{2}\%$ semiannually, what income will his stock produce? What per cent is this on his investment?

258. The following are some of the quotations of the New York stock market for one day:

CONSOLIDATED EXCHANGE SALES

	OPEN	HIGH	LOW	LAST
23,830 Amal. Cop.	57 $\frac{3}{4}$	57 $\frac{3}{4}$	55 $\frac{1}{4}$	55 $\frac{7}{8}$
230 Am. Cen. Fnd.	30 $\frac{1}{4}$	30 $\frac{1}{4}$	29 $\frac{1}{2}$	29 $\frac{1}{2}$
170 Am. Ice	18	19 $\frac{1}{8}$	17 $\frac{3}{4}$	18
90 Am. Locom.	37	37 $\frac{1}{2}$	36 $\frac{3}{4}$	36 $\frac{3}{4}$
23,120 Am. Smelt.	69	69 $\frac{1}{8}$	66 $\frac{1}{4}$	67 $\frac{1}{8}$
2,110 Am. Sugar	119 $\frac{1}{2}$	121 $\frac{1}{2}$	118 $\frac{3}{4}$	119 $\frac{1}{8}$
1,770 Anacon. Cop.	37 $\frac{1}{2}$	37 $\frac{3}{8}$	36	36 $\frac{1}{8}$
860 A. T. & S. F.	73 $\frac{3}{8}$	74	72	72 $\frac{1}{8}$
190 Balt. & Ohio	82	82 $\frac{1}{4}$	80 $\frac{5}{8}$	80 $\frac{3}{8}$
10,730 Brook. R. T.	46	47	45	45 $\frac{1}{4}$
140 Can. Pac.	146	146 $\frac{1}{2}$	145	146
30 Ch. & Gt. W.	5	5	4 $\frac{3}{4}$	4 $\frac{3}{4}$

Make and solve ten problems on the above.

COMMON AND PREFERRED STOCK

259. Many corporations issue two kinds of stock, called **common** and **preferred**. The difference is in the obligations as to dividends. Preferred stock is guaranteed to pay a specified dividend, if there is any profit with which to pay dividends. This must be paid before common stock can receive any dividend. But preferred stock can never receive more than the amount specified, while the common can get all the surplus profit, if any.

WRITTEN EXERCISES

260. Solve:

1. A publishing company was incorporated with \$100,000 capital, in 1,000 shares. Of these, 250 shares were preferred 6% shares. The remainder were common. At the end of the second year, the common stock drew 3%. How much was paid in dividends in all?

2. At that rate how much more would a share of preferred stock be worth than a share of common?

3. Later the common stock paid 4% semiannually. At that rate, how much more would a share of common be worth than a share of preferred?

4. Following is a list of quotations of railroad stocks made at two different times. At the rate of dividend mentioned, what per cent will a purchase of a share of each stock yield at each price quoted?

RAILWAY SHARES

NAME	YEARLY DIVIDEND PER CENT	FIRST PRICE	LAST PRICE
(1) Atchison	6	110½	72½
(2) Atchison, pf.	5	106	85
(3) Baltimore & Ohio	6	125½	80½
(4) Baltimore & Ohio, pf.	4	99½	82
(5) Central of New Jersey	8	239½	170
(6) Chesapeake and Ohio	1	65½	29½
(7) Chi., Mil. & St. Paul	7	199½	118½
(8) Chicago & Northwestern	7	240	146
(9) Delaware & Hudson	9	234½	154
(10) Denver & Rio Grande, pf.	5	91½	49
(11) Great Northern, pf.	7	348	121½
(12) Illinois Central	7	184½	125
(13) Louisville & Nashville	6	156½	96½

5. Mr. Butler needed \$6,000 to use in his business. He gave his note for \$6,000 to the Liberty Bank, for 90 days, at 5%. What was the interest?

6. He deposited as security with his note 100 shares of N. Y. Central stock, quoted at 102. What was the par value of the security? The market value?

7. The bank discounted the note; that is, deducted the interest from the \$6,000. The balance was placed to Mr. Butler's credit in the bank. How much was added to his bank account?

8. When the note was due, Mr. Butler gave a new note for \$3,500, for 90 days, and his check for the balance and the interest on the new note. What was the amount of his check?

9. When the note was due, Mr. Butler, having met losses, was unable to pay the note, and the bank refused to renew it. They then sold enough of the security to pay the note. This was at that time quoted at 96.

(1) How many shares were sold?

(2) What was the par value of the stock returned to Mr. Butler?

(3) How much did he lose by the decline in value of the stock?

BONDS

261. Corporations, as well as individuals, borrow money to meet the needs of business. Sometimes the corporation can raise money by simply giving its note signed by the officers of the company. But usually security is required.

The most usual form of security for a corporation to give is a **bond**. A bond is really a mortgage on the property of the corporation, empowering the purchasers of the bond, who are the actual money lenders, to take possession of the property if the corporation fails to meet its obligations.

Bonds of large corporations, like stocks, are sold on the market and are quoted daily.

262. Stocks are certificates of ownership of a share in the property of a corporation. Their returns, in the form of dividends, depend wholly upon the success of the business of the corporation. The returns can be paid only after all expenses have been met.

263. Bonds are certificates of a share in a mortgage on the corporation's property. Their returns, in the form of interest, are fixed in the bond, and must be paid before dividends are paid.

Governments also, national, state, county, and city, raise money by issuing bonds, which are a mortgage on the property of the government.

Most bonds have **coupons** attached to them, which are cut off and presented as proofs that interest is due.

WRITTEN EXERCISES

264. Solve:

1. If U. S. 3% coupon bonds are quoted at $101\frac{1}{8}$, how many can a bank purchase with \$100,000? What will be the annual income from the investment?

2. If Japanese Imperial Government $4\frac{1}{2}\%$ bonds are quoted at 86, how many can be purchased for \$24,000?

3. What will be the annual income from the investment? What rate of interest on the price will that represent?

4. Which is the better investment; New York City $4\frac{1}{2}\%$ bonds at $102\frac{3}{8}$, or B. & O. Railway 4% bonds, at $99\frac{1}{2}$? How much per bond?

UNITED STATES OF AMERICA

STATE OF NEW JERSEY

76

\$1000

BOROUGH OF NORTH FAIRFIELD WATER BOND

Know all men by these Presents

That The Mayor and Council of the Borough of North Fairfield, a municipal corporation of the County of Bergen and State of New Jersey, do hereby certify and receive hereby promising to pay to the bearer or if registered to the registered owner hereof the sum of

ONE THOUSAND DOLLARS

in gold coin of the United States of America of the present standard weight and fineness of the New Haven County Trust Company of New Jersey on the first day of April 1907 with interest thereon at the rate of five and the half percent per annum payable in like gold coin semi-annually on the first days of October and April in each year at the said Trust Company on the presentation and surrendering the unpaid coupons as they may respectively become due.

This bond is one of a series of similar bonds of like tenor, date and amount numbered from one to seven five each inclusive issued pursuant to the authority of an act of the Legislature of the State of New Jersey entitled "An Act Relating to the Borough of North Fairfield" approved March 22nd 1892 and all such amendments thereto and amendments thereto and amendments thereof and the issue of bonds having been approved by the majority of the electors voting in force thereof as an action duly called for the purpose in said Borough on the first day of December A. D. 1891 and for the purpose herein said principal and interest the full and true and valid of the Borough of North Fairfield are hereby irrevocably pledged.

In Witness Whereof the said Mayor and Council of the Borough of North Fairfield have caused this bond to be executed under its corporate seal, and to be signed by the Mayor and Clerk of said Borough and the annexed coupons to be executed with the following tenor: The first day of April A. D. 1900.

Attest

William Lyons

Mayor of the Borough of North Fairfield

Mark Ewing

Clerk of the Borough of North Fairfield

INSURANCE**TERMS DEFINED**

265. **Insurance** is a form of provision against loss from certain causes, such as fire, death, shipwreck, and various accidents.

An **insurance policy** is an agreement to pay to the loser, in case of loss from a specified cause, a sum either stated in the policy or to be determined on some fixed plan.

The price paid for insurance is called the **premium**.

The price or **rate** of insurance is usually stated as such a per cent on \$100 or \$1000.

There are two classes of insurance, — **property insurance** and **personal insurance**.

PROPERTY INSURANCE

266. The most common forms of property insurance are **fire insurance**, **marine insurance**, **burglar insurance**, and **plate glass insurance**. There are various other kinds, however.

PERSONAL INSURANCE

267. Common forms of personal insurance are **life insurance**, **accident insurance**, and **credit insurance**.

Of these the most important is life insurance.

268. **Life insurance** is a form of contract in which the insurer, a life insurance company, agrees to pay a fixed amount upon the death of the insured person. The person to receive the payment is called the **beneficiary**.

The terms of the contract, including the name of the beneficiary, are set forth in a document called the **policy**.

Sometimes it is specified in the policy that, if the insured live to a fixed date, the money shall be paid to him.

The insured person pays the company a specified amount, in most cases annually in advance. This is called a **premium**.

In some policies the premium is to be paid during the life of the insured. In others it is to be paid for a definite number of years, as ten or twenty.

The amount of the premium varies according to the age of the insured when the policy is taken out and the number of years during which payments are to be made.

In the following tables rates are given for ages from 21 to 35 for payments during life, and for a 20-year period.

Observe the gradual increase of the premiums with the increase of the age. Note also the difference between the annual premiums of the twenty-year policy and those of the "straight life" policy.

PREMIUM FOR \$1000 INSURANCE

ANNUAL PAYMENTS

Age	DURING LIFE Premium	DURING TWENTY YEARS Premium
21	\$ 15.91	\$ 23.49
22	16.26	23.86
23	16.62	24.26
24	17.00	24.67
25	17.40	25.09
26	17.83	25.55
27	18.29	26.02
28	18.76	26.51
29	19.27	27.03
30	19.80	27.56
31	20.36	28.12
32	20.94	28.70
33	21.57	29.30
34	22.53	29.93
35	22.92	30.39

WRITTEN EXERCISES

269. Solve :

1. Mr. Weathesby owns two houses. One is built of fire-proof material throughout. The other is frame. He insured the fire-proof house for \$5,000 and its contents for \$3,000, at a premium of \$3.50 per \$1,000 per year. The frame house he insured for \$6,500 at a rate of \$7.50 per \$1,000. What was the premium on each?

2. Bullard Brothers insured their brick store for \$50,000 at \$12 per \$1,000. Their stock they insured against loss by fire for \$75,000 at \$10.50 per \$1,000, the insurance on the store and stock being divided equally among three companies. They also carried burglar insurance for \$25,000 at \$6 per \$1,000. Their plate glass windows they insured for \$500 at \$1.75 per \$100. What was their total annual expenditure for insurance premiums?

3. The building was gutted by fire. Much of the stock was injured or destroyed, and, during the fire burglars carried away a case of diamonds. The *adjusters* determined that the loss on the building was 75 % of the amount insured, on the stock by fire 45 %, and by burglary \$4,500. The windows were a total loss. How much did each company lose?

4. Mr. Johnson, when 30 years old, took out a life insurance policy for \$5,000, on the 20 annual payment plan, naming Mrs. Johnson as beneficiary. What were his annual premiums?

5. After 12 years he died. How much had he paid in premiums?

6. How much did Mrs. Johnson receive in excess of the premiums paid?

Make and solve ten life insurance problems.

REVIEW

270. Solve :

1. Mr. Lane gave Mr. Berry his note for \$2,000 for four months with interest at 6 %. At the end of the four months how much was due Mr. Berry ?

2. If Library Bureau stock is quoted at 120, what will be the cost of 23 shares ?

3. If this stock pays 8 %, what is the rate of income ?

4. Mr. White bought 150 shares of Illinois Central Railway stock at 111. In two weeks, the price of the stock had risen to 119, and he sold his shares. He paid his broker $\frac{1}{8}$ % for buying and $\frac{1}{8}$ % for selling. How much did he gain ?

5. At the age of 25 Mr. Day had his life insured for \$2,000, taking out a twenty-year policy, the rate for which was \$25.09. At the end of the 20 years, he received the \$2,000. How much more was this than he had paid in ?

6. How much will Mr. Daniels receive at the bank on a three months' note for \$600, the interest being at 6 % ?

7. Write an interest-bearing note at 5 %. What amount must be paid when the note falls due ?

8. A man had a house which cost him \$3,480. He had it insured for $\frac{3}{4}$ of this amount, at a cost of \$6.40 per thousand. The insurance ran out and was not renewed. Three days afterward the house was totally destroyed by fire. How much greater was the man's loss than it would have been if the house had been insured at the time of the fire ?

9. A man has \$12,000 which he wishes to invest. Will it be better for him to buy 6 % stock at 120, or 5 % stock at 110 ? How much better ?

10. When Delaware & Hudson R. R. stock is quoted at 225, how many shares can be bought for \$11,500 ? What is the rate of income if this is 7 % stock ?

11. Mr. James invested \$ 27,390 in an 8 % stock which he bought at 114, brokerage $\frac{1}{8}$ %. What was his yearly income from this stock?

12. Later he sold his stock at 118. What did he receive for it? How much did he gain by the transaction? What would have been his loss if he had sold it for 111?

13. How much greater was Mr. James' rate of income from this stock than the rate of income of the one to whom he sold it?

14. Which would you rather buy, 5 % stock at 75, or 7 % stock at 105? Why?

15. Find the proceeds of a 60-day note for \$ 925, bearing interest at 6 %, discounted 24 days after it was made.

16. Mr. Brown bought a farm for \$3,400. He paid \$1,300 cash. For the balance he gave a mortgage with interest at 5 %. This mortgage was not paid until the end of 5 years. What was the entire amount, including interest, that he had to pay for the farm?

SUMMARY OF CHAPTER V

BANKING.

- Banks.
- Clearing Houses.
- Savings Banks.
- Trust Companies.
- Loans.
- Mortgages.
- Bank Discount.
- Exchange.

BUSINESS PRACTICE.

- Stocks.
- Bonds.
- Insurance.

CHAPTER VI

HOW TO SOLVE A PROBLEM—ELEMENTS OF ALGEBRA

THE THREE METHODS

271. The ratio of 12 to 6 = the ratio of 8 to what number?

Read the problem carefully, to see

(1) What is given?

There is given a ratio, 12 : 6, which is 2, and one term, 8, the dividend of an equal ratio.

(2) What is to be found?

There must be found the second term, or the *divisor*, of the equal ratio.

(3) What is their relation?

The ratio is the quotient of one term divided by the other. 8, the first term, is the dividend. 2 is the ratio or quotient. The relation is that of dividend to quotient.

(4) What principle must you employ to find the unknown term from the known?

The principle is this:

Dividend $\div$ quotient = divisor.

$8 \div 2 = 4$ = the divisor or second term of the incomplete ratio.

That is, $12 : 6 = 8 : 4$.

How can you prove the correctness of your answer?

Divisor $\times$ quotient = dividend. $4 \times 2 = 8$.

Give other proofs.

Name the steps in the solution of a problem.

Select ten problems in your book and name these steps for each one.

Naming the steps in the solution of a problem is called **stating the problem**. It is important to state all problems before beginning to solve them.

Several methods of solving problems are used in arithmetic. One is **analysis**, sometimes called **unitary analysis**. Another makes use of **ratio**, and a third employs the **equation**. Frequently the equation includes one or both of the other methods.

272.

SOLVING BY ANALYSIS

1. If 2 apples cost 4 cents, what will 6 apples cost?

2 apples cost 4 cents.

1 apple costs $\frac{1}{2}$ of 4 cents = 2 cents.

6 apples cost 6×2 cents = 12 cents.

2. What are three eggs worth at 24 cents a dozen?

If 12 eggs cost 24 cents, 1 egg will cost $\frac{1}{12}$ of 24 cents = 2 cents. 3 eggs will cost 3×2 cents = 6 cents.

3. $\frac{3}{4}$ of Daisy's money is \$15. How much money has she?

$\frac{3}{4}$ of the money is \$15.

$\frac{1}{4}$ of the money is $\frac{1}{3}$ of \$15 = \$5.

$\frac{4}{4}$ of the money is $4 \times \$5 = \$20$.

273.

SOLVING BY RATIO

1. If 2 apples cost 5 cents, what will 6 apples cost?

The ratio of 6 apples to 2 apples is 3.

Therefore, 6 apples cost 3×5 cents = 15 cents.

2. What are 3 eggs worth at 24 cents a dozen?

The ratio of $3:12 = \frac{1}{4}$; $\frac{1}{4}$ of 24 cents = 6 cents.

3. $\frac{3}{4}$ of Ruth's money is \$20. How much money has she?

The ratio of $\frac{4}{4} \div \frac{3}{4} = \frac{4}{3}$. $\frac{4}{3}$ of \$20 = $\frac{80}{3} = \$26.66\frac{2}{3}$.

WRITTEN EXERCISES

274. Solve the following by any method:

1. If $\frac{5}{8}$ of an acre of land was sold for \$1,200, what is the value of $2\frac{1}{2}$ acres at that price?

2. If $\frac{3}{8}$ of Mr. Lanning's crop of wheat is worth \$840, what is the value of the crop?

3. If $12\frac{1}{2}$ yards of silk cost \$20, how much will 100 yards cost? How many yards can be bought for \$80? For \$95? For \$105?

4. A jobber bought 100 books for \$16. What was the cost of 75 books at that rate?

5. A farmer had 1,000 bushels of potatoes. He sold 750 bushels for \$810. What is the value of his crop at that rate?

6. Mr. Rankin deposited \$2,400 in the bank, which was $\frac{3}{4}$ of his money. How much had he?

7. A farmer sold $37\frac{1}{2}$ pounds of butter for \$7.50. What are 100 pounds worth at that rate?

8. If $62\frac{1}{2}\%$ of a number is 15, what is the number?

9. $\frac{5}{8}$ of a number is 80. What is the number?

10. $87\frac{1}{2}\%$ of my bank account is \$3,500. What is $12\frac{1}{2}\%$ of it?

11. $62\frac{1}{2}\%$ of Mr. Randolph's wheat is 2,500 bushels. He sold $37\frac{1}{2}\%$. How many bushels did he sell?

12. If $\frac{5}{8}$ of a barrel of apples are worth \$2, what are $\frac{3}{10}$ of a barrel worth?

13. A real estate agent sold a house for \$4,200 and made 40% on it. What did it cost?

14. He sold another for the same price and lost 40%. What did it cost?

15. What was the gain or loss on both sales?

SOLVING BY EQUATIONS—THE USE OF LETTERS

275. A still better method for the solution of different problems is by the use of the equation, using a letter or letters, as x , or y , to represent the quantity to be found.

Numbers are often represented by letters of the alphabet instead of by the ordinary numerals. Numbers represented by letters are called **literal quantities**.

Thus we may speak of a apples, b boys, x hats.

We can use the letters in calculation as if they were figures. They may represent any numbers that we choose and then may be added, subtracted, multiplied, or divided, as we please.

Suppose you say that $a = 6$, then 6 boys may be written a boys.

2×6 boys may be written $2 \times a$ boys.

Suppose you decide that you will use b for 2. Then 2×6 boys may be written $a \times b$ boys, etc.

$2 \text{ boys} + 6 \text{ boys}$ will be $a \text{ boys} + b \text{ boys}$.

In writing numbers by the use of letters, use the same signs that you do with numerals. In representing multiplication, however, the sign may be omitted. $ab = a \times b$; $2a = 2 \times a$; $2bc = 2 \times b \times c$. All other signs must always be used to indicate the operations to be performed.

In the expression, $2abc$, 2 is called the **coefficient** of abc .

When a literal quantity is represented as multiplied by an Arabic numeral, or figure, the figure is called the **coefficient** of the literal quantity.

276. Letting $a = 2$, $b = 3$, $c = 4$, and $d = 5$, find the values of :

1. $a + b + c$.

5. $ab + cd$.

2. $2b - d$.

6. $cd - b$.

3. $3a + 2b - 2c$.

7. $2bc$.

4. $2d + a - 3b$.

8. $3abc$.

277. POSITIVE AND NEGATIVE QUANTITIES

$$5 + 4 - 4 = \quad 3 + 2 - 2 =$$

$$6 - 5 + 5 = \quad 7 - 6 + 6 =$$

In each of the above examples the amount added equals the amount subtracted. $+4$ and -4 offset each other. So do -5 and $+5$; $+2$ and -2 ; -6 and $+6$.

$$278. \quad 1. \quad 5 + 3 - 4 = \quad 5. \quad 7 - 7 + 8 =$$

$$2. \quad 4 + 6 - 5 - 3 = \quad 6. \quad 5 - 3 + 2 + 3 =$$

$$3. \quad 5 - 4 + 3 - 2 = \quad 7. \quad 4 - 6 + 8 - 9 + 17 - 20.$$

$$4. \quad 8 - 6 + 10 - 11 = \quad 8. \quad 6 + 6 + 6 - 7 - 7 - 7 - 8.$$

In each of the above we can perform each operation separately; or we can find the result by subtracting the sum of the numbers having the *minus* sign from the sum of the numbers having the *plus* sign. (A quantity having neither the $+$ sign nor the $-$ sign is supposed to have the $+$ sign.) Thus, in example 2, we can say $4 + 6 = 10$; $10 - 5 = 5$; $5 - 3 = 2$. Or, we can say $4 + 6 = 10$; the sum of the minus quantities $= 8$; $10 - 8 = 2$.

Quantities having no sign, or the sign $+$, are called **positive quantities**.

Quantities written with the sign $-$ are called **negative quantities**.

Sometimes the result of combining the positive and negative terms of a quantity is less than nothing; that is, it is negative.

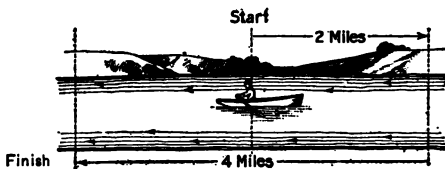
A man at the end of a year found that money was due him to the amount of \$2526 and he owed debts amounting to \$3186. What was the excess of his debts over his credits?

Write the difference with a minus sign as a negative quantity. Thus: $2526 - 3186 = -660$.

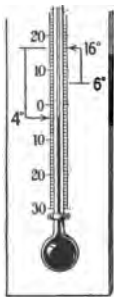
WRITTEN EXERCISES

279. A boy started to row his boat up the stream at the rate of 2 miles an hour. The stream flowed down at the rate of 4 miles an hour. At the end of an hour was he above or below his starting place, and how far?

He had rowed up 2 miles and been carried down 4 miles, thus:



At the end he was 2 miles below the starting place. The answer, then, is a negative quantity, -2 . That is, it could be represented by an equation, $2 - 4 = -2$. His course up the stream had been 2 less than nothing.



One morning the thermometer showed 6 degrees above zero. During the day the mercury rose 10 degrees, and at night fell 20 degrees. How far below zero was it at night? Write this as a negative quantity.

$$6 + 10 - 20 = -4$$

Write as negative quantities the results of the following:

1. $6 - 9 =$

4. $-3 - 4 - 5 + 8 =$

2. $2 + 3 - 4 - 4 =$

5. $6 + 3 - 7 - 4 - 8 =$

3. $8 - 1 + 9 - 20 =$

6. $-1 + 2 - 3 + 1 =$

7. $5 + 6 - 7 + 8 - 9 + 16 - 20 =$

8. $10 - 100 - 90 + 675 - 840 =$

9. $56 + 75 + 173 - 1000 - 20 =$

280.**LITERAL QUANTITIES**

2 boys + 2 boys = 4 boys.

2 × 2 girls = 4 girls.

 $2a + 2a = 4a.$ $2 \times 2a = 4a.$

3 books - 2 books = 1 book.

6 horses + 2 = 3 horses.

 $3b - 2b = 1b.$ $6b + 2 = 3b.$

What is the numeral written before a literal quantity called?

A literal quantity having no coefficient is understood to have 1 for a coefficient. $b = 1b.$ $c = 1c.$

As letters stand for quantities, they can be added, subtracted, multiplied, and divided like the quantities they stand for.

WRITTEN EXERCISES

281. Perform the operations indicated by the signs in the following problems:

1. $2a + 3a =$

7. $2d + 5d - 6d =$

2. $3b - b =$

8. $2c - 4c =$

3. $b - 3b =$

9. $5c - 6c =$

4. $6a - 4a =$

10. $18e - 21e + 6e =$

5. $15b - 12b =$

11. $7f - 8f + 9f - 10f =$

6. $3c - 4c =$

12. $2a + 3a - 4a - 2a =$

ADDITION

282. Add 2 dogs and 3 cats.

The answer is merely 2 dogs + 3 cats.

You cannot state your answer in a single term.

Add a and b .

The answer is necessarily just $a + b$.

The sum of a and b , less c , $= a + b - c$.

Unlike literal quantities to be added or subtracted are merely written with the signs to show the operation.

Add: a , b , and c .

Add: $a + b + 2b$. $b + 2b = 3b$. *Ans.* $a + 3b$.

Add: a , $2a$, $3a$, $2b$, $3b$.

These terms may be arranged in this way:

SOLUTION:

$$\begin{array}{r} a \\ 2a + 2b \\ 3a + 3b \\ \hline 6a + 5b \end{array}$$

Add: $2a$, $-3a$, $-b$, $2b$.

SOLUTION:

$$\begin{array}{r} 2a - b \\ -3a + 2b \\ \hline -a + b \end{array}$$

Add:	(1)	(2)	(3)	(4)
	$2b$	$-3c$	$-d$	x
	$-b$	c	$3d$	$-4x$
	$3b$	$4c$	$5d$	$5x$
	$-2b$	$-5c$	$-8d$	$-10x$
	<u>$4b$</u>	<u>$6c$</u>	<u>$10d$</u>	<u>$6x$</u>

In adding algebraic quantities place like quantities in the same columns, with the proper signs, and add.

283. Add: (1)	(2)	(3)
$2a + 3b - 3c$	$6b - 7c - 8a$	$5x + 3y - z$
<u>$3a + 3b - 2c$</u>	<u>$3b - 2c - 9a$</u>	<u>$2x - 6y - 3z$</u>
(4)		(5)
$7m + 3n - p - 2$		$2c - d + x$
<u>$14m - 6n + p - 7$</u>		$c - 2x$
		<u>$a + 3c + d - 5x$</u>

How do you add two quantities, one positive and one negative?

WRITTEN EXERCISES

284. Arrange the following quantities in columns and add:

6. $2b - 3 + 4c - 6b - 5c + b - c + 5.$
7. $6a - 7c + 2d - 5c + 6d - a + 2a - 7d + 6c.$
8. $8x - 5x - 3x + 2y - 7c - 6y - 4y + 8y - 24.$
9. $15 + 20 + 2a - 3a - 6d + 4a - 5d + c + 6d - 20c.$
10. $2z - x + 3y - 6z - y - 6x + 8y - 6 - z + 3 - 5z + x.$
11. $2ab + 2ac + 2d - x - 3ab - 4d + 3ac + 6x.$
12. $4xy + 4xz - 2y + 2z + 5xy + 6xy - 3xz - 2xz.$
13. $y + x + xy + z + xyz + 2xy + 3xyz + 2y + 3x + 4z.$
14. $abc + bcd + cde + 2abc + 3cde + 4bed + 5 + a + 2a.$
15. $20xy + 15x + 30y + 40x + 6xy + 7xy - x - y - 2x.$
16. $3abx - 2by + 4cd - 2abx - 4abx + 2by - 54cd.$
17. $x + y - z + a - z + x + a - y.$
18. $15xyz - 7abc - abc - xyz - xy + ab + ax - 6d.$

SUBTRACTION

285. John had \$15. He owed \$20.

Represent his condition as a negative quantity.

$$15 - 20 = -5.$$

He received \$25. How much was he now worth?

$$-5 + 25 = +20.$$

That is, you add 25 to his credits.

Subtract $2x$ from $3x$.

$$3x - 2x = x.$$

Subtract 2 from 1.

$$1 - 2 = -1.$$

Subtract $3x$ from $2x$.

$$2x - 3x = -x.$$

Subtract $-2x$ from $3x$.

$$3x - (-2x) = +5x.$$

Subtract $-4ab$ from $6ab$.

$$6ab - (-4ab) = 10ab.$$

Subtract $-3mn$ from $6mn$.

$$6mn - (-3mn) = 9mn.$$

$$\begin{array}{r} \text{Add:} \quad -5 \\ \quad +25 \\ \hline \quad +20 \end{array}$$

$$\begin{array}{r} \text{Add:} \quad -4 \\ \quad +12 \\ \hline \quad +8 \end{array}$$

$$\begin{array}{r} \text{Subtract:} \quad -4 \\ \quad -12 \\ \hline \quad +8 \end{array}$$

Subtracting a negative quantity is the same as adding a positive quantity, and subtracting a positive quantity is the same as adding a negative quantity.

To subtract one quantity from another, change the sign of the subtrahend and proceed as in addition.

WRITTEN EXERCISES

286.. Subtract:

(1)	(2)	(3)	(4)	(5)
-30	-2	$-2a$	$-6b$	$-20abc$
$\underline{12}$	$\underline{16}$	$\underline{5a}$	$\underline{4b}$	$\underline{45abc}$

(6)	(7)	(8)	(9)	(10)
-5	16	-18	17	20
$\underline{-8}$	$\underline{-7}$	$\underline{-6}$	$\underline{-18}$	$\underline{-30}$

(11)	(12)	(13)	(14)	(15)
-40	16	-17	30	$-16y$
$\underline{-35}$	$\underline{-2}$	$\underline{-20}$	$\underline{-30}$	$\underline{12y}$

$$\begin{array}{r} 16. \quad 3x - 5y + z - 6 \\ \quad 2x - 47 + z - 4 \\ \hline \end{array}$$

$$\begin{array}{r} 20. \quad 4x + 6y + z - 8 \\ \quad 8x + 2y - 3z - 4 \\ \hline \end{array}$$

$$\begin{array}{r} 17. \quad 2ab + 3cd - 2b + a - 6 \\ \quad 3ab - 2cd + b - 3a + 10 \\ \hline \end{array}$$

$$\begin{array}{r} 21. \quad 15ab - 2 + 13ac - 4d \\ \quad 6ab - 4 + 7ac + 8d \\ \hline \end{array}$$

$$\begin{array}{r} 18. \quad 9a - 10b + 3c + 6abc \\ \quad 10a + 6b + 5c - 2abc \\ \hline \end{array}$$

$$\begin{array}{r} 22. \quad 25d - 13a + 6bc - 8b \\ \quad 16d + 14a + 7bc - 2b \\ \hline \end{array}$$

$$\begin{array}{r} 19. \quad 4x - 5y + 8z + 14xy \\ \quad 5x + 10y - 8z + 5xy \\ \hline \end{array}$$

$$\begin{array}{r} 23. \quad 20ab + 30b - 60cd - 18 \\ \quad 10ab + 40b - 20cd + 28 \\ \hline \end{array}$$

MULTIPLICATION

287. Multiply:

$a \times 2.$

$a \times 2 = 2a.$

$a \times b.$

$a \times b = ab.$

$a \times b \times c.$

$a \times b \times c = abc.$

$2a \times 3b.$

$2a \times 3b = 6ab.$

When letters are written one after another, without a sign, multiplication is indicated.

WRITTEN EXERCISES

288. Write in different ways the products of:

1. $a, b,$ and $c.$

6. $d, c,$ and $e.$

2. $b, e,$ and $f.$

7. $2a \times 2b = 4ab.$

3. $b, e,$ and $a.$

8. $3a \times 4b =$

4. $x, y,$ and $z.$

9. $5c \times 6d =$

5. $a, b, c, d, e,$ and $f.$

10. $7c \times 5c =$

289. MULTIPLYING A QUANTITY BY ITSELF.

$2 \times 2 = 2^2 = 4.$

$a \times a = a^2.$

A quantity, whether numeral or literal, when multiplied by itself is **squared**. If letters are used, the squaring is merely indicated by the sign. This sign is called an **index** (plural, *indices*). Every number has an index written or expressed. If no index is expressed, 1 is understood.

a is said to be in the first degree and its index, not expressed, is 1. It could be written a^1 . a^2 is read " a square," or " a in the second degree." a^4 is read " a fourth power," or " a in the fourth degree."

Read: a ; a^2 ; b^3 ; c^4 ; d^5 .

$2 \times 2 = 2^2 = 4.$

$a \times a = a^1 \times a^1 = a^{1+1} = a^2.$

$2^2 \times 2 = 2^3 = 8.$

$a^2 \times a = a^{2+1} = a^3 = a \times a \times a.$

A quantity is squared by multiplying it by itself. It is cubed by multiplying it twice by itself. It is raised to the fourth power by multiplying it three times by itself.

To indicate the multiplication of a quantity by itself, add its indices.

WRITTEN EXERCISES

290. Multiply:

1. $2a \times a \times ab.$

5. $a \times 2b \times a^2c.$

2. $a \times b \times cd \times ab.$

6. $a^3 \times ab \times b^2.$

3. $ax \times bx.$

7. $xy \times x^2y^2 \times xyz.$

4. $2c \times abc.$

8. $abc \times 2a^2 \times 3b \times 3c^2.$

291. EXPRESSING MULTIPLICATION.

Multiply $2 + 3$ by 3.

SOLUTION: $(2 + 3) = 5; 5 \times 3 = 15.$

$2 + 3$

Or; $3 \times 2 = 6; 3 \times 3 = 9; 9 + 6 = 15.$

$\frac{3}{9}$

$6 + 9 = 15.$

We can add the quantities and then multiply the sum; or we can multiply each quantity and add the products.

Multiply $a + b$ by 2.

$(a + b) \times 2 = 2a + 2b; \text{ or,}$

$a + b$

$\frac{2}{2a + 2b}$

$(a - b) \times 3 = 3a - 3b.$

292. In using the signs of multiplication, or division, with quantities connected by $+$ or $-$, it is necessary to inclose these quantities in brackets (), to show that the entire quantity is to be multiplied or divided.

$(a + b)2$ means that the quantity $a + b$ is to be multiplied by 2; $a + b2$ would mean the b alone was to be multiplied.

What is the meaning of $(a - c) + b$?

WRITTEN EXERCISES

293. Express by signs:

1. $x + y$ multiplied by 3.
2. $a + b + c$ multiplied by x .
3. $3a - 4 + c$ multiplied by $2c$.
4. $a - y$ multiplied by $c + d$.
5. $4a - 3$ multiplied by $3a - 4x$.
6. $2a - z$ divided by $a - b$.
7. $4 - 5x + 3y$ divided by $2 - 2a$.

8. The sum of a , b , and c divided by the difference of x and y .

294. MULTIPLICATION OF NEGATIVE QUANTITIES.

$$2 \times -a = -2a. \quad a \times -a = -a^2. \quad 2 \times -a = -2a.$$

If Mr. Jones owes \$10, it is represented algebraically as $-\$10$. If he owes \$10 to each of two pupils, it is represented algebraically as $2 \times -\$10$, or $-\$20$.

$$-a \times -a = +a^2.$$

In multiplication in algebra, the product of two negative quantities or of two positive quantities is positive; the product of a negative quantity and a positive quantity is negative.

WRITTEN EXERCISES

295. Multiply:

$$1. a \times -a. \quad 2. b^2 \times b^2. \quad 3. -cd \times cd. \quad 4. x^2 \times -x^3.$$

296. MULTIPLYING QUANTITIES OF SEVERAL TERMS.

Multiply $a + b + 2c$ by 2.

$$\begin{array}{r} a + b + 2c \\ 2 \\ \hline 2a + 2b + 4c \end{array}$$

 $a + b - c - d$ by a .

$$\begin{array}{r} a + b - c - d \\ a \\ \hline a^2 + ab - ac - ad \end{array}$$

In multiplying a quantity composed of numbers connected by + or -, multiply each number separately, giving the product in each case the proper sign.

297. WHEN THE MULTIPLIER HAS SEVERAL TERMS.

Multiply $a + b$ by $a + b$.

$$\begin{array}{r} a + b \\ a + b \\ \hline a^2 + ab \\ \hline ab + b^2 \\ \hline a^2 + 2ab + b^2 \end{array}$$

$x + y - z$ by $x + 2y$.

$$\begin{array}{r} x + y - z \\ x + 2y \\ \hline x^2 + xy - xz \\ \hline 2xy + 2y^2 - 2yz \\ \hline x^2 + 3xy - xz + 2y^2 - 2yz \end{array}$$

When the multiplier has more than one term, multiply each term of the multiplicand by each term of the multiplier and add the results.

WRITTEN EXERCISES

298. Multiply:

1. $a + b + c$ by $a + b$.
2. $x - 2y - 2z$ by $2z - 2y$.
3. $m + n + p$ by $2 - m$.
4. $a - b - 2c + cd$ by $2a - 3$.
5. $ab - ac - ad$ by $2ab - 3ac$.
6. $3 + x - y$ by $3 + x - y$.

DIVISION

299. In division we reverse the process of multiplication.

Divide 4 by 2. $4 \div 2 = 2$.

Divide $4a$ by 2. $4a \div 2 = 2a$.

Divide a by b .

$a \div b = \frac{a}{b}$. The result can only be indicated.

$$2^2 = 2 \times 2; 2^2 \div 2 = 2 \times 2 \div 2 = 2.$$

$$a^2 = a \times a = a^{1+1}.$$

How do we multiply literal quantities?

In dividing a^2 by a we subtract the indices; $2 - 1 = 1$.

$$a^2 \div a = a^{2-1} = a^1 = a.$$

Divide a by a . $a \div a = a^{1-1} = 1 a^0 = 1$.

$$\frac{2c}{c} = 2 c^{1-1} = 2 c^0 = 2.$$

$$\frac{2a}{2b} = \frac{2}{2} \times \frac{a}{b} = \frac{a}{b}. \quad \text{Ans.}$$

In dividing literal quantities having coefficients, the coefficients may be divided, while the division of the literal numbers is only indicated.

$$\text{Divide } 4a \text{ by } 2b. \quad 4a \div 2b = \frac{4a}{2b} = \frac{2a}{b}.$$

WRITTEN EXERCISES.

300. Solve:

$$1. \quad \frac{2a^2}{2} =$$

$$5. \quad \frac{50c^3}{10c^2} =$$

$$9. \quad \frac{4x^3y}{2xy} =$$

$$2. \quad \frac{2a^2}{a^2} =$$

$$6. \quad \frac{2ab}{2a} =$$

$$10. \quad \frac{9ca^2}{3ca^2} =$$

$$3. \quad \frac{36a}{4a} =$$

$$7. \quad \frac{2a^2b^2}{2ab} =$$

$$11. \quad \frac{15bc^4}{5bc} =$$

$$4. \quad \frac{48b^2}{4b} =$$

$$8. \quad \frac{3a^2c}{3a} =$$

301. NEGATIVE QUANTITIES IN DIVISION.

$$a^3 \div a = a^2; \quad -a^3 \div -a = a^2.$$

$$a^4 \div -a^2 = -a^2; \quad -a^3 \div a = -a^2.$$

In division, if both dividend and divisor are either positive or negative, the quotient is positive; if one is positive and the other negative, the quotient is negative.

WRITTEN EXERCISES

302. Divide:

- | | | |
|--------------------|--------------------|-----------------------|
| 1. $2x + x.$ | 4. $-6x \div -3.$ | 7. $8c \div -2c.$ |
| 2. $3a \div 3.$ | 5. $14b \div 7.$ | 8. $15x \div -3.$ |
| 3. $-4y \div -26.$ | 6. $18ax \div 9a.$ | 9. $14axy \div -2xy.$ |

303. $3a + 6ab - 9a^2$ by $3a$.

$$\begin{array}{r} 3a \overline{) 3a + 6ab - 9a^2} \\ 1 \quad + \quad 2b - 3a. \end{array}$$

If the dividend contains several terms, divide each separately and give each quotient its proper sign.

Divide: $a^2 + 2ab + b^2$ by $a + b$.

$$\begin{array}{r} a + b \overline{) a^2 + 2ab + b^2} \\ \underline{a^2 + ab} \\ ab + b^2 \\ \underline{ab + b^2} \\ 0 \end{array}$$

If each dividend and divisor has more than one term, divide as in long division in arithmetic, giving each term of the quotient its proper sign.

WRITTEN EXERCISES

304. Divide:

- 1.
- $a^3 + 3a^2b + 3ab^2 + b^3$
- by
- $a + b$
- .

$$\begin{array}{r} a + b \overline{) a^3 + 3a^2b + 3ab^2 + b^3} \\ \underline{a^3 + a^2b} \\ 2a^2b + 3ab^2 \\ \underline{2a^2b + 2ab^2} \\ ab^2 + b^3 \\ \underline{ab^2 + b^3} \\ 0 \end{array}$$

2. $a^2 - b^2$ by $a + b$; by $a - b$.
 3. $a^3 + 3a^2b + 3ab^2 + b^3$ by $a^2 + 2ab + b^2$.
 4. $x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$ by $x + y$.
 5. $4a^2 + 8ab + 4b^2$ by $2a + 2b$.

EQUATIONS

ORAL EXERCISES

305. Read the following equations:

- | | |
|-------------------------------|-------------------------------------|
| 1. $4 + 5 = 9.$ | 8. $\frac{1}{2}^8 = \frac{2}{8}^4.$ |
| 2. $3 + 4 = 2 + 5.$ | 9. $2 \times 3 = \frac{1}{8}^8.$ |
| 3. $6 - 2 = 7 - 3.$ | 10. $\frac{4}{8}^8 = 2 \times 4.$ |
| 4. $8 + 1 = 12 - 3.$ | 11. $\frac{5}{6}^9 = 40 \div 4.$ |
| 5. $20 - 5 = 11 + 4.$ | 12. $30 \div 5 = 2 \times 3.$ |
| 6. $6 \times 2 = 3 \times 4.$ | 13. $3 \times 4 = 6 + 6.$ |
| 7. $25 \div 5 = 15 \div 3.$ | 14. $8 - 2 = 3 \times 2.$ |

306. In each of the above expressions the quantities separated by the sign $=$ are equal.

Each of the expressions is called an **equation**.

An equation indicates the equality of two quantities.

In equation (2) the sum of 3 and 4 = the sum of 2 and 5.

$3 + 4$ and $2 + 5$ are called **members** of the equation.

Name the members of the other equations.

Subtract 4 from each member of the second equation.

The answer is $3 = 2 + 1$. Is this an equation?

Add 3 to each member of equation (3).

You have $6 - 2 + 3 = 7 - 3 + 3$. Is this an equation?

Subtract 5 from each member in equation (4). Perform the operations indicated. Have you still an equation?

Multiply each member of equation (6) by 2, thus:

$$6 \times 2 \times 2 = 3 \times 4 \times 2. \text{ Is the result an equation?}$$

Divide each member of equation (4) by 3, thus:

$$\frac{8 + 1}{3} = \frac{12 - 3}{3}.$$

Perform the operations. Have you still an equation?

Does multiplying or dividing each member of an equation by the same number affect the equality?

WRITTEN EXERCISES

307. Perform the operations indicated in section 305 and see if the result in each case is an equation:

1. Add 3 to each member of equations (6), (7), and (8).
2. Subtract 4 from each member of equations (9) and (10).
3. Multiply by 2 each member of equations (11) and (14).
4. Divide by 3 each member of equations (12) and (13).

UNKNOWN QUANTITIES

308. In problems stated as equations the unknown quantity, that is, the number to be found, is often represented by one of the last letters of the alphabet, as x , y , or z .

Thus, $a + b + c = x$, letting $a = 2$, $b = 3$, and $c = 4$, would read $2 + 3 + 4 = x$.

Adding, we find $2 + 3 + 4 = 9$. Therefore in this example $x = 9$.

$2b - c = x$. That is, $2 \times 3 - 4 = x$.

Solving: We have $6 - 4 = 2$.

$x = 2$ in this example.

WRITTEN EXERCISES

309. Find the value of x in each of the following: if $a = 2$, $b = 3$, $c = 4$, $d = 5$:

1. $2a + 3b - c = x$.

2. $2d + c = x$.

3. $ab + cd - 2a = x$.

4. $\frac{c}{a} = x$.

5. $\frac{a+c}{b} = x$.

6. $\frac{ad+c}{c} = x$.

7. $\frac{3c}{b} + d = x$.

8. $\frac{2d-a}{c} = x$.

9. $\frac{4b+c}{a} - d = x$.

SOLVING EQUATIONS

310. FINDING THE VALUE OF AN UNKNOWN QUANTITY.

Find the value of x in

$$x + 5 = 10.$$

A short method of subtracting 5 from both members is to move the 5 to the other member with its sign changed, thus:

$$x = 10 - 5 = 5.$$

Find the value of x in $x - 3 = 6$.

Moving 3 to the other member with its sign changed, gives:

$$x = 6 + 3 = 9.$$

Moving a term from one member of an equation to the other, with its sign changed, is called **transposition**.

Transposing terms in this way does not affect the equality.

311. Find the value of x in the following equations :

$$4x - 10 = 3x + 6.$$

Transposing $3x$ from the second member of the equation to the first, we have

$$4x - 10 - 3x = 6, \text{ or } x - 10 = 6.$$

Transposing 10 to the second member, we have

$$x = 6 + 10. \quad x = 16.$$

Both transpositions may be made at once, thus:

$$4x - 3x = 6 + 10. \quad x = 16.$$

PROOF: $x = 16$; $4x = 64$; $64 - 10 = 48 + 6$; $54 = 54$.

Finding the value of the unknown quantity in an equation is called **solving the equation**.

To solve an equation, first place the known quantities in one member and the unknown in the other. Then combine the unknown quantities into a single quantity and divide both members by its coefficient.

WRITTEN EXERCISES

312. Solve the following equations and state what you do to both members of the equation in each problem :

- | | | |
|-------------------|---------------------|--------------------|
| 1. $4 + x = 16.$ | 6. $x + 20 = 45.$ | 11. $20 = 10 + x.$ |
| 2. $x - 5 = 18.$ | 7. $20 + x = 25.$ | 12. $35 = 20 + x.$ |
| 3. $x - 12 = 20.$ | 8. $x - 8 = 9.$ | 13. $18 + x = 28.$ |
| 4. $8 + x = 24.$ | 9. $14 + x = 36.$ | 14. $28 = 15 + x.$ |
| 5. $40 + x = 50.$ | 10. $x + 20 = 100.$ | 15. $50 = 20 + x.$ |
16. If $x = 8$, what is the value of $5x$?
17. If $x = 20$, what is the value of $3x$?
18. If $x = 10$, what is the value of $8x$?

How did you find the value of x in Examples 16, 17, and 18?

19. If $5x = 40$, what is the value of x ?
20. If $3x = 60$, what is the value of x ?
21. If $8x = 80$, what is the value of x ?

313. If $\frac{1}{2}x = 5$, 2 times $\frac{1}{2}x = \frac{2}{2}x$, or $x = 2$ times $5 = 10$.

If $\frac{1}{3}x = 6$, what is the value of x ?

If $\frac{1}{4}x = 3$, what is the value of x ?

What has been done to both members of the equation to give an equation whose first member is x ?

Both members of an equation may be multiplied or divided by the same number without destroying the equality.

Give and solve an equation in which the value of x may be found by addition.

Give and solve an equation in which the value of x may be found by subtraction.

Give and solve an equation in which the value of x may be found by multiplication.

Give and solve an equation in which the value of x may be found by division.

WRITTEN EXERCISES

314. Solve the following equations :

1. $20 + x = 41$.
2. $x - 30 = -6$.
3. $x + 15 = 12$.
4. $x + 7 = -8$.
5. $20 - x = -10$.
6. $2x + x = 30$.
7. $3x - 10 = 2$.
8. $4x + 12 = 4$.
9. $5x - 15 = 5$.
10. $14 - x = 13$.

Transpose x when, in the final result, it would be negative if not transposed.

11. $4x - 21 + 3x = ?$
12. $10x + 16 - 7x = ?$
13. $16x - 4 - 6x + 8 = 34$.
14. $4x - 16 = 2x + 16$.
15. $15x - 15 - 3x + 5 = 4x + 6$.
16. $20 - 3x + 15 - 2x = 6 - 4x$.
17. $7 + 7x - 8 - 8x = -2$.
18. $2 + 3 - 18 - 30 = -11x$.
19. $x + 3 - 30x + 40 - 6x - 5 = 3x$.
20. $20 + 1 - 6x + 2 - 7x - 3 = 13 + 6x$.

SOLUTION OF PROBLEMS BY EQUATIONS

315. How many dollars added to \$10 will give \$15?

Let x = the number of dollars that added to \$10 will give \$15. Then $\$10 + x = \$15$. $\$10 + x = \$15$ is an equation.

What is the first member of this equation? The second?

If 10 is subtracted from each member of the equation, will the equality be preserved? What will the equation be?

What have we found the value of x to be?

If 16 be added to a number, it will equal 85. What is the number?

Let x = the number.

Then $x + 16 = 85$.

Subtracting 16 from both numbers; that is, transposing 16, $x = 69$.

WRITTEN EXERCISES

316. Solve:

1. Henry bought 27 marbles. He then had 100 marbles. How many had he at first?

2. After losing 30 marbles Fred had 115 left. How many had he at first?

3. The number of telephone calls allowed for a certain amount a year is 600. 375 have been made. How many calls are still due?

4. 9 times the number of girls in a class is 288. How many girls are in the class?

5. Jane was hunting eggs. Had she found 35 more she would have had 12 dozen. How many did she find?

6. A fox pursued by a hound has run 310 rods, which is 42 rods more than the hound has run. How far has the hound run?

7. 4 times the distance around a certain block in a city is 64 rods. What is the distance around the block?

8. 8 times the distance around a race track is 280 rods. How far is it around the track?

9. If 3 times a certain number be added to it, the sum will equal 36. What is the number?

10. After buying $\frac{1}{2}$ as much wheat as he raised, Mr. Huyer has 450 bushels. How many bushels did he raise?

11. Five times a number, less 40, equals three times the number, plus 20. What is the number?

12. If one half of a number be added to itself, the sum will be 48. What is the number?

13. Three times Mr. Holbrook's money, less 10 dollars, equals his money plus 25 dollars. How much money has he?

14. A peach grower bought of one man as many peaches as he had of his own ; of another one half as many. He then had 2,000 baskets. How many did he buy ?

15. Six times the distance run by some boys in a race, less 60 rods, was 4 times the distance, plus 30 rods. How far did they run ?

16. After selling 50 sheep, a farmer had three fifths of his flock left. How many sheep had he ?

17. Luther said, "If I had 60 cents more than I have, I should have three times as much as I now have." How much has he ?

18. Amos said if he had twice as much money and one half as much as he now had, he would have \$500. How much has he ?

19. A farmer raised a certain number of bushels of rye, twice as many bushels of oats, and three times as many bushels of wheat. He had 1,200 bushels in all. How many bushels of each had he ?

20. Horace has 15 cents more than Charles. If each is given 5 cents, Horace will then have twice as much as Charles. How many cents have each ?

21. A mason and a plumber together earn \$9 a day. The mason works 18 days and the plumber 23 days. They earn together \$182. What does each earn per day ?

22. Mrs. Lynn paid 72 cts. for flour and sugar, buying the same number of pounds of each. For the flour she paid 3 cts. per pound and for the sugar 6 cts. per pound. How many pounds of each did she buy ?

23. Mr. Leonard's wages were \$4 per day and his son's wages were \$2 per day. Both worked the same number of

days during March. The amount of their earnings was \$144. How many days did each work?

24. The earnings of Mr. Danforth and his son amounted to \$180. Mr. Danforth worked three times as many days as his son. Mr. Danforth's wages were \$5 per day and his son's \$3 per day. How many days did each work?

317. FRACTIONAL EQUATIONS.

Find the value of x in the equation $\frac{x}{2} = 4$.

$\frac{x}{2}$ is the same as $\frac{1}{2}$ of x , which equals $\frac{1}{2} \times x$, or $\frac{1}{2}x$.

That is, in $\frac{x}{2}$, the coefficient of x is $\frac{1}{2}$.

The equation, then, is $\frac{1}{2}x = 4$.

(1) When x , having a coefficient, stands alone as one member of an equation, its value is found by dividing both members by the coefficient.

In $\frac{1}{2}x = 4$, divide both numbers by $\frac{1}{2}$.

We have $(\frac{1}{2} \div \frac{1}{2})x = 4 \div \frac{1}{2}$; or,

$$(\frac{1}{2} \times \frac{2}{1})x = 4 \times \frac{2}{1}.$$

$$x = 8.$$

(2) A shorter way to find the value of x is to multiply each member of the equation by the denominator. Thus (remembering that $\frac{1}{2}x = \frac{x}{2}$):

$$\frac{x}{2} = 4.$$

$$\frac{x}{2} \times 2 = 4 \times 2; \quad \frac{2x}{2} = 8.$$

$$x = 8.$$

(3) If there is a fraction in both members find a common denominator, and multiply both members by it.

Find the value of x in the equation $\frac{x}{2} = \frac{2}{3}$.

Here both members are fractions.

$$\begin{array}{r} 3 \quad 4 \\ 6x = \frac{12}{2} \\ 3x = 4. \\ x = \frac{4}{3} = 1\frac{1}{3}. \end{array}$$

Removing the fractions from an equation is called **clearing the equation of fractions**.

Make a rule for clearing an equation of fractions.

WRITTEN EXERCISES

318. Solve:

1. $\frac{x}{2} = 30.$

2. $\frac{x}{3} = 20 - 2x.$

3. $\frac{2x}{4} = 15.$

4. $\frac{3x}{5} = 24 - x.$

5. $\frac{5x}{7} = 5 - x.$

6. $\frac{x}{4} = \frac{6}{12}.$

7. $\frac{x+3}{5} = \frac{4}{5}.$

8. $\frac{2x-2}{4} = \frac{1}{2}.$

9. $\frac{3x-6}{8} = \frac{3}{4}.$

10. $\frac{2-x}{16} = \frac{1}{16}.$

11. $\frac{x+2}{3} = \frac{x-5}{2}.$

12. $\frac{2x-1}{4} = \frac{3x-2}{4}.$

13. $\frac{1+5x}{6} = \frac{3+x}{2}.$

14. $\frac{4x-6}{2} = \frac{2x+9}{5}.$

15. $\frac{5x+3}{7} = \frac{6x+6}{9}.$

16. $\frac{1}{x+2} = \frac{2}{6}.$

17. $\frac{3}{2x-1} = \frac{4}{x+2}.$

18. $\frac{4}{5x+8} = \frac{5}{7x+7}.$

319. SOLUTION OF PROBLEMS INVOLVING FRACTIONAL EQUATIONS.

The sum of two numbers is 320, and the quotient of the greater number divided by the less is 3. What are the numbers?
Ans. 240 and 80.

SUGGESTION :

x = smaller.

$320 - x$ = larger.

WRITTEN EXERCISES

320. Solve :

1. Mr. Reed paid for his carriage $\frac{5}{8}$ as much as he paid for his horse. The horse cost \$64 more than the carriage. What was the cost of each?

2. Five times Anne's money increased by $\frac{4}{5}$ of her money is equal to \$464. How much money has she?

3. The difference between $\frac{3}{4}$ of a man's age and $\frac{2}{3}$ of his age is 4 years. How old is he?

4. Mr. Hudson, Mr. Childs, and Mr. Lewis engaged in business with a capital of \$14,000. Mr. Childs put $\frac{1}{2}$ as much money in the business as Mr. Hudson, and Mr. Lewis put in $\frac{1}{2}$ as much as Mr. Childs. What part of the capital did each provide?

HINT : x = Mr. Lewis's money.

5. The weight of a team with a load of coal is 5500 lb. The team weighs $\frac{4}{5}$ as much as the coal, and the wagon weighs $\frac{1}{2}$ as much as the team. What is the weight of each?

6. Harry and his sister Celia have together \$.64. Celia has 12 cts. more than Harry. How much have each.

7. Mr. Kennedy sold two plots of land containing 80 acres in all. One lot is 10 acres larger than the other. What is the size of each plot?

8. In a school of 1,200 pupils the girls outnumbered the boys by 120. How many were there of each?

9. Mr. Harris spent \$2 for 5-cent stamps, 2-cent stamps and postal cards. He bought 30 more 2-cent stamps than 5-cent stamps, and 30 more postal cards than 2-cent stamps. How many did he buy of each?

Let x = number of 5-cent stamps.

$x + 30$ = number of 2-cent stamps.

$x + 60$ = number of postal cards.

Then $5x + 2(x + 30) + x + 60 = \$2.$

10. The sum of two numbers is 30. Three times the first number added to four times the second is 104. What are the numbers?

x = the first number.

$30 - x$ = the second number.

$$3x + 4(30 - x) = 104.$$

11. There are 865 pupils in a school. The girls exceed the boys by 15. What is the number of each?

12. \$2,400 is divided among three children. The second receives twice as much as the first, and the third receives five times as much as the first. What is the share of each?

13. 175 is the difference between six times a certain number and eleven times the same number. What is the number?

14. The sum of three numbers is 240. The second is double the first and the third is four times the sum of the other two. What are the numbers?

15. Frank and Edward had together 72 marbles. Edward gave Frank four of his marbles. Frank then had twice as many as Edward. How many marbles had each?

16. The sum of the ages of a father and a son is 68 years. In 6 years the father's age will be three times the age of the son. What is the age of each?

17. On Saturday, Sept. 14, 1907, Mr. Dalrymple deposited 45 bills in a bank. Some of them were \$5 bills and the others \$10 bills. The amount deposited was \$325. How many of each did he deposit?

18. Mr. Macy sold 4 baskets of apples and 8 baskets of peaches for \$13.80, and 8 baskets of apples and 4 baskets of peaches for \$9.60. How much did he receive for a basket of apples? Of peaches?

19. The ages of Mr. Matthews and his son together make 98 years. Ten years ago Mr. Matthews was twice as old as his son. What is the age of each now?

SUMMARY OF CHAPTER VI

HOW TO SOLVE A PROBLEM.

Stating Problems.

Solving by Analysis.

Solving by Ratio.

Solving by the Equation.

ELEMENTS OF ALGEBRA.

Solving by the Use of Letters.

Positive and Negative Quantities.

Addition.

Subtraction.

Multiplication.

Division.

Unknown Quantities.

Equations.

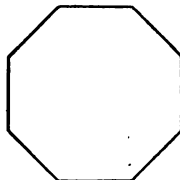
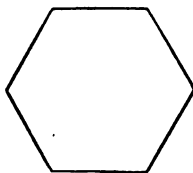
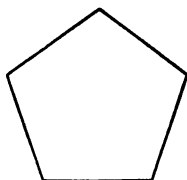
CHAPTER VII

MENSURATION

PLANE FIGURES

POLYGONS

321. What is a plane figure? Name five plane figures.
A plane figure bounded by straight lines is called a **polygon**.
Squares, triangles, hexagons, are examples of polygons.



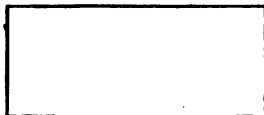
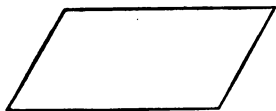
A polygon is named from the number of its sides or angles.

How many sides has a triangle? A quadrangle? A pentagon? A hexagon? An octagon? How many angles has each figure named?

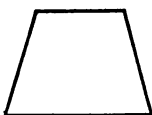
In previous lessons, the following rules for finding the areas of plane figures have been learned:

1. The area of a triangle is equal to $\frac{1}{2}$ the product of the base by the altitude.

2. The area of a rectangle is equal to the product of the length by the breadth.



3. The area of a parallelogram is equal to the product of the base by the altitude.

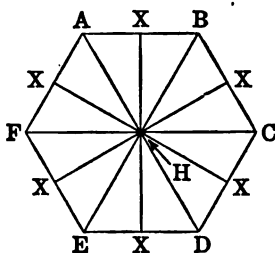
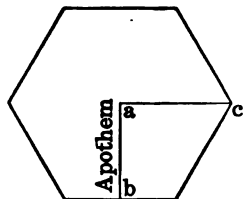


4. The area of a trapezoid is equal to $\frac{1}{2}$ the sum of the two parallel sides multiplied by the altitude.

A polygon having all its sides equal and all its angles equal is called a **regular polygon**.

Equilateral triangles and squares are examples of regular polygons.

A line drawn from the center of a regular polygon, perpendicular to any side, is called its **apothem**. The line AB is the apothem of this hexagon. The line AC is the radius of the hexagon. Make a definition for the radius.



322. Into how many equal triangles do the radii of the hexagon divide it?

The apothem is the altitude of each triangle. How do you find the area of each triangle?

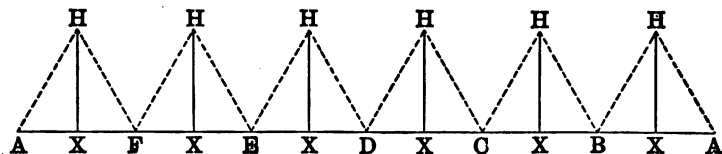
Area of the triangle AHB = $AB \times \frac{1}{2} HX$.

Area of the hexagon

$$= (AB + BC + CD + DE + EF + FA) \times \frac{1}{2} HX$$

$$= \text{the perimeter, } ABCDEF \times \frac{1}{2} HX, \text{ or}$$

$$\frac{1}{2} \text{ of } ABCDEF \times HX.$$



The area of a regular polygon = $\frac{1}{2}$ the product of its perimeter by its apothem.

WRITTEN EXERCISES

323. Solve :

1. Find the area of a regular hexagon 6 ft. on a side, its apothem being 5.2 ft.

2. An eight-sided fountain is 9 ft. on a side. The perpendicular distance from the center to the middle of each side is 10.86 ft. What is the area of the fountain?

3. A barn is 40 feet wide. The distance from the peak to the beams or plate is 14 feet. What is the area of the gable end? Draw a figure showing the gable end.

4. A department store 60 x 40 feet has four floors. What is the total floor area?

5. A piece of land in the form of a parallelogram contains 2 acres. It is 20 rods between the two parallel sides. What is the length of the other two sides?

6. A tapering board is 12 feet long. It is 14 inches wide at one end and 8 inches at the other. How many square feet does it contain?

7. The new park is in the shape of a trapezoid. One of the parallel sides is $\frac{1}{2}$ mile long, the other 120 rods. It is 85 rods in width. How many acres does it contain?

8. The cellar door is $5 \times 3\frac{1}{2}$ ft. How many square feet of boards in it?

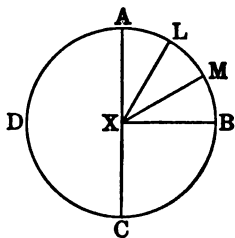
9. It slopes from the house, the upper end being 3 ft. above the ground. How many square feet in the triangular supports on each side, the base lines being 4 ft.?

10. Draw diagrams of the top and the supports on the scale of $\frac{1}{2}$ in. to 1 ft.

11. The pages of a book are $7\frac{1}{2}$ in. $\times$ 5 in. There is a margin of 1 in. at top and bottom and of $\frac{3}{4}$ in. on each side. What is the length of a line of print? A page?

MEASUREMENT OF ANGLES

324. The circumference of every circle, large or small, is supposed to be divided into 360 equal parts, called **degrees** ($^{\circ}$); each degree into 60 **minutes** ($'$); and each minute into 60 **seconds** ($''$).



Thus: the circumference of the circle $ABCD = 360^{\circ}$.

A half of the circumference $AC = 180^{\circ}$.

A quarter of the circumference $AB = 90^{\circ}$.

What kind of angle is AXB ? What part of the circumference measures it? How many degrees is it? How many degrees in a right angle?

All angles are measured in degrees, minutes, and seconds, by the parts of the circumference opposite them, the center being at the apex of the angle.

A part of the circumference of a circle is called an **arc**.

Thus, the angle AXB is an angle of 90° and is measured by the arc B .

The angle AXM is an angle of 60° . Name its arc.

The angle AXL is an angle of 30° .

TABLE

60 seconds = 1 minute; $60'' = 1'$

60 minutes = 1 degree; $60' = 1^{\circ}$

360 degrees = 1 circle

4 right angles, $360^{\circ} = 1$ circle

What are degrees of longitude?

Where is the apex of the angle that is measured by a degree of longitude?

CIRCLES

325. A circle is a plane figure bounded by a curved line, every point of which is equally distant from a point within called the center.

Draw a circle. What is its circumference? Draw a radius; a diameter.

What is the ratio of the circumference of a circle to its diameter?

Explain the formula $c = 2\pi r$.

What does π equal?

1. What is the circumference of a circle whose diameter is 6 in.?

SOLUTION: $c = d \times \pi$.

$$3.1416 \times 6 \text{ in.} = 18.8496 \text{ in., circumference.}$$

2. What is the circumference of a circle whose radius is 2.5 in.?

SOLUTION: $c = 2r \times \pi$.

$$2 \times 2.5 \times 3.1416 = ?$$

What is the radius of a circle whose circumference is 15.708 in.?

SOLUTION: $c \div \pi = 2r$.

$$15.708 \div 3.1416 = 5 = 2r.$$

$$5 \div 2 = 2\frac{1}{2} = r.$$

WRITTEN EXERCISES

326. Solve:

	GIVEN	FIND
1.	$c = 47.124 \text{ ft.}$	d .
2.	$c = 78.53 \text{ in.}$	r .
3.	$r = 12 \text{ in.}$	c .
4.	$d = 16 \text{ ft.}$	c .
5.	$r = 10.5 \text{ in.}$	c .

6. The bottom of my ink bottle measures 3 in. across. What is its circumference?

7. The minute spaces on the tower clock face are 1 ft. apart. What is the diameter of the face?

8. What is the diameter of a circular one-mile race track? Of a water main 3 ft. in circumference?

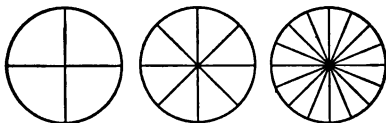
9. Miss Jewett had a new circle painted on the floor of the kindergarten, large enough to provide room for 25 children, allowing 2 ft. for each child. What was the diameter of the circle?

10. Ten times around the track in Madison Square Garden, New York, make 1 mile. What is the diameter of the track?

11. A smokestack $3\frac{1}{2}$ ft. in diameter is made of sheet iron. How wide must the sheets be cut to make it, allowing 10 in. for lapping?

AREA OF A CIRCLE

327. Draw a circle having a radius of 3 in. and divide it into 16 equal parts by drawing 8 diameters at right angles, thus:—



Each section may be divided into still smaller sections.

It is plain that the circumference may be thought of as unrolled into a straight line, and hence as made up of any number of very short straight lines, which may be arranged in any form of polygon.

Hence a circle may be thought of as a regular polygon having a great number of very short sides, with the radius for its apothem.

How do you find the area of a regular polygon?

Area of a regular polygon = $\frac{1}{2}$ perimeter $\times$ apothem.

Hence, *area of a circle* = $\frac{1}{2}$ *circumference* $\times$ *radius*.

Circumference = $2\pi \times r$.

Area = $\frac{1}{2} 2\pi \times r \times r = \frac{1}{2} 2\pi \times r^2 = \pi \times r^2$, or πr^2 .

The area of a circle is found by multiplying the square of the radius by 3.1416.

Find the area of a circle of 3" radius.

$3''^2 = 9$ sq. in.

9 sq. in. $\times$ 3.1416 = 28.2744 sq. in., area.

WRITTEN EXERCISES

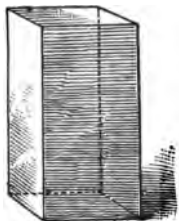
328. Solve:

1. Find the area of a circle whose radius is 4 in.
2. Find the area of a circle whose diameter is 10 in.
3. How many square feet of boards are required to floor a circular summer house whose diameter is 16 ft.?
4. How many square feet are there in the surface of a circular pond having a circumference of 6,854 ft.?
5. How many square feet in the surface of the stump of a redwood whose diameter is 4 ft. 6 in.?
6. How many square feet in a right triangle whose base is 17 in. and altitude 21 in.?
7. What is the area of a cross section of a 1-inch pipe?
8. What is the area of a cross section of a 2-inch pipe? Note that the area of the cross section of the 2-inch pipe is 4 times as great as that of the 1-inch pipe.
9. What is the area of a cross section of a 3-inch pipe? This is 9 times as great as the area of a cross section of the 1-inch pipe. How many times as great would a 4-inch pipe be? A 5-inch pipe?

MEASUREMENT OF SOLIDS

329. A solid is a figure having 3 *dimensions*,—length, breadth, and thickness.

The space occupied by a solid is called its **solid contents** or **volume**.



Parallelopipeds. A solid having all its edges straight and all its sides parallelograms is called a **parallelopiped**.

A square box, the schoolroom, a book, are **parallelopipeds**.

The solid contents of a parallelopiped are found by multiplying together its three dimensions.

PRISMS

330. A prism is a solid whose sides are parallelograms and whose two ends, or bases, are equal polygons.

A prism is named from the shape of its bases, as, *triangular*, *square*, *hexagonal*, *octagonal*.

The altitude of a prism is the perpendicular distance between the bases.

A prism whose altitude is at right angles to its bases is called a **right prism**.

The total area of the sides is called the **convex surface**.

The area of the convex surface plus the area of the bases is the total area.

WRITTEN EXERCISES

331. Solve :

1. How many cubic feet in a stick of timber 40 ft. long, 9 in. wide, and 5 in. thick?
2. Name some objects that are good examples of regular parallelopipeds.
3. Find the cubical contents of one.

Area of the Surface of a Prism.

How many square inches in the convex surface of the prism represented by this cut which is drawn on the scale of 1 in. to 6 in.?

How many square inches in each side? In all the sides?

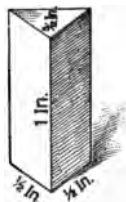
$$3 \times 3 \times 2 = 9 \times 2 = 18. \text{ Ans. } 18 \text{ sq. in.}$$

How many square inches in the ends?

How do you find the area of a triangle?

The area of each end = $\frac{1}{2}$ base $\times$ apothem = $\frac{1}{2} 3 \times 2.6 = 3.9$.

The area of the surface of the prism = 18 sq. in. + 3.9 sq. in. + 3.9 sq. in. = 25.8 sq. in.



Area of surface of a prism = perimeter of base $\times$ altitude $\times$ base $\times$ apothem.

1. Find the area of the surface of a prism 6 ft. high with a base of 2 ft. square.

2. Find the area of the surface of a hexagonal monument in the shape of a prism 10 ft. high and 3 ft. on a side. The apothem of the base is 4.12 ft.

CYLINDERS

332. What is a circle?

Define circumference, radius, diameter.

Explain the formula, circumference = $2\pi r$.

A **cylinder** is a solid having a uniformly curved surface and its ends similar and equal.

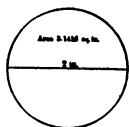
The **altitude** of a cylinder is the distance between the bases.

In regular cylinders the altitude is at right angles to the base.

NOTE. The pupils should each be furnished with a cylinder 2 in. in diameter and 4 in. high; a circular box will answer the purpose.

333. TO FIND THE AREA OF THE SURFACE OF A CYLINDER.

From a sheet of paper cut circles to fit the top and bottom of the cylinder. Then fit a piece of paper around the curved surface. Mark it and cut it out.



To find the area of each end.

How do you find the area of a circle?

$$\text{Area} = \pi r^2.$$



The diameter of the cylinder given is 2 in.

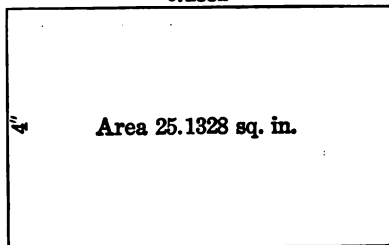
$$r = 1 \text{ in. } r^2 = 1.$$

$$\text{The area} = 3.1416 \times 1 = 3.1416.$$

To find the area of the curved surface.

Spread out the paper that fits around the curved surface of the cylinder. What is its shape?

Its length corresponds with the circumference of the cylinder.
6.2832"



Mark the length on the rectangle.

With what does its width correspond? Mark it on the rectangle.

How do you find the area of a rectangle?

Area of the curved surface of a cylinder = the circumference $\times$ the altitude.

Find the entire area.

3.1416 sq. in. in top.

3.1416 sq. in. in bottom.

25.1328 sq. in. in curved surface.

31.4160 sq. in. in surface of cylinder.

SUGGESTION TO TEACHER. Hold up a large cylinder and give dimensions, as diameter 4", altitude 8". Let the pupils draw to scale.

334. To find the area of the surface of the cylinder.

(1) Find the areas of the ends:

$$\pi r^2 = 2^2 \times 3.1416.$$

$$4 \times 3.1416 = 12.5664.$$

$$12.5664 = \text{sq. in. in top.}$$

$$12.5664 = \text{sq. in. in bottom.}$$

(2) Find the area of the curved surface:

$$12.5664 \text{ in. in circumference.}$$

$$\frac{8 \text{ (height)}}{}$$

$$100.5312 \text{ sq. in. in curved surface.}$$

(3) Find the entire area:

$$12.5664 \text{ sq. in. in top.}$$

$$12.5664 \text{ sq. in. in bottom.}$$

$$100.5312 \text{ sq. in. in curved surface.}$$

$$125.6640 \text{ sq. in. in surface of cylinder.}$$

1. Take the necessary measures of some cylinder. Draw a diagram of it to scale.

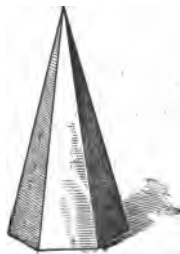
2. Find the areas of the ends; of the curved surface; the area of the entire surface.

THE PYRAMID

335. A *pyramid* is a solid having a polygon for its base and its surface made of triangles meeting at a point, called the **apex**.

The triangles making the surface of a *regular pyramid* are all *isosceles*.

The perpendicular distance from the apex of the pyramid to the base is called its **altitude**. The altitude of one of its triangles is called the **slant height** of the pyramid.



336. TO FIND THE AREA OF THE SURFACE OF A PYRAMID.

How do you find the area of an isosceles triangle?

How, then, can you find the area of the surface of a pyramid?

The area of the surface of a regular pyramid = the perimeter of its base $\times \frac{1}{2}$ its slant height.

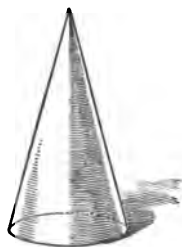
1. How many square feet in the surface of a square pyramid each of whose sides has a base of 3 ft. and a slant height of 6 ft.?

SOLUTION: Perimeter $\times \frac{1}{2}$ slant height $= 4 \times 3 \times 3 = 36$.

2. How many square feet are there in the surface of a pyramid, the base of which has 6 edges, each 30 ft. long, and whose slant height is 20 ft.?

3. A pyramid with a square base is 20 ft. on a side. The slant height is 12 ft. The pyramid is to be covered with paper at \$.05 a square foot. How much will it cost?

THE CONE



337. A cone is a solid having a curved base and tapering evenly to a point called the *apex* or *vertex*.

Most cones have circles for bases and are called *circular cones*.

As the circumference of a circle may be thought of as made up of a very great number of very short straight lines, so the surface of a cone may be thought of as made of a very large number of very narrow isosceles triangles. Hence:



The area of the surface of a cone = the perimeter of its base by $\frac{1}{2}$ its slant height.

WRITTEN EXERCISES

338. Solve:

1. Find the area of the surface of a cylinder having a diameter of 6 inches and an altitude of 8 inches. Represent by drawing.

2. Helen has a cylindrical box 8 inches in diameter and 5 inches high. She wishes to line this with silk and give it to her brother for a collar box on his birthday. How many square inches of silk will she need for the lining if she adds 60 extra square inches for turning in?

3. How many square feet of galvanized iron will it take to make a boiler $2\frac{1}{2}$ feet in diameter and 7 feet long?

4. The Pennsylvania Railroad Company erected a water tank 12 feet in diameter and 22 feet high. How many square feet of sheet iron were required to line it?

5. How many square feet of sheathing does it take to cover a six-sided steeple 5 feet on a side at the base, having a slant height of 75 feet?

6. The fountain in the park is hexagonal, 8 feet on a side. In the winter it is covered with a board cover in the shape of a pyramid whose slant height is 16 feet. How many feet of boards does it contain?

7. Find the area of the surface of a square pyramid, 10 ft. $\times$ 10 ft. at the base, and having a slant height of 18 feet.

8. At 32 cts. a square yard what will it cost to paint a round steeple whose slant height is 80 feet, and its diameter 8 feet at the base?

9. How much cardboard will be required to make a cornucopia 3 inches in diameter at the mouth and 10 inches deep, measured on the outside?

THE SPHERE

339. A sphere is a solid bounded by a curved surface, every point of which is equally distant from a point within called the center.



The **diameter** of a sphere is any straight line passing through the center and ending at opposite points of the surface.

The **radius** of a sphere is any straight line from the center to the surface.

A section of a sphere passing through the center is called a **great circle** of the sphere.

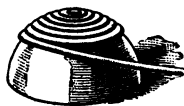
Every great circle of a sphere divides it into two equal parts, or **hemispheres**.

The **circumference** of a sphere is the circumference of one of its great circles.

340. TO FIND THE AREA OF THE SURFACE OF A SPHERE.

NOTE. If possible, get one or more models of hemispheres and wind the curved and flat surfaces, as shown in the picture. Then measure the cords used. It is well to wax the cords.

The length of the cord used to cover the curved surface of a hemisphere, as shown in the picture, is exactly twice that used to cover the flat surface. Hence:



The area of the curved surface of a sphere is 4 times the area of its great circle.

How do you find the area of a circle?

The area of a circle = πr^2 .

The curved surface of a sphere = $4 \pi r^2$.

Find the convex surface of a ball 4 in. in diameter.

SOLUTION:

$$r^2 = 2^2 = 4$$

$$\pi = 3.1416$$

$$4 \times 4 \times 3.1416 = \text{Ans.}$$

WRITTEN EXERCISES

341. Solve:

1. How much leather will it take to cover a push ball 8 ft. in diameter?

2. The gilded ball on the flagpole of one of the high buildings in Seattle is $2\frac{1}{2}$ ft. in diameter. How much will it cost to gild it at $12\frac{1}{2}$ cts. a square foot?

3. How many square feet in the surface of an 18-in. globe?

4. What is the convex surface of a cannon ball 6 in. in diameter?

5. How many square inches of leather does it take to cover a baseball $3\frac{1}{2}$ in. in diameter?

VOLUMES OF SOLIDS

342. Lay cubical 1-inch blocks to form a parallelopiped $3 \times 3 \times 2$ in. How many cubic inches does it contain?

To find the volume of a parallelopiped, multiply together its three dimensions.

WRITTEN EXERCISES

343. Solve:

1. Find the cubic contents of a parallelopiped 6 inches long, 4 inches wide, and 3 inches high.

2. How many cubic inches in a 4-inch cube? In a 5-inch cube? In a 6-inch cube? In an 8-inch cube?

3. What is the ratio of a 6-inch cube to a 3-inch cube?

4. What is the ratio of an 8-inch cube to a 4-inch cube?

VOLUME OF A PRISM

344. Lay four 4-inch cubes in a square. How many cubic inches in the square? $4 \times 1 = 4$.

Lay another row. How many cubic inches have you now? $4 \times 2 = 8$.

Lay three more rows. How many cubic inches have you now? $5 \times 4 = 20$.

What kind of a solid have you made?

How do you find the volume of a regular prism?

Area of base $\times$ altitude = volume.

WRITTEN EXERCISES

345. Solve:

1. A certain pillar is 8 ft. high and 2 ft. square. What is its volume?

2. An office building is 150 ft. high and 80×40 ft. on the ground. What are its solid contents?

3. I desire to build a triangular corner cupboard, 6 ft. high, extending 3 ft. along each wall. What will be its solid contents? Draw a diagram.

SUGGESTION: How do you find the area of a right triangle?

4. The hexagonal base of the Soldiers' Monument is 6 ft. high. Its sides are each 9 ft. and its apothem is 10.86 ft. How many cubic feet of stone were required for its construction?

5. Is a brick an example of a square prism? Is a foundation wall?

6. How many bricks 8 in. $\times$ 4 in. $\times$ 2 in. will be required for a solid wall 10 ft. long, 8 ft. wide, and 8 ft. high?

7. If a person requires 50 cu. in. of air per minute, how long will the air in a schoolroom 30 ft. by 24 ft. by 14 ft. last 39 pupils and the teacher?

VOLUME OF A CYLINDER

346. The figure represents a cylinder whose base has an area of 4 square inches. How many cubic inches in one layer 1 inch high?

If the cylinder is 4 inches high, how many cubic inches does it contain?

$$\begin{array}{l} 4 \text{ cu. in. in 1 layer.} \\ \frac{4 \text{ (layers)}}{16 \text{ cu. in. in the cylinder.}} \end{array}$$

Volume of a cylinder = area of the base $\times$ height (h) = $\pi r^2 \times h$.

Find the volume of a cylinder 2 in. in diameter and 4 in. high.

πr^2 (the area of the base) = 3.1416 sq. in.

4 in. (height) $\times$ 3.1416 = 12.5664 cu. in., solid contents, or volume.

Find the volume of a cylinder having a diameter of 8 in. and altitude of 12 in.

SUGGESTION: $\pi r^2 = 3.1416 \times 16 = 50.2656 =$ sq. in. in base.

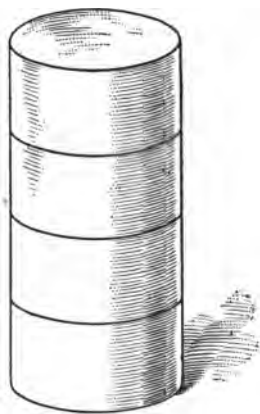
$$\pi r^2 \times h = ?$$

1. A cylinder is 4 in. in diameter and 12 in. high. What is its volume?

2. If its diameter and its height were both doubled, what would the volume be?

3. Which of the following are cylinders: a cheese, stove pipe, a silver dollar, a lead pencil, a piece of pipe?

4. Take the necessary measurements of some cylinder in your schoolroom. Draw a diagram of it to scale, and find its volume.



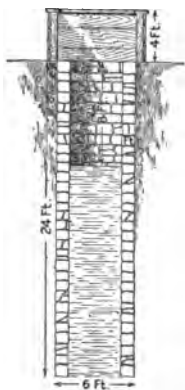
WRITTEN EXERCISES

347. Solve:

1. Find the number of cubic feet of earth that will be removed in digging a cistern 4 feet in diameter and 18 feet deep.

2. A circular pond was 2,500 feet in diameter, and had an average depth of 5.5 feet. How many cubic feet of water did it contain?

3. How many cubic yards of earth will be removed in digging a circular pit 12 feet in diameter and 3 feet deep?



4. In a church there were 12 stone pillars, each 18 feet high and 7.854 feet in circumference. How many cubic feet of stone in all the pillars?

5. How many cubic feet of water in a cylindrical boiler 4 feet in diameter and $6\frac{1}{2}$ feet long?

6. Mr. Westcott dug a well 24 ft. deep and 6 ft. in diameter. How many cubic feet were excavated?

7. He lined it with a stone wall 1 ft. thick. What was the diameter of the well then?

8. When water stands in it 8 ft. from the top, how many gallons does it contain?

9. A square curb is built over the well, 4 ft. high and just reaching to the outside of the stone lining. How many feet of boards are required for the curb?

10. Mr. Horton built a silo* near his barn. It was 10 ft. in diameter and 20 ft. high. How many square feet of lumber were required to cover the sides?

* NOTE. A silo is a cylindrical building used to hold certain kinds of feed for cattle, called *ensilage*.

11. It had a flat tin roof. How many square feet of tin were required to cover it?

12. $\frac{1}{2}$ of the silo was below the surface of the ground. How many cubic feet of earth were excavated for it?

13. How many bushels of ensilage will the silo hold?

A bushel contains 2150.42 cu. in.

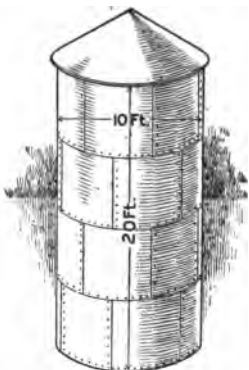
14. How many cubic feet of gas will a pipe 6 in. in diameter and 1 mile long hold?

15. The new gas house is 25 ft. in diameter and 40 ft. high when fully distended. How many cubic feet of gas will it contain?

16. Mary's mother made a cake for her on her tenth birthday. Its diameter was 10 inches and its height 3 inches. What was its circumference? How many square inches in its top? How many cubic inches of cake were there in all?

17. If the cake was cut into eight equal pieces, what was the length of the arc of a circle represented by the rounded part of each piece?

18. How many degrees in the acute angle of each piece?



CAPACITY IN GALLONS *

348. How many cubic inches are there in a gallon?
How many cubic inches are there in a cubic foot?

* Represent 1 gallon and 1 cubic foot, either by having the actual measures, or something about the same size, as a waste basket, hat box, can, etc.

$$231 \overline{) 1728} \text{ cu. in. } \overset{7\frac{111}{231} \text{ times}}{=} = 7\frac{111}{231}, \text{ nearly } 7\frac{1}{2}. \text{ There are ap-}$$

 proximately $7\frac{1}{2}$ gallons in 1 cubic foot. In problems use $7\frac{1}{2}$.

1. How many gallons in 2 cubic feet?
2. How many gallons in 10 cubic feet?
3. How many gallons in 20 cubic feet?

How many gallons of water will a tank hold that is 5 feet in diameter and 6 feet deep?

$$\text{area} = \pi r^2.$$

$$\frac{1}{2} \text{ of } 5 = 2.5, \text{ radius.}$$

$$2.5 \times 2.5 = 6.25 = \text{radius squared.}$$

$$3.1416 \times 6.25 = 19.635000 \text{ sq. ft., area of base.}$$

$$19.635 \text{ sq. ft in base.}$$

$$\underline{6 \text{ (altitude)}}$$

$$117.810 \text{ cu. ft. in tank.}$$

$$7.5 \times 117.81 = 883.575 \text{ gallons of water in the tank.}$$

WRITTEN EXERCISES

349. Solve:

1. How many gallons of water can be let into a cylindrical swimming tank 40 feet in diameter and 8 feet deep? Into a tank $36\frac{1}{2}$ feet in diameter and 9 feet deep?
2. How many gallons of water will be contained by a round boiler that is $3\frac{1}{2}$ feet in diameter and 8 feet high? By one that is 5.5 feet in diameter and 12.4 feet high?
3. How many gallons of oil will the cylindrical tank on an oil wagon hold, if it is 43.9824 feet in circumference and 15 feet long?
4. How many hours will it take to empty a cylindrical tank whose diameter is 10 feet and altitude 12 feet, if the pipe that carries off the water empties 12 gallons per minute?

5. How many gallons of water will a cylindrical cistern contain, whose diameter is 3 feet and depth 15 feet?

6. How many gallons of paint will a cylindrical vat contain that is $6\frac{1}{2}$ feet in diameter and 4 feet deep?

7. The Central Railroad of New Jersey erected a cylindrical water tank to supply their trains. This tank was 12 feet in diameter and 20 feet high. How many gallons of water would it hold?

8. How many gallons of water can be held at one time in a hose whose diameter is 2 inches and length 500 feet? (Use 231 inches to a gallon.)

9. In a factory there are six boilers, each 5 feet in diameter and 9 feet long. How many gallons of water will all six contain?

10. There are $31\frac{1}{2}$ gallons in a barrel. How many barrels will 63 gallons fill? 126 gallons? 315 gallons?

11. How many barrels can be filled with oil from a tank 14 feet in diameter and 7 feet deep?

12. The Northern Pacific Railroad erected a water tower for supplying passing trains. This tower was 47.124 feet in circumference and 22 feet high. How many square feet of sheet iron were required to line it?

13. How many gallons of water will this tower hold?

14. How many cubic feet of ice can be cut from a circular pond 120 feet in diameter, if the ice is frozen to the depth of $2\frac{1}{2}$ feet?

15. A New Orleans mill had a vat 12 feet in diameter and 4 feet deep. How many barrels of molasses could be filled from the contents of this tank?

16. How many cubic feet of earth will be removed in digging a well $3\frac{1}{2}$ feet in diameter and 10 feet deep?

VOLUME OF PYRAMID AND CONE

350. How do you find the volume of a prism? Of a cylinder?

Take a hollow prism and a hollow pyramid with the same base and altitude. Fill the pyramid with sand or water. Empty it into the prism. How many times do you repeat the operation to fill the prism?

Perform the same experiment with a cone and a cylinder of equal base and altitude.

If these measures are not at hand, you can easily make them from cardboard.

The volume of a prism or a cylinder = area of base $\times$ altitude.

The volume of a pyramid or a cone = approximately $\frac{1}{3}$ area of base $\times$ altitude.

WRITTEN EXERCISES

351. Solve:

1. Find the volume of a cone 6 ft. high with a diameter at the base of 4 ft.

2. How many cubic feet are there in a monument in the shape of a triangular pyramid 20 ft. high, the base being 3 ft. on a side?

3. How many loads are there in a conical pile of sand 20 feet across at the base and 10 ft. deep? (1 load = 1 cu. yd.)

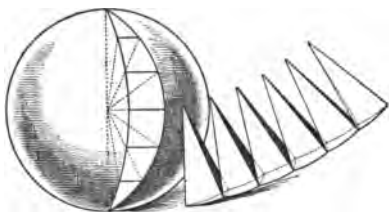
4. How many cubic feet of stone are required to build a solid steeple 40 ft. high with a square base 8 ft. on a side?

5. How many cubic feet are there in a conical pile of coal 150 ft. high and 125 ft. in diameter at the base?

6. How many cubic feet in a conical ant hill 2 ft. deep at the center, and 3 ft. across at the base?

VOLUME OF A SPHERE

352. As suggested in the picture, a sphere may be thought of as made up of a very large number of very small pyramids.



The sum of the bases of these pyramids equals the surface of the sphere.

The altitude of each of them is the radius of the sphere.

How do you find the volume of a pyramid?

How, then, do you find the volume of a sphere?

The volume of a sphere = $\frac{1}{3}$ surface $\times$ radius.

The surface of a sphere = $4 \pi r^2$.

Then $\frac{1}{3} (4 \pi r^2 \times r) = \text{Volume}$.

The volume of a sphere equals $\frac{1}{3}$ its radius $\times$ its surface.

WRITTEN EXERCISES

353. Solve:

1. Find the volume of a baseball 3 in. in diameter; a globe 2 ft. in diameter; 6 in. in diameter; 1 ft. in diameter.

2. What are the solid contents of a ball for bowling, 8 in. in diameter? 1 ft. in diameter?

3. How many times larger is a ball 3 in. in diameter than a ball 1 in. in diameter?

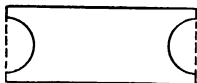
4. How many times larger is a 4-in. ball than a 1-in. ball? Than a 2-in. ball?

5. Find the volume of each of these balls.

REVIEW

354. Solve :

1. What is the approximate diameter of a circle whose circumference is $44''$? (Use $\frac{7}{4}$ for the ratio.)
2. What is the area of a trapezoid whose parallel sides are $11''$ and $17''$, and the distance between them $10''$?
3. What is the area of a right triangle whose base is $16''$ and altitude $30''$?
4. What is the area of an isosceles triangle whose base is $22''$ and altitude $21''$?
5. What is the approximate circumference of a circle whose diameter is $21''$?
6. How many cubic feet of earth will be removed in digging a well $5\frac{1}{2}$ feet in diameter and 15 feet deep?
7. How thick is the trunk of a tree that is 7.854 feet in circumference?
8. Mr. Walsh has a square farm $\frac{1}{4}$ mile on a side. How many acres in the farm?
9. A circular lawn in a park is 120 feet in diameter. This has a border of geraniums $2\frac{1}{2}$ feet wide. How many yards of wire fencing will be required to inclose the geraniums on the outside?
10. (a) How many cubic feet of ensilage will a silo contain if it is 12 feet in diameter and 20 feet high? (b) How many bushels? (There are 2150.42 cubic inches in a bushel.)



11. Each of a class of 16 boys made a fish line reel.

They used this drawing, which is on a scale of 4 in. to 1 in.

- (a) How many square inches of wood were required?
- (b) How many square inches were cut out of the ends of each reel? From all together?

(c) What did the wood cost at \$.04 a square foot.

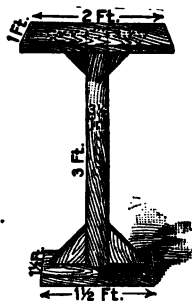
12. A seventh grade manual training class made for the dictionary in the schoolroom a rectangular table, with a single round leg 3 ft. high, on a base.

The top of the table was 2×1 ft.

The base was $1\frac{1}{2} \times 1\frac{1}{2}$ ft.

Four triangular brackets of wood were made, 6 in. on each side about the right angle, to support the top, and a like number to strengthen the base.

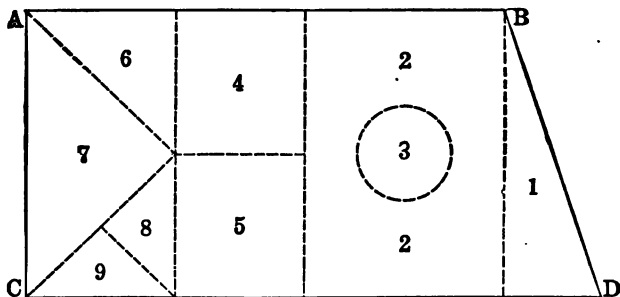
How many square feet of board were used in top, base, and brackets?



13. How many feet of board 1 foot wide were required to make the top, base, and brackets?

14. From a square piece of timber $3\frac{1}{2} \times 3\frac{1}{2}$ in., and of the required length, a round leg $3\frac{1}{2}$ in. in diameter and of the desired length was turned. How much wood was there in the timber? How much in the leg? How much was cut off?

How much did the wood for the table cost, boards being 5 cents a foot and the timber 5 cents a linear foot?



15. This diagram represents Mr. Gordon's farm, drawn on a scale of 1 inch to 80 rods. What figure is represented?

How does it differ from a rectangle?

How do you find the area of such a figure?

16. The line AB represents what fraction of a mile? The line CD? The line AC?

How many acres are there in the farm?

What kind of a figure is 1?

How do you find the area of such a figure?

17. What is its area?

18. The number of acres in this field is what per cent of the number of acres in the farm?

19. Mr. Gordon paid \$14,437.50 for the farm. What was the average price per acre?

20. He paid $62\frac{1}{2}$ per cent cash and gave a mortgage for the remainder. What was the amount of the mortgage?

21. He paid interest semiannually at 5 per cent per year. How much interest did he pay each time?

22. In the center of field 2 there is a lake $\frac{1}{2}$ inch (by scale) in diameter. How many acres in the surface of the lake?

If the average depth of the water is 10 ft. in the lake, what is the approximate number of gallons in it, allowing 7.5 gallons to the cubic foot?

23. How many acres of land in field 2? What per cent of the farm is in the field?

24. What kind of a triangle does field 6 represent? What is its area?

25. What per cent is its area of the area of the farm?

26. Find the sum of the areas of all the fields.

27. What should this sum equal?

28. If the farm is assessed for 75 per cent of its value and taxed at the rate of \$1.65 per hundred of assessed valuation, how much is the tax?

29. If the house on the farm is insured for \$5,000 at the rate of $2\frac{1}{2}$ per cent per year, what is the premium?

30. Field 4 was a wood lot. Mr. Gordon cleared 43.5 per cent of the wood off the lot. How many acres of woods were left?

31. $12\frac{1}{2}$ per cent of the 1,200 trees still standing are oak, $66\frac{2}{3}$ per cent chestnuts, 15 per cent pine, and the remainder maples. How many trees were there of each kind?

32. Mr. Gordon sold a pile of oak wood 48 feet long, 4 feet wide, and 3 feet 6 inches high, for \$6.75 per cord; a pile of chestnut 66 feet long, 4 feet wide, and 4 feet 6 inches high, for \$4.87 $\frac{1}{2}$ per cord; and a pile of pine wood 30 feet long, 4 feet wide, 3 feet 9 inches high, for \$5.25 per cord. How much did he receive for the wood?

33. In the southeast corner of the wood lot he fenced off a plot containing one acre, on which he built his house. This plot was 16 rods long. How wide was it?

34. He inclosed his house lot with a board fence 4.5 feet high, using 308 posts which were worth $18\frac{1}{2}$ cents each. The boards used were worth \$35 per thousand feet. What was the value of the posts and boards used in the fence?

35. He built a silo 12 feet in diameter and 18 feet high. How many cubic feet of fodder would it hold?

36. He covered the bottom and sides with sheet iron. How many square feet were required?

SEEDS REQUIRED PER ACRE FOR PLANTING (APPROXIMATE)

	BUSHELS
Oats	$2\frac{1}{2}$
Rye	$1\frac{1}{2}$
Wheat	$2\frac{1}{2}$

37. What was the area of field No. 1?

38. Field 1 was sowed with oats. The cost of plowing was \$3 an acre. The cost of the seed oats was 50 cts. per bushel. Fertilizer, costing \$30 per ton, was used at the rate of 400 lb. to the acre. Labor for sowing and harrowing was \$2 per acre; for harvesting, \$1 per acre; for threshing, \$1.25 per acre. The crop, 30 bushels per acre, was sold at 40 cts. per bushel. The straw averaged 1 ton per acre, and was sold at \$10 per ton.

What was the profit on the oat field?

OAT FIELD			Dr.	Cr.
Apr.	20	To plowing 15 A., at \$3	45	
"	28	To 3 tons fertilizer, at \$30	90	
"	29	To 37.5 bu. seed oats, at \$.50	18 75	
"	30	To sowing and harrowing, at \$2 per A.	30	
Aug.	10	To harvesting 15 A., at \$1	15	
Oct.	11	To threshing 15 A., at \$1.25	18 75	
Nov.	21	By 450 bu. of oats, at \$.40		180
Dec.	12	By 15 tons of straw, at \$10 per ton		150
		Balance,	112 50	
			330 00	330 00

39. Field 7 was sowed with rye.

The plowing cost \$3 per acre.

The fertilizer cost \$30 per ton, and 500 pounds were used per acre.

The seed rye cost 80 cts. per bushel (see table).

The labor for sowing, harrowing, and rolling cost \$3 per acre.

The harvesting cost \$1.50 per acre and the threshing \$3 per acre.

The yield was 28 bushels per acre, which was sold for

80 cts. per bushel, and $1\frac{1}{4}$ tons of straw per acre which was sold for \$16 per ton.

What was the profit on the rye field?

40. Make and balance the account for the rye field.

41. Stull Brothers, dealers in grain, hay, and feed, have a gasoline engine 35 horse power to run their mill. It takes one tenth of a gallon of gasoline for a horse power to run it an hour, at a cost of \$.13 per gallon. What will the gasoline cost to run the engine 5 hr. 30 min.

42. If the mill grinds 400 pounds of corn in 5 minutes, how long will it take to grind a car load of 600 bushels? (Allow 56 lb. to the bushel.)

43. Lubricating oil, bought in quantities of 5 gallons, costs \$.45 per gallon. If bought by the barrel of 50 gallons, it costs \$.29, 10% off for cash and an allowance of \$.75 for the barrel if returned. What is the cost per gallon, by the barrel, if all the above conditions are complied with?

44. What per cent is gained by purchasing by the barrel instead of in 5-gallon quantities?

45. The driving wheel of the engine makes 235 revolutions per minute, and that of the mill 1,750. The number of revolutions of the mill wheel is what per cent of the number of revolutions of the engine wheel?

46. Equal numbers of bushels of corn weighing 56 lb. per bushel and of oats weighing 32 lb. per bushel were ground together. The mixture weighed 5,280 lb. How many pounds of corn did the mixture contain? Of oats?

47. Mr. Baldwin has 275 bushels of potatoes, in bags of $2\frac{3}{4}$ bushels each. What are the potatoes worth at \$2.25 per bag?

48. A grocer put 315 pounds of sugar in packages of $3\frac{1}{2}$ pounds each, which he sold at 23 cents per package. The sugar cost him $5\frac{1}{2}$ cents per pound. How much did he make?

49. The distance from New York to New London is 110 miles. How long will it take a steamer sailing at the rate of $18\frac{1}{2}$ miles an hour to go from New York to New London?

50. The product of the length and width of a corn crib is equal to $68\frac{1}{2}$ square feet; the cubic contents are $426\frac{1}{4}$ cubic feet. How high is the crib?

51. The length of a room is 30 feet, the width 25 feet, and the cubic contents 9,095 cubic feet. How high is the ceiling?

52. What is the value of 18 tons of coal, at the rate of $4\frac{1}{2}$ tons for \$29.50?

53. If $37\frac{1}{2}\%$ of the number of cubic feet of air in a room is equal to 4,218.75 cubic feet, how many cubic feet of air are there in the room?

54. There are 40 pupils in this room. How many cubic feet of air does that give to each pupil?

55. A grocer sold 18 pounds 12 ounces of butter on Monday; $27\frac{3}{4}$ pounds Tuesday, 36.375 pounds on Wednesday. How many pounds did he sell? What is the value of the butter at \$.26 a pound?

56. What is the sum of 16.5 feet, $25\frac{3}{4}$ feet, $42\frac{2}{3}$ feet, and 15 feet 6 inches?

57. Mr. Hawkins laid .375 of a mile of sidewalk. How much did it cost him at \$13.20 a rod?

58. Mr. Wells bought a barrel of cranberries, containing 3 bushels, at \$2 $\frac{1}{2}$ per bushel. He sold them at 12 $\frac{1}{2}$ cents per quart. How much did he make on the berries?

59. What is the diameter of a sphere whose surface is 50.2656 square miles?

60. If a cannon ball weighs 36 pounds, what will one weigh whose diameter is three times as great?

61. A field in the form of a trapezoid contains 25 acres. One of its parallel sides is 90 rods, the other 105 rods. What is its width?

62. If a circus ring is 75 feet in diameter, how many times must a horse go around it to travel a mile?

63. How many 2-inch spheres does it take to equal in volume a 4-inch sphere? an 8-inch sphere?

SUMMARY OF CHAPTER VII

MENSURATION.

Measurement of Plane Figures.

Measurement of Solids.

Surfaces of Solids.

Volumes of Solids.

CHAPTER VIII

POWERS AND ROOTS

POWERS

354a. $2 \times 2 = 4$; $2 \times 2 \times 2 = 8$; $2 \times 2 \times 2 \times 2 = 16$.

$3 \times 3 = 9$; $3 \times 3 \times 3 = 27$.

In problem 1, how was 4 obtained? 8? 16?

In problem 2, how was 9 obtained? 27?

Note that in each case the product was obtained by multiplying together equal factors.

A product, or number produced by multiplying a number by itself, is called a **power** of that number.

The number itself is called the **first power**.

The product resulting from multiplying the number by itself once is called the **second power** or **square**; that resulting from multiplying the number by itself twice, the **third power** or **cube**; 4 is the second power of 2, and 8 is the third power of 2.

What is the second power of 3? Of 4? Of 6? Of 8?

What is the third power of 3? Of 4? Of 6? Of 8?

$2 \times 2 \times 2$ may be written 2^3 .

3×3 may be written 3^2 .

$5 \times 5 \times 5 \times 5$ may be written 5^4 .

The small figures, 3, 2, and 4, are called exponents.

An **exponent** shows how many times the number is used as a factor. It is written at the right of the number and a little above it. $4^3 = 4 \times 4 \times 4$.

What does 3^5 equal? 6^3 ? 7^2 ? 8^3 ? 9^4 ?

355. POWERS OF NUMBERS TO 9

First power = 1, 2, 3, 4, 5, 6, 7, 8, 9.

Second power = 1, 4, 9, 16, 25, 36, 49, 64, 81.

Third power = 1, 8, 27, 64, 125, 216, 343, 512, 729.

Study this table until you can readily give the second and the third powers of the numbers in the first row.

What number multiplied by itself will produce 36?

What number multiplied by itself will produce 100?

What number multiplied by itself will produce 121?

ROOTS

356. What are the equal factors of 36? Of 49? Of 27?
Of 8? Of 216? Of 64?

A number which is multiplied by itself to produce a given number, or which is one of the equal factors of the number, is called a **root** of that number.

If the root is one of two equal factors, it is called the **square root**; if it is one of three equal factors, it is called the **cube root**.

357. In the problem, $4 = 2 \times 2$, 2 is the square root of 4, because it is one of the two equal factors.

ORAL EXERCISES

358. Find by inspection the square root of:

225	324	1,296	64	900	144
169	625	121	196	256	400

359. In the problem, $8 = 2 \times 2 \times 2$, 2 is one of the three equal factors and is the cube root of 8.

Find the cube root of the following numbers by finding one of their three equal factors:

343, 216, 1,000, 1,728, 2,744, 512.

360. Instead of writing the words square root, cube root, etc., mathematicians use a sign to indicate roots. They use the sign $\sqrt{}$, $\sqrt[3]{}$, $\sqrt[4]{}$. It is called the **radical**, or root sign. When it is used without the figure at the opening, the square root is indicated; when the figure 3 is used, the cube root is indicated. In arithmetic, roots above the cube root are not considered.

$\sqrt{9}$ indicates the square root of 9.

$\sqrt[3]{64}$ indicates the cube root of 64.

361. The following method enables one, in many cases, to determine at a glance the square root of a number. It grows out of the method of multiplying in which every complete product is put down, instead of adding and combining as one multiplies.

ILLUSTRATIONS: 12

12

4 = units squared = u^2 .

20 = tens $\times$ units = $t \times u$.

20 = tens $\times$ units = $t \times u$.

100 = tens squared = t^2 .

144 = units squared + 2(tens $\times$ units) +
tens squared = $u^2 + 2(t \times u) + t^2$.

16

24

16

24

36 = u^2 .

16 = u^2 .

60 = $t \times u$.

80 = t^2 .

60 = $t \times u$.

80 = $t \times u$.

100 = t^2 .

400 = $t \times u$.

256 = $u^2 + 2(t \times u) + t^2$.

576 = $u^2 + 2(t \times u) + t^2$.

NOTE TO TEACHERS: Two methods of finding square root are here given, the "Multiplication method" and the "Geometric method." Teachers should select one and teach that alone, — not both.

FINDING ROOTS

THE MULTIPLICATION METHOD

362. These multiplications teach us how to proceed without factoring if we wish to find the square root of 144, or the number which multiplied by itself makes 144.

It will be seen that 144 is made up of two squares, $10^2 + 2^2$, and twice the product of the numbers making the squares, $2 \times 10 \times 2$. It can be separated into these parts as follows:

ILLUSTRATIONS:	$144 = 100 + 4 + 40.$	TEST: $10^2 = 100$
The square root of 100	$= 10$	$2^2 = 4$
The square root of 4	$= 2$	$2 \times 10 \times 2 = 40$
Their sum	$= \underline{12}$, the square root.	Total $= \underline{144}$

$169 = 100 + 9 + 60.$	TEST: $10^2 = 100$
$\sqrt{100} = 10$	$3^2 = 9$
$\sqrt{9} = 3$	$2 \times 10 \times 3 = 60$
Their sum $= \underline{13}$, the square root.	Total $= \underline{169}$

$625 = 400 + 25 + 200.$	TEST: $20^2 = 400$
$\sqrt{400} = 20$	$5^2 = 25$
$\sqrt{25} = 5$	$2 \times 20 \times 5 = 200$
Their sum $= \underline{25}$, the square root.	Total $= \underline{625}$

$6889 = 6400 + 9 + 400.$	TEST: $80^2 = 6400$
$\sqrt{6400} = 80$	$3^2 = 9$
$\sqrt{9} = 3$	$2 \times 80 \times 3 = 480$
Their sum $= \underline{83}$, the square root.	Total $= \underline{6889}$

WRITTEN EXERCISES

363. Find the square root of the following numbers by the method just given:

1. 361. 2. 729. 3. 7,225. 4. 5,929. 5. 1,296. 6. 1,681.

364.

$$1 \times 1 = 1$$

$$9 \times 9 = 81$$

$$10 \times 10 = 100$$

$$99 \times 99 = 9,601$$

$$100 \times 100 = 10,000$$

$$999 \times 999 = 998,001$$

9, the largest number expressed by one figure, when squared, gives 81, a number of two figures.

10, the smallest number expressed by two figures, when squared, gives a number of three figures; and 99, the largest number expressed by two figures, when squared, gives 9,601, a number of four figures.

100, the smallest number expressed by three figures, when squared, gives 10,000, a number of five figures; and 999, the largest number expressed by three figures, when squared, gives 998,001, a number of six figures.

Therefore, if the number of which the square root is to be found contains one or two figures, there will be one figure in the root; if it contains three or four figures, there will be two figures in the root; if it contains five or six figures, there will be three figures in the root.

ORAL EXERCISES

365. How many figures will there be in the square root of each of the following numbers?

1. 9.

3. 225.

5. 2,597,586.

7. 2,965.

2. 64.

4. 46,765.

6. 34,758,267.

8. 868,675.

366. The first step in extracting the square root of a whole number is to divide it into periods of two figures each, beginning at the right.

In the numbers that you have produced by squaring other numbers, you will observe that the tens squared can always

be found in the left-hand period. If you can determine the square, you can find what the tens' figure is by extracting the square root of this square. For example, in the number 1,024, the greatest square is 900. The square root of 900 is 30, which represents the tens.

ORAL EXERCISES

367. Find the greatest squares and the tens' figure in each:

- | | | | |
|-----------|------------|------------|-------------|
| 1. 121. | 3. 5,929. | 5. 9,601. | 7. 348,767. |
| 2. 2,116. | 4. 14,641. | 6. 21,345. | 8. 3,446. |

368. In the following, after finding the greatest square and its root, divide the number by twice the root and find in the quotient the other root from the formula $u^2 + 2 \times u \times t + t^2$.

169, 576, 1089, 256, 625, 196, 2025, 361, 2500, 1296, 4096.

Extract the square root of 1,024.

SOLUTION: $30 + 2 = 32$, the square root.

$$t^2 + 2(t \times u) + u^2 = 1,024$$

$t^2 = 900$, the greatest square in the left-hand period.

$$\begin{array}{rcl} t & = & 30 \\ 2t & = & 60 \\ 60 \times u + u^2 & = & 124 \\ u & = & 2 \\ 60 \times 2 + 2^2 & = & 124 \end{array}$$

Extract the square root of 2,116.

SOLUTION: $40 + 6 = 46$, the square root.

$$t^2 + 2(t \times u) + u^2 = 2,116$$

$t^2 = 1,600$, the greatest square in the left-hand period.

$$\begin{array}{rcl} t & = & 40 \\ 2t & = & 80 \\ 80 \times u + u^2 & = & 516 \\ u & = & 6 \\ 80 \times 6 + 36 & = & 516 \end{array}$$

Extract the square root of 6,889.

SOLUTION: $80 + 3 = 83$, the square root.

$$t^2 + 2(t \times u) + u^2 = 6,889$$

$t^2 = 6,400$, the greatest square in the left-hand period.

$$t = 80$$

$$2t = 160$$

$$160 \times u + u^2 = 489$$

$$u = 3$$

$$160 \times 3 + 3^2 = 489$$

Extract the square root of 55,225.

SOLUTION: $200 + 30 + 5 = 235$, the square root.

$$t^2 + 2(t \times u) + u^2 = 55,225$$

$t^2 = 40,000$, the greatest square in the left-hand period.

$$t = 200$$

$$2t = 400$$

$$400 \times u + u^2 = 15,225$$

$$u = 30$$

$$400 \times 30 + 30^2 = 12,900$$

$$t = 230$$

$$2t = 460$$

$$460 \times u + u^2 = 2,325$$

$$u = 5$$

$$460 \times 5 + 5^2 = 2,325$$

the portion of the root already found.

the portion of the root already found.

WRITTEN EXERCISES

369. Extract the square root of the following numbers:

1. 6,724.

3. 374,544.

5. 4,624.

2. 60,516.

4. 23,104.

6. 59,136.

If the number of which the square root is to be found is a decimal, begin at the decimal point and divide into periods, both right and left. If necessary, add a cipher to the right

hand period of the decimal so that each period may have two figures. Then proceed as before.

When it is required to extract the square root of a common fraction, extract the square root of the numerator and denominator separately if they are perfect squares. If they are not squares, change the fraction to a decimal before finding the square root.

When it is required to extract the cube root of a common fraction, extract the cube root of the numerator and denominator separately if they are perfect cubes. If they are not perfect cubes, change the fraction to a decimal before finding the cube root.

WRITTEN EXERCISES

370. Solve:

1. $\sqrt{2.56}$. 5. $\sqrt{36.3}$. 9. $\sqrt{.001764}$. 13. $\sqrt{3.285}$.

2. $\sqrt{56.25}$. 6. $\sqrt{\frac{4}{49}}$. 10. $\sqrt{401.2009}$. 14. $\sqrt{\frac{1}{2}}$.

3. $\sqrt[3]{64}$. 7. $\sqrt[3]{125}$. 11. $\sqrt[3]{\frac{1}{27}}$. 15. $\sqrt[3]{\frac{8}{27}}$.

4. $\sqrt{\frac{1}{8}}$. 8. $\sqrt[3]{\frac{1}{84}}$. 12. $\sqrt[3]{216}$. 16. $\sqrt[3]{\frac{64}{125}}$.

17. How many rods of fence will it require to fence one side of a square field containing 10 acres?

18. Find the square root of 1.21.

371. THE GEOMETRIC METHOD.

Draw a line 5 inches long. See diagram, p. 228.

Draw a square upon it.

How many square inches in the square?

$$5 = 3 + 2.$$

What is the square of 3? Of 2?

How much less than 5^2 is $3^2 + 2^2$?

Draw and cut out a 3-inch square; a 2-inch square.

Place them on the 5-inch square, as shown in the diagram.

5 in.	
3 in.	2 in.
$3 \text{ in.}^2 \text{ or } a^2$ 3 in. square 9 sq. in.	$2 \times 3 \text{ in. or } a \times b$ $a \times b$
$2 \times 3 \text{ in.}$ $a \times b$	$(2 \text{ in.})^2 \text{ or } b^2$ 2 in. square 2 sq. in.

Call 3 inches a , and 2 inches b .

How large is each of the sections of 5 in.^2 not covered by 2 in.^2 and 3 in.^2 ?

We see that the squares of two numbers taken together are not equal to the square of their sum.

As each section left over is 2×3 square inches, their sum is $2 \times (2 \times 3)$ square inches.

That is, $5 \text{ in.}^2 = 2 \text{ in.}^2 + 3 \text{ in.}^2 + 2 \times 2 \times 3 \text{ in.}$

Or, the square of 2 and 3 and twice the product of 2 and 3 = the square of 5.

Since $3 \text{ in.} = a$ and $2 \text{ in.} = b$, then $(a + b)^2 = a^2 + b^2 + 2ab$.

Prove it by finding the square of $a + b$ by multiplication.

The square of the sum of two numbers is equal to the sum of the squares of the numbers plus twice their product.

$15 = 10 + 5$. Then $15^2 = 10^2 + 5^2 + 2 \times 5 \times 10 = 225$.

Find the square of 16.

$16^2 = 10^2 + 6^2 + 2 \times 10 \times 6$.

Find at sight the squares of 17, 18, 19, 21, 22, 23, 24, 25.

372. The square root of a number too large to be factored at sight is found by reversing the process of squaring given above; that is, by separating the number into two smaller squares, as a and b , and finding the roots of these separately, making allowance for the $2ab$, or twice the product of the roots.

Find the square root of 196.

Since 196 is larger than the square of 10, it is evident that

the square root contains two figures, a unit's figure and a ten's figure. Call the ten's figure a and the unit's figure b .

Then $196 = \text{the square of } (a + b) = a^2 + 2ab + b^2$.

$a^2 = \text{the ten's figure squared.}$

$b^2 = \text{the unit's figure squared.}$

$2ab = \text{twice the product of } a \times b.$

$196 = 100 + 96 = 100 + 16 + 80.$

$100 = a^2$; $16 = b^2$; $80 = 2ab.$

100 is the square of 10; and 16 is the square of 4.

$2 \times 4 \times 10 = 80.$

The square root of 196 is $10 + 4 = 14.$

As we cannot always tell at sight the parts into which a number may be divided for the purpose of extracting the square root, we simplify the process by dividing it into periods which suggest the proper division.

Thus, in finding the square root of 225, since 225 is larger than the square of 9, we can see that there must be in the root a ten's and a unit's figure. So we mark it thus, 2/25, or thus, 2 25. This means $200 + 25$. The 200 must contain a^2 , the square of the ten's figure of the root. The square of 20 (two tens) is not contained in 200. So the largest ten whose square is contained in 200 is 10 (1 ten).

Taking 10^2 from 225 leaves 125.

$225 - 100 = 125$; $a^2 + b^2 + 2ab - a^2 = b^2 + 2ab.$

125 must contain the square of the unit's figure, plus $2 \times$ the product of the unit by the ten's figure.

To find the unit's figure, we divide 125 ($2 \times 10 \times b$) by (2×10).

$$\frac{(b^2 + 2ab)}{2a} = \frac{125}{20} = 5 \text{ with a remainder of } 25; 25 = 5^2.$$

Hence $125 = 20 \times 5 + 5^2$ ($2a \times b + 5^2$).

That is, $225 = 10^2 + 2 \times 10 \times 5 + 5^2 = (10 + 5)^2$, and $\sqrt{225} = 10 + 5 = 15.$

Observe that $\frac{125}{20} = 6\frac{5}{20}$; that is, the quotient is 6 rather than 5. The reason we use 5 is this: 125 contains not only $2ab$, but b^2 besides ($2ab + b^2$). Hence allowance must be made for the b^2 , the product of the unit's figure by itself.

We commonly do not put down the cipher in the unit's place of either the quotient, or the divisor, but leave the place vacant and write in it the unit's figure when found. Thus:

$$\begin{array}{r} 225 \overline{)1} \\ 100 \\ \underline{2} \end{array}$$

The Process :

$$\begin{array}{r} \overset{'}{2} \overset{' }{25} \overline{)1} \end{array}$$

We divide 225 into 2 periods, 2 (hundred) and 25.

We say: The greatest square in 2 is 1. Put 1 in the root.

Subtract its square, 1, from 2.

$$\begin{array}{r} 225 \overline{)15} \\ 1 \\ \underline{25} \overline{)125} \\ 125 \\ \underline{} \end{array}$$

Double 1 for a trial divisor and divide.

It appears to go 5 times. Annex 5 to 2 and also place it in the root and multiply. 15 is the square root of 225.

1. To find the square root, beginning at the right, divide the number into periods by marking every alternate figure. (If the number is or contains a decimal, begin at the point.)

2. Find the greatest square in the first, or left-hand period. Place its square root in the quotient, subtract the square, and bring down the next period.

3. Double the root found for a trial divisor. Find a trial quotient and place it in the unit's place of the quotient, and also of the trial divisor.

4. Multiply the trial divisor by the trial quotient and subtract the product. (If the subtrahend is larger than the minuend, try again.) Bring down the remainder, if any, and proceed as before.

WRITTEN EXERCISES

373. Solve:

1. What is the length of 1 side of a square rug containing 64 square yards?
2. Our athletic field is square and contains 16 acres. How many rods long is one side?
3. In a square field there are 625 square rods. What is the length of one side of the field in rods?
4. Mr. Williams has a square field containing 400 square rods. How long is each side of the field? What is the perimeter of the field?
5. Mr. Thurston has a cubical box containing 216 cubic feet. How long, wide, and high is the box? How many square inches in the surface of the box? How many inches in the edges of the box?
6. A farmer has a bin whose length, breadth, and depth are the same. There are 729 cubic feet in the bin. What are its dimensions?
7. How many square feet of boards did it take to build the bin with a cover?
8. There are 1,000 cubic feet in a small room whose three dimensions are the same. How high is the ceiling of the room? How many square feet in the floor?
9. The number of square feet in the floor of a room is 225. The length and the breadth are the same. What are its dimensions? What is the perimeter of the room? How many yards in the perimeter?
10. Mr. Hudson dug a cellar; the length, breadth, and depth were the same. There were 8,000 cubic feet in the cellar. What did it cost to cement the cellar at 5 cents per square foot?

RIGHT TRIANGLES

374. Draw a square on the scale of $\frac{1}{4}$ in. to 1 in. to represent a 15-in. square.

a^2	$a \times b$
$a \times b$	b^2

Divide it as the square is divided and write in each section the number of square inches it contains and the formula representing it.

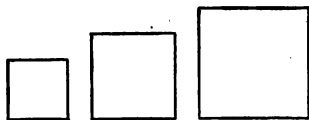
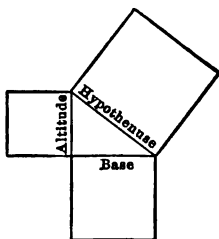
In Section 3 of the rule, why is the root already found doubled to make the trial divisor?

What part of the square of 15 is shown by a^2 ? b^2 ?

375. TO FIND THE HYPOTENUSE OF A RIGHT TRIANGLE.

The side of a right triangle opposite the right angle is called the **hypotenuse**.

Draw a right triangle on a base 4 inches long, with an altitude of 3 inches.



Measure the hypotenuse.

Draw squares on lines equal to the base, altitude, and hypotenuse of the triangle.

How many square inches in each square?

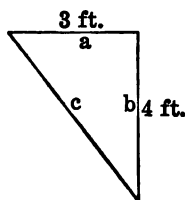
Compare the sum of the squares of the base and altitude with the square of the hypotenuse.

Calling the base b , altitude a , and hypotenuse h , we have equation,

$$a^2 + b^2 = h^2; \text{ or}$$

$$a^2 = h^2 - b^2;$$

$$b^2 = h^2 - a^2.$$



In the above triangular bracket what is the length of c ?

$$a^2 + b^2 = c^2$$

$$9 + 16 = c^2$$

$$c^2 = 25$$

$$c = \sqrt{25} = 5$$

WRITTEN EXERCISES

376. Solve:

1. Find the missing side of each of the following triangles:

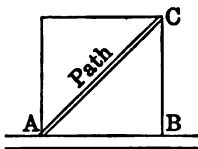
	ALTITUDE	BASE	HYPOTENUSE
(1)	3		5
(2)		6	8
(3)	5	12	
(4)	13		14

2. A pole is 12 ft. high. It is supported by a guy rope fastened to a stake 9 ft. from its foot. How long is the guy rope?

3. A ladder is 17 ft. long. To reach a window 15 ft. from the ground, how far from the base of the wall must the foot of the ladder be placed?

4. A man fell into the water from a pier 15 ft. above the water. A rope was thrown to him. It required 20 ft. of rope to reach him. How far was he away from the pier?

5. Mr. Simmons' ice house has an inclined shoot for taking in ice. It extends from a door 20 ft. above the ground to the edge of the pond, 60 ft. from the building. How long is the shoot?



6. The lot at the corner of 2d St. and Fourth Ave. is 120×50 ft. A path ran diagonally across it. The owner built upon it, closing the path. How much farther must people now go to get from A to C?

CUBICAL CONTENTS

377. How many dimensions must you know in order to calculate the contents of a parallelopiped?

What are the factors? What is the multiple?

The contents of a bin are 9,800 cubic feet; the length 28 feet, and the width 25 feet. What is the height?

$$28 \times 25 = 700; 9,800 \div 700 = 14.$$

When the cubic contents and two of the dimensions of a parallelopiped are given, the third dimension is found by dividing the cubic contents by the product of the given dimensions.

WRITTEN EXERCISES

378. Solve:

1. There are 8,424 cubic feet in a room. The length of the room is 27 feet; the width is 24 feet. What is its height?

2. Mr. Doremus dug a cellar, from which he removed 9,180 cubic feet of dirt. The length of the cellar was 36 feet; the depth was 8.5 feet. What was the width of the cellar?

3. What did it cost to dig the cellar at $12\frac{1}{2}$ cents per cubic yard? (A cubic yard is 3 feet long, 3 feet wide, and 3 feet high.)

4. There were taken from a marl pit 1,560 cubic feet of marl. The width of the pit was 8 feet; the depth was 12 feet. How long was it?

5. What was the value of the marl at \$1.25 per load, a load containing 40 cubic feet?

Find the missing dimensions of the following:

6. Contents 240 cubic feet, length 8 feet, width 5 feet.

7. Contents 880 cubic feet, length 11 feet, depth 8 feet.

8. Contents 1,880 cubic feet, width 10 feet, depth 9 feet.

9. Contents 3,200 cubic feet, length 25 feet, width 16 feet.

10. Contents 8,400 cubic feet, length $33\frac{1}{3}$ feet, width 21 feet.

11. Contents 4,500 cubic feet, length 18 feet, width $16\frac{2}{3}$ feet.

SUMMARY OF CHAPTER VIII

POWERS AND ROOTS — APPLICATIONS.

Powers.

Roots by method of factoring.

Methods of finding roots.

Multiplication Method.

Geometric Method.

APPLICATIONS OF SQUARE AND CUBE ROOT.

In finding the sides of a right triangle.

In finding the dimensions of a cube.

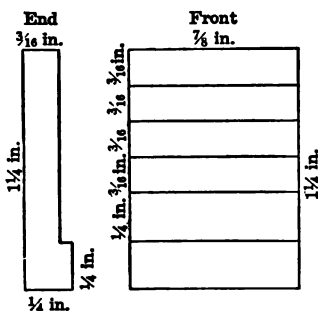
GENERAL REVIEW

379. Solve:

1. From a farm containing 135 acres 60 square rods, there were sold 72 acres 100 square rods. How many acres remained?

2. A man has purchased three varieties of strawberry plants; of the first he has 210 plants, of the second 168 plants, and of the third 231 plants. He wishes to set them in rows, each variety by itself, so that there will be the same number of plants in each row. What is the greatest number that he can put in a row? How many rows of each variety will there be?

3. In their manual training class, the eighth grade made a set of bookshelves for the schoolroom. The teacher gave them drawings, on a scale of $\frac{1}{4}$ in. to 1 ft., like the copies given here. Measure these copies and make estimates as follows:



Number of board feet in the sides.

Number of board feet in the back.

Number of board feet in the shelves.

If boards 13 inches wide are bought, how much is wasted?

4. At $\$.05\frac{1}{2}$ a square foot, what did the wood cost?

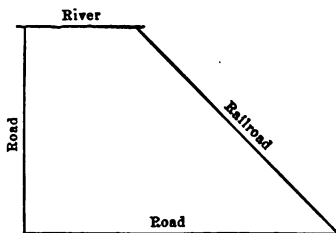
5. What width of board would be better to use? How much less would it cost?

6. How many books $1\frac{3}{4}$ inches thick could be placed on the shelves?

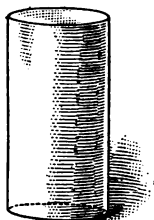
7. The Equator is 24,899 miles long. How far is it to the center of the earth from a point on the Equator? How far through the earth?

8. New York is almost exactly opposite the island of St. Paul in the Pacific Ocean south of the Equator. How far is it in a direct line on the surface of the earth from New York to St. Paul Island? How far through the earth?

9. This is a diagram of Mr. Preston's farm. The longer road extends along by the side of the farm for a distance of 960 rods, the shorter one for a distance of 720 rods. The side of the farm that is bordered by the river measures 360 rods.

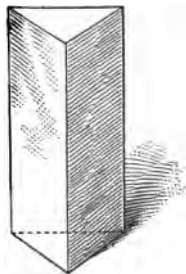


Find the length of the side bordered by the railroad. How many acres are there in the farm?



10. This cylinder is drawn on a scale of 5 feet to an inch. Find the area of the curved surface. What is the area of the bases?

11. This figure represents a stone that is 4 feet long and whose equilateral base is $1\frac{1}{2}$ feet high. Find the number of cubic feet in the stone.



12. How many square feet of tin will be required to line the flat roof of a circular summer house having a circumference of 68.54 feet?

13. How many acres are there in a circular park having a diameter of 120 rods?

14. A California redwood tree measures 17.2788 feet around the trunk. What is the diameter of the tree? What is the area of a cross section?

15. Philadelphia is 75° west of London. What is the difference in time between the two cities?

16. New York is $74^{\circ} 3'$ west longitude and Chicago is $87^{\circ} 38'$ west. What is the difference in time?

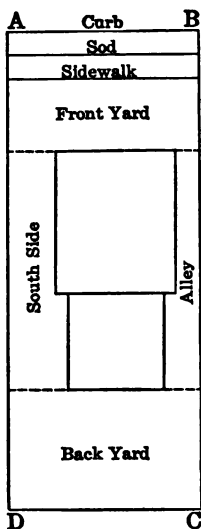
17. St. Petersburg is 30° east longitude. Washington is 77° west. When it is 9 A.M. in Washington, what time is it in St. Petersburg?

18. When it is 8 P.M. at St. Petersburg, what time is it in Washington?

19. Mr. Wilson had a winter cover for his fountain made in the shape of a square pyramid 6 feet on a side with a slant height of 10 feet. How many feet of boards did it take?

20. The upright post of a derrick 20 feet high is supported by guy cables attached to its top and fastened at the ground to stakes 20 feet from the base of the post. How long are the cables?

21. The rectangle $ABCD$ represents a building lot. It is drawn on a scale of 80 feet to 1 inch. What are its dimensions?



22. The figure within the rectangle represents the ground floor of the house, drawn on the same scale. How wide is the front of the house? How wide is the rear of the house? How long is the house? How many square feet does it cover?

23. The sidewalk is made of paving stone 5 feet wide, that cost \$.14 a square foot, laid. What did the sidewalk cost?

24. The remaining part of the front lot is sodded. The sod cost \$.25 per square yard, laid. What did it cost to sod the front yard?

25. The alley is paved with brick and stone at an average cost of \$.13 $\frac{1}{2}$ per square foot. What did it cost to pave the alley?

26. The south side and back yard are sodded. The sod cost \$.25 per yard laid. What did it cost to sod them?

27. The fence on the south and west sides cost, on the average, \$.18 $\frac{1}{2}$ per foot. What was the cost of the fence if it was built to the front of the house?

28. The cellar wall extends entirely around the house. It is 7 $\frac{1}{2}$ feet high and 18 inches thick. How many cubic feet in the wall? Allowing 24 bricks to a cubic foot, how many thousand bricks in the wall?

29. The carpenter's contract for building the house was \$3,694. The mason's contract was \$1,340, the plumber's \$325. The electric wiring cost \$110. What was the amount of the contracts for the house?

30. The carpenter's work was what per cent of the amount of the contracts? The mason's? The plumber's? The painter's?

31. The mason's contract was what per cent of the carpenter's? The plumber's contract was what per cent of the mason's?

32. The drawing-room on the second floor is 25 feet long and 13 feet 6 inches wide. What did the floor boards cost at \$45 per thousand feet? Laying the breadths across the room, what would it cost to carpet the room with Brussels carpeting $\frac{3}{4}$ yard wide at \$2.25 per yard, allowing 2 $\frac{1}{2}$ % for matching?

33. The bedrooms on the southeast corner in the rear of the house are 15 feet long, 13 feet 6 inches wide, and the walls are 9 feet high. Allowing 12 $\frac{1}{2}$ % for doors and windows, what would it cost to plaster each room at \$.30 per square yard?

34. On the walls of the house 300 rolls of paper were used, at an average marked price of \$.40 per roll. The paper was bought for 5 %, 10 %, and 15 % off for cash. What was the cost of the paper?

35. 1150 feet of molding were furnished. The average price was \$.05 a foot, with 10 % off for cash. What was the cash price for the molding? If it cost \$.01½ a foot to put on the entire molding, what was the entire cost?

36. An allowance of 25 % was made on 2 mantels for which \$75 each was asked. What was the cash price of the mantels?

37. The first payment to the carpenter was 25 % of his contract price, due when the building was inclosed. The man building the house had his note discounted at 5 % for 30 days to get the money to pay for it. What were the proceeds of the note?

38. The second payment, 33⅓ % of what was still due on the contract, was paid when the partitions were up and the floors were laid. How much was it?

39. The third payment, 50 % of what still remained to be paid on the contract, was paid when the house was trimmed. How much was it?

40. When the carpenter's work was completed, the owner gave a 90-day note at 5 % for the amount still due to the carpenter. What was the amount of the note when due?

41. According to the items given, what was the entire cost of the house?

42. The cost of the lot was \$1,200. Allowing \$25 a year for water, \$15 a year for insurance, \$75 for repairs, and 5 % for the use of the money invested, what will be the net profit for a year if the house is rented at \$80 per month?

43. The ball on an engine-house tower is 2 feet 6 inches in diameter. How many square inches of gilding will be required to gild it?

44. What will it cost to paint a smokestack 15 feet high and 9 feet in circumference at \$.04 per square foot?

45. A square field contains 9 acres. What is the length of its diagonal?

46. Mr. Baker had a farm having two parallel sides, one, $\frac{1}{2}$ of a mile and the other $\frac{5}{16}$ of a mile long. The distance between them was $\frac{1}{4}$ of a mile. Draw to scale, 1 inch to 80 rods.

47. How many acres were there in the farm?

48. Letter the short parallel side AB and the longer side CD , A and C being the vertices of the right angles. Connect BC , thus dividing the farm into two parts. In ABC wheat was planted; in BCD corn was planted. Find the number of acres in the wheat field.

49. Find the number of acres in the corn field.

50. In a park there is a circular fountain 35 feet in diameter. Encircling this is a gravel walk 7 feet wide. What is the distance around the outer edge of the walk? What is the area of the fountain?

51. A race course is half a mile in circumference. How many feet is it across the course?

52. Mr. Small bought $33\frac{5}{8}$ acres of land at $\$42\frac{7}{8}$ an acre. How much did the land cost him?

53. He sold the land for $\$57\frac{5}{8}$ an acre. How much did he gain?

54. If it takes $4\frac{1}{4}$ yards of cloth to make a coat, $1\frac{3}{4}$ yards to make a vest, and $3\frac{3}{4}$ yards to make a pair of trousers, how many yards are required for the suit?

55. Mrs. Doane bought $3\frac{3}{4}$ pounds of rice at \$.08 a pound, $2\frac{1}{2}$ pounds of tea at \$.58 a pound, 20 pounds of sugar at $\$.04\frac{3}{4}$ a pound, and $1\frac{3}{8}$ pounds of cheese at \$.22 a pound. What was the entire amount of her bill?

56. Mr. Green paid \$1,680 for 16 acres of land. How much would he pay for 12 acres at the same rate?

57. From a barrel of gasoline, a dealer sold 5 gal. 3 qt. to one man and 10 gal. 2 qt. to another. How many gallons remained?

58. Mr. Doremus sold 52 bu. of wheat, which was 13 % of his whole crop. How many bushels had he left? What was its value at \$.87½ a bushel?

59. Mr. Pierson's house is insured for \$4,500 and his furniture for \$800. The insurance on the house is 60 % of its value, and that on the furniture is 66⅔ % of its value. What is the value of the house and furniture?

60. In 35 % of an acre there are 15,246 sq. ft. How many square feet are there in an acre?

61. From a cask 15 gallons of molasses were drawn, which was 24 % of its contents. What was the value of the whole cask at \$.75 per gallon?

62. What is the gain or loss per cent on an article bought for ¼ of a dollar and sold for \$.37½?

63. A man increased his money 37½ %. He then had \$121. How much had he at first?

64. Mr. Ames paid \$8,260 for a house, which was 3⅓ % of his money. How much money had he?

65. Mr. Morgan lost \$45,500, which was 2½ % of his money. How much had he at first?

66. A bookseller sold 2 books for \$.96 each. On one he made 20 % and on the other he lost 20 %. Did he gain or lose on both, and what per cent?

67. Three men engaged in business. Mr. Tuttle put in \$7,250 which was 6¼ % of his money; Mr. Rice put in \$9,500, which was 14⅔ % of his money; and Mr. Ludlow put in \$20,500, which was 83⅓ % of his money. How much money had each? How much money had they together?

68. A speculator bought a tract of land containing a square mile at \$24 an acre. He sold it to Mr. Davis at a profit of $16\frac{2}{3}\%$. Mr. Davis divided it into farms of 160 acres each and sold each farm for \$5,000. How much did Mr. Davis make? How much did the speculator make?

69. A wholesale dealer in jewelry sold a watch that cost him \$80 to a retail dealer at a profit of 40%. The retail dealer sold it to a customer for 30% more than it had cost him. Which man made the more. How much more?

70. Mr. Andrews bought 4,500 lb. of Timothy hay at \$.0 $\frac{3}{4}$ per pound, and sold it for \$20 per ton. What was his gain per cent?

71. Helen's age is 6 yr. 4 mo. 20 da. which is $12\frac{1}{2}\%$ of her father's age. How old is her father?

72. Mr. Durand and Mr. Lawson each bought a tract of land, paying the same price. Mr. Durand sold his tract for \$7,200 and made 25%. Mr. Lawson sold his tract for \$6,336. How much did he make! What per cent did he make?

73. Find the difference between $12\frac{1}{2}\%$ of 4,864 feet and $\frac{1}{8}\%$ of 48,640 feet.

74. Find the difference between $37\frac{1}{2}\%$ of 9,728 minutes and $\frac{3}{8}\%$ of 97,280 minutes.

75. Mr. Sartolia sold 25 bu. 3 pk. 6 qt. of peanuts. This was $6\frac{1}{4}\%$ of all that he had. How many bushels did he have at first?

76. If John had 25% more money than he has, he would have \$400. How much has he?

77. Jan. 1, 1908, Mr. Riley borrowed \$360 of Mr. Hawkins, agreeing to pay him 5% for the use of the money. How much did Mr. Riley owe Mr. Hawkins Aug. 16, 1909?

78. What is the interest on \$480 at 6% from February 12, 1907, to April 18, 1908?

SUPPLEMENT

INTERNATIONAL METRIC SYSTEM

SYNOPSIS OF THE SYSTEM

1. The fundamental unit of the Metric System is the **meter** — the unit of length. From this the units of capacity (**liter**) and of weight (**gram**) were derived. All other units are the decimal subdivisions or multiples of these.

These three units are simply related; *e.g.* for all practical purposes one **cubic decimeter** equals one **liter** and one **liter** of water weighs one **kilogram**.

The metric tables are formed by combining the words “**meter**,” “**gram**” and “**liter**” with the six numerical prefixes, as in the following tables.

The tables and descriptions in the following are taken from the pamphlet on the Metric System issued by the Department of Commerce and Labor of the United States government.

PREFIXES	MEANING	UNITS
milli- = one thousandth	$\frac{1}{1000}$	.001
centi- = one hundredth	$\frac{1}{100}$	.01
deci- = one tenth	$\frac{1}{10}$	.1
unit = one		1
deka- = ten	10	10
hecto- = one hundred	100	100
kilo- = one thousand	1000	1000
		“meter” for length
		“gram” for weight or mass.
		“liter” for capacity

2.

UNITS OF LENGTH

milli-meter	.001 meter, mm.
centi-meter	.01 " cm.
deci-meter	.1 " dm.
meter	1 " m.
deka-meter	10 " Dm.
hecto-meter	100 " Hm.
kilo-meter	1000 " Km.

Where miles are used in England and the United States for measuring distances, the kilometer (1,000 meters) is used in metric countries. The kilometer is about 200 rods.

There are about 1,600 meters in a statute mile, 20 meters in a chain, and 5 meters in a rod. One meter = 39.37 inches exactly.

3. The **meter** is used for dry goods, merchandise, engineering construction, building, and other purposes where the yard and foot are used. The meter is about a tenth longer than the yard.

The **centimeter** and **millimeter** are used instead of the inch and its fractions in machine construction and similar work. The centimeter, as its name shows, is the hundredth part of a meter. It is used in cabinet work, in expressing sizes of paper and books, and in many cases where the inch is used. The centimeter is about two-fifths of an inch and the millimeter about one twenty-fifth of an inch. The millimeter is divided for finer work into tenths, hundredths, and thousandths.



4. If a number of distances in millimeters, meters, and kilometers are to be added, reduction is unnecessary. They are added as dollars, dimes, and cents are now added. For example, "1,050.25 meters" is not read "1 kilometer, 5 dekameters, 2 decimeters, and 5 centimeters," but "one thousand and fifty meters, twenty-five centimeters," just as "\$1,050.25" is read "one thousand and fifty dollars, twenty-five cents."

Complete the table :

— mm.	= 1	—
— cm.	= 1	—
— dm.	= 1	—
— m.	= 1	—
— Dm.	= 1	—
— Hm.	= 1	—

ORAL EXERCISES

5. Solve :

1. What is the ratio of 1 Km. to 1 Hm.? To 1 Dm.? To 1 M.? How many Dm. in 6 Km.? In 8 Km.? In 4.5 Km.? In 9.25 Km.? In 12.75 Km.?

2. How many M. in 1 Km.? In 5 Km.? In 7 Km.? In $7\frac{1}{2}$ Km.? In $4\frac{1}{4}$ Km.? In $7\frac{3}{4}$ Km.?

3. How many M. in 2 Dm. 5 M.? In 9 Dm. 6 M.? In 8 Dm. 4 M.? In 1 Hm. 1 Dm. 2 M.? In 5 Hm. 3 Dm. 6 M.? In 6 Km. 5 Hm. 4 Dm. 3 M.? In 12 Km. 8 Hm. 5 M.?

4. How many inches in 1 Dm.? In 1 Hm.? In 1 Km.?

5. How many inches in 1 dm.? In 1 cm.? In 1 mm.?

6. Change 3,645 M. to Dm.; to dm.; to Hm.; to cm.; to Km.; to mm.

7. Change 8,716 dm. to m.; to Dm.; to cm.; to Hm.; to Km.

WRITTEN EXERCISES

6. Solve:

1. Express in one number:

(1) 15 Km. 9 Hm. 8 Dm. 5 m.

(2) 4 dm. 3 cm. 2 mm.

(3) 84 Km. 5 Hm. 6 Dm.

2. Express in units of each of the other denominations:

46,875.435 m.; 125,248.06 m.; 33,205.504 m.

3. Express the answers to the following in denominations of the Metric System:

(1) $486.5 \text{ m.} \times 25$; by 2.5; by .25.(2) $745.6 \text{ m.} \times 8\frac{1}{2}$; by $8\frac{1}{2}$; by $6\frac{1}{2}$.(3) $375.5 \text{ m.} \div 25$; by 2.5; by .25.4. A kilometer is about $\frac{5}{8}$ of a mile. Find the approximate number of miles in 15.5 Km.; in 36.05 Km.; in 88.75 Km.

5. Find the approximate number of kilometers in 15.9375 miles; in 31.875 miles; in 26.5625 miles.

6. Change the following to its approximate equal in our common system:

(1) 12 Km. 8 Hm. 6 Dm. 4 m.

SOLUTION: 12 Km. 8 Hm. 6 Dm. 4 m. = 12.864 Km.

 $12.864 \times .625 = 7.7184 \text{ miles.}$ $.7184 \times 320 = 246.9888 \text{ rods.}$ $.9888 \text{ rod} \times 16.5 = 14.3142 \text{ feet.}$ $.3142 \times 12 = 3.7704 \text{ inches.}$

8 miles, 246 rods, 14 feet, 3.7704 inches.

(2) 23 Km. 4 Hm. 3 Dm. 5 m.

(3) 34 Km. 7 Hm. 3 Dm.

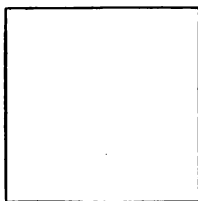
7. The length of a cable is 2 Km. 5 Hm. What is its approximate length in miles and rods?

AREA

7. The table of areas is formed by squaring the length measure, as in our common system.

For land measure 10 meters square is called an "are" (meaning "area"). The side of one are is about 33 feet. The hectare is 100 meters square, and, as its name indicates, is 100 ares, or about $2\frac{1}{2}$ acres. An acre is about 0.4 hectare. A standard United States quarter section contains almost 64 hectares. A square kilometer contains 100 hectares.

For smaller measures of surface the square meter is used. The square meter is about 20 per cent larger than the square yard. For still smaller surfaces the square centimeter is used. A square inch contains about $6\frac{1}{2}$ square centimeters.



Sq. In.



Sq. Cm.

8. METRIC SQUARE MEASURE

1 square kilometer (sq. Km.) = 1,000,000 sq. m.

1 square hectometer (sq. Hm.) = 10,000 sq. m.

1 square dekameter (sq. Dm.) = 100 sq. m.

1 square meter (sq. m.)

1 square decimeter (sq. dm.) = .01 sq. m.

1 square centimeter (sq. cm.) = .0001 sq. m.

1 square millimeter (sq. mm.) = .000001 sq. m.

1. How many sq. mm. in 1 sq. Km. ?
2. How many sq. cm. in 1 sq. Hm. ?
3. How many sq. dm. in 1 sq. Dm. ?

Complete the table :

— sq. mm. = 1 sq. —
— sq. cm. = 1 sq. —
— sq. dm. = 1 sq. —
— sq. m. = 1 sq. —
— sq. Dm. = 1 sq. —
— sq. Hm. = 1 sq. —

By what ratio do the units of the denominations of this table increase and decrease?

9. ORAL EXERCISES

1. What are the dimensions in meters of a square dekameter?

2. A square dekameter equals how many square meters?

3. How many meters are there in the perimeter of a rectangle 3 dekameters long and 2 dekameters wide?

4. How many square meters are there in the rectangle? In a rectangle 8 dekameters long? 5 dekameters long?

5. What are the dimensions of a square hectometer in meters?

6. How many square meters in a square hectometer? How many square dekameters?

7. A rectangle whose sides are 4 hectometers and 3 hectometers, respectively, equals how many square dekameters? How many square meters?

8. What are the dimensions of a square kilometer in hectometers? In dekameters? In meters?

9. How many square hectometers in a square kilometer? How many square dekameters? How many square meters?

10. The side of a square decimeter equals what decimal of a meter?

11. A square decimeter equals what decimal of a square meter?

12. The side of a square centimeter equals what decimal of a meter?

13. A square centimeter equals what decimal of a meter?

14. A square millimeter equals what decimal of a meter?

15. How many square meters in 8 square kilometers? In 6 square hectometers? In 25 square dekameters? In 3 square decimeters? In 4 square dekameters, 8 square meters? In 5 square hectometers, 5 square dekameters?

10.

WRITTEN EXERCISES

1. Express 5 sq. Hm. 18 sq. Dm. in square meters.

2. Express 25 Km. 15 Hm. 9 cm. in meters.

3. How many square meters in a rectangle 12 dm. long and 8 meters wide?

4. Mr. Milbrook has a yard 5 Hm. long and 18 Dm. wide, how many square meters in the yard?

5. Mr. Fairbank had a field 12 Hm. long and 25 Dm. wide; he sold 125,000 sq. m. How many square meters had he left?

11.

METRIC LAND MEASURE

1 hectare (ha.) = 100 ares

1 are (a.)

1 centare (ca.) = .01 are.

The standard unit of metric land measure is an are, which is equivalent to 100 square meters, or approximately to .025 or $\frac{1}{40}$ of an acre.

1. What is the side of a square that contains 100 sq. m.?

2. Name a rectangle that contains 100 sq. m.

3. What is the approximate number of acres in 45 a.? In 86 a.? In 97 a.? In 36 a.? 25 ca.? In 53 a.? 17 ca.?

4. What is the approximate value of 15 ca. at \$17 an acre? Of 265 a. at \$50.75 an acre? Of 312 a. at \$125.25 an acre?

5. Find the approximate value of 18 A. of land at \$3.25 an a. Of 24 A. at \$5.75 an a. Of 36 A. 80 sq. rd. at \$4.80 an a. Of 25 A. 40 sq. rd. at \$4 $\frac{1}{2}$ an a. Of 16 A. 32 sq. rd. at \$3 $\frac{1}{2}$ an a.

NOTE. There are 40 times as many acres as ares, approximately.

6. Mr. Pierre has a tract of land 250 m. long 150 m. wide. What is its value at \$650 an A.?

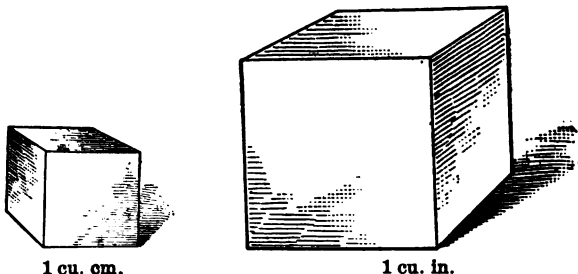
SOLUTION: $250 \times 150 = 37,500$ sq. m.;
 $37,500$ sq. m. $\div 100 = 375$ a.;
 375 a. $\times .025 = 9.375$ A.
 $\$650 \times 9.375 = \$6,093.75$. *Ans.*

7. Find the approximate value of the following tracts of land:

LENGTH	WIDTH	PRICE PER A.
(1) 350 m.	210 m.	\$400
(2) 175 "	140 "	\$820
(3) 400 "	300 "	\$260
(4) 550 "	400 "	\$350
(5) 600 "	35 Dm.	\$500
(6) 40 Dm.	30 Dm.	\$525
(7) 80 rd.	60 rd.	\$420
(8) 120 "	80 "	\$500
(9) 96 "	60 "	\$300
(10) 585 Dm.	170 Dm.	\$100
(11) 210 m.	35 m.	\$500
(12) 25 Hm.	13 Dm.	\$400

VOLUME

12. The cubic measures are the cubes of the linear units. The cubic meter (sometimes called the *stere*, meaning "solid") is the unit of volume. A cubic meter of water weighs a metric ton and is equal to 1 kiloliter. The cubic meter is used in place of the cubic yard and is about 30 per cent larger. This is used for "cuts and fills" in grading land, in measuring timber, expressing contents of tanks and reservoirs, flow of rivers, dimensions of stone, tonnage of ships, and other places where the cubic yard and foot are used.



The thousandth part of the cubic meter (1 cubic decimeter) is called the *liter*. (See table of capacity units).

For very small volumes the *cubic centimeter* (cu. cm. or cm.³) is used. This volume of water weighs a *gram*, which is the unit of weight or mass. There are about 16 cubic centimeters in a cubic inch. The cubic centimeter is the unit of volume used by chemists as well as in pharmacy, medicine, surgery, and other technical work.

One thousand cubic centimeters make 1 liter.

1. Approximately 1 cu. ft. = how many cm.³?
2. 1 cu. yd. = how many cm.³?
3. 1 m.³ = how many cu. ft.?
4. 1 cu. yd. = how many m.³?

13. METRIC CUBIC MEASURE

1 cubic kilometer (cu. Km.)	1,000,000,000 cu. m.
1 cubic hectometer (cu. Hm.)	1,000,000 cu. m.
1 cubic decameter (cu. Dm.)	1,000 cu. m.
1 cubic meter (cu. m.)	
1 cubic decimeter (cu. dm.)	.001 cu. m.
1 cubic centimeter (cu. cm.)	.000001 cu. m.
1 cubic millimeter (cu. mm.)	.000000001 cu. m.

By what ratio do the units of this measure increase?
Complete the table.

— cu. mm.	= 1 —
— cu. c.	= 1 —
— cu. d.	= 1 —
— cu. m.	= 1 —
— cu. D.	= 1 —
— cu. H.	= 1 —

NOTE. A cubic meter, or its equivalent, is called a stere (S.). A cubic decimeter is called a liter.

The standard unit of metric wood measure is a stere, which is $\frac{1}{4}$ of a cord, approximately.

14. METRIC WOOD MEASURE

1 dekastere (Ds.)	= 10 steres.
1 stere (s.)	
1 decistere (ds.)	= .1 of a stere.

ORAL AND WRITTEN EXERCISES

15. Solve orally :

1. How many cubic decimeters in 5 cu. m.? In 7 cu. m.? In 9 cu. m.? In 4 s.? In 6 s.? In 8 s.? In 2 cu. m.? In 300 cu. dm.? In 15 cu. m. In 750 cu. Dm.?

2. How many cubic meters in 8,000 cu. dm.? In 35,000 cu. dm.? In 75,000 cu. dm.? How many steres in 4,000 cu. dm.? In 16,000 cu. dm.?

3. How many liters in 1 cu. dm.? In 5 cu. cm.? In 9 cu. cm.? In 500 cu. dm.? In 6 cu. cm.? In 250 cu. dm.? In 5 s.? In 3 s.? In 20 s.?

4. How many cords of wood in 15 s.? In 12 s.? In 10 s.?

5. A pile of wood 4 m. long, 3 m. wide, and 12 m. high contains how many steres? How many cords?

6. How many cords of tan bark in a pile 5 m. long, 4 m. wide, and 2 m. high? In a pile 6 m. long, 3 m. wide, and 2 m. high.?

16. Solve on paper:

1. Five boxes, each holding a stere, are full of packages of cereal. Each package holds a liter. What is the value of the cereal at 15 cents a package?

2. What the value of 2 steres of "Grape Nut," each package containing a liter, at the rate of two packages for 25 cents? Of $2\frac{1}{2}$ steres? Of 5.5 steres? Of 8.25 steres?

3. What is the approximate value of a pile of tan bark 8 m. long, 4 m. wide, and 2 m. high, at \$6.25 a cord? Of a pile 10 m. long, 6 m. wide, and 4 M. high, at \$5.75 a cord? At \$7.50 a cord?

4. About how many cubic feet are there in a stere? In $7\frac{1}{4}$ s.? In 12.25 s.? In 15.5 Ds.? In 10.5 s.? In 17.25 s.? In 8.35 s?

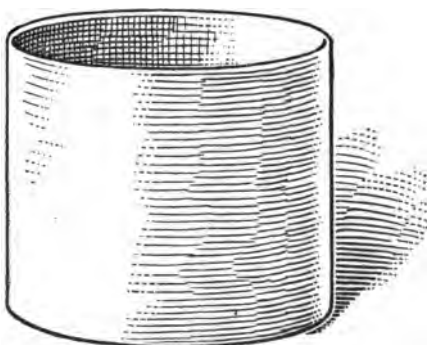
5. What is the value of a pile of wood 5 Dm. long, 2 m. wide, and 2 m. high, at \$3.98 a cord? Of a pile 4 Dm. long, 1 m. wide, and 2 m. high, at \$6.70 a cord?

17.

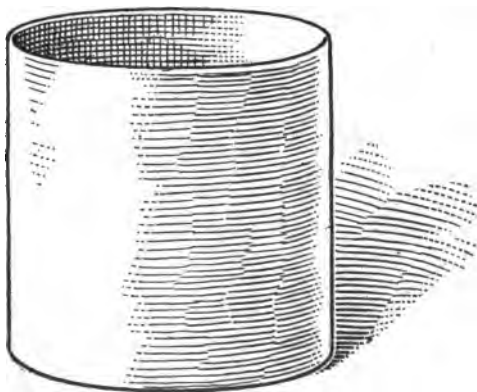
UNITS OF CAPACITY

milli-liter =	.001 liter, ml.
centi-liter =	.01 " cl.
deci-liter =	.1 " dl.
liter =	1 " l.
deka-liter =	10 " Dl.
hecto-liter =	100 " Hl.
kilo-liter =	1,000 " Kl.

The *hectoliter* (100 liters) serves the same purpose as the United States bushel (2,150.42 cubic inches), and is equal to about 3 bushels, or a barrel. A peck is about 9 liters. The liter is used for measurements commonly given in the gallon, and the liquid and dry quarts, a liter being 5 per cent larger than our liquid



1 dl.



1 gi.

quart and 10 per cent smaller than the dry quart. A liter of water weighs exactly a kilogram, *i.e.* 1,000 grams. A thousand liters of water weigh 1 metric ton.

One liter equals 1.05658 liquid quarts or 0.9081 dry quart.

One kilogram equals 2.204622 avoirdupois pounds.

ORAL EXERCISES

18. What is the ratio of :

- | | |
|---------------------|----------------------|
| 1. 1 Kl. to 1 Hl. ? | 7. 1 Kl. to 1 Dl. ? |
| 2. 1 Hl. to 1 Dl. ? | 8. 1 Hl. to 1 l. ? |
| 3. 1 Dl. to 1 l. ? | 9. 1 cl. to 1 l. ? |
| 4. 1 l. to 1 Dl. ? | 10. 1 Ml. to 1 Dl. ? |
| 5. 1 Dl. to 1 cl. ? | 11. 1 Dl. to 1 Kl. ? |
| 6. 1 cl. to 1 ml. ? | 12. 1 Ml. to 1 ml. ? |

WRITTEN EXERCISES

19. Solve as directed :

1. Express in one number :

4 Kl., 5 Hl., 2 Dl., 6 l., 3 dl., 7 cl., 8 ml.

2. Express 6275.325 l. in units of each of the other denominations.

3. Write and add :

8 Kl., 0 Hl., 5 l., 3 dl., 0 cl., 4 ml., 3 Kl., 6 Hl., 9 Dl., 2 l., 0 dl., 4 cl., 5 Kl., 7 Hl., 0 dl., 3 l., 5 dl., 0 cl., 5 ml.

4. From 8,645.5 l. take 5,468.15 l.

5. $575.25 \text{ l.} \div 5$.

20. The liter is equal to about .9 of a quart dry measure and 1.05 quarts liquid measure.

How many quarts dry measure in 4 l. ?

Since there are .9 quart in 1 l., there are in 4 l. .9 quart $\times 4 = 3.6$ quarts.

ORAL EXERCISES

21. How many quarts dry measure are there in:

6 l.? 8 l.? 11 l.? 12 l.? 9 l.? 5 l.? 1 Dl.? 2 Dl.? 5 Dl.?
8 Dl.? 3 Dl.? 7 Dl.? 1 Hl.? 5 Hl.? 4 Hl.? 9 Hl.? 6 Hl.?
3 Hl.? 1 Kl.? 5 Kl.? 4 Kl.? 8 Kl.? 9 Kl.?

WRITTEN EXERCISES

22. 1. Find the number of quarts, liquid measure, that each one of the above metric expressions equals.

2. Mr. Winthrop sold 4 Dl., 4 l. of syrup. What is its value at \$.18 per quart?

SOLUTION: 5 Dl., 4 l. = 54 L.; 1.05 quarts $\times$ 54 = 56.7 quarts.

$$\$.18 \times 56.7 = \$ 10.206.$$

3. What is the value of 6 Dl., 5 l. of alcohol at \$.80 per quart?

4. What are 2 Dl., 3 l. of chestnuts worth, at \$.12 per quart?

5. What are 1 Hl., 5 Dl. of beans worth, at \$.15 per quart?

6. A grocer sold 4 Hl., 4 Dl. of vinegar at \$.32 per gallon. How much did he receive?

7. A vendor sold 5 Hl., 6 Dl. of peanuts at \$.80 a peck. What did he receive for them?

8. Change 3 bushels, 1 peck, 5 quarts, 1 pint to liters.

SOLUTION: 3 bushels, 1 peck, 5 quarts, 1 pint = 109.5 quarts.

$$109.5 \div .9 = 121.66 \text{ liters.}$$

9. How many liters are there in 28 bushels, 2 pecks, 4 quarts, 1 pint?

10. How many liters are there in 16 bushels, 3 pecks, 3 quarts, 1 pint?

11. How many liters are there in 48 bushels, 1 peck, 7 quarts, 1 pint?

12. How many liters are there in 12 gallons, 3 quarts, 1 pint, 3 gills?

SOLUTION: 12 gallons, 3 quarts, 1 pint, 3 gills = 51.875 quarts.
 $51.875 \div 1.05 = 49.404$ liters.

13. How many liters are there in 85 gallons, 2 quarts, 1 pint, 2 gills?

14. How many liters are there in 64 gallons, 3 quarts, 0 pints, 3 gills?

15. How many liters are there in 15 gallons, 0 quarts, 1 pint, 1 gill?

16. A grocer bought 20 bushels, 3 pecks, 6 quarts of hickory nuts for 8 cents a quart and sold them for 8 cents a liter? How much did he make?

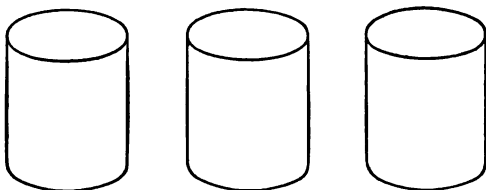
17. If 80 gallons, 3 quarts of molasses were bought at 16 cents a quart and sold at 20 cents per liter, how much would be made?

23. UNITS OF WEIGHT (OR MASS)

milli-gram =	.001	gram, mg.
centi-gram =	.01	“ cg.
deci-gram =	.1	“ dg.
gram =	1	“ g.
deka-gram =	10	“ Dg.
hecto-gram =	100	“ Hg.
kilo-gram =	1,000	“ Kg.

Measurements commonly expressed in gross tons or short tons are stated in metric tons (1,000 kilograms). The metric ton comes between our long and short tons and serves the purposes of both.

The kilogram and "half kilo" serve for everyday trade, the latter being 10 per cent larger than the pound. The kilogram is approximately 2.2 pounds.



Relative size of avoirdupois ounce, 30-gram, and Troy ounce (brass) weights.
(Actual size.)

The gram and its multiples and divisions are used for the same purpose as ounces, pennyweights, drams, scruples, and grains. For foreign postage, 30 grams is the legal equivalent of the avoirdupois ounce.



Gram



Scruple

Relative size of
gram and scruple
(brass) weights.
(Actual size.)

Complete the table :

—	mg.	= 1	—
—	cg.	= 1	—
—	dg.	= 1	—
—	g.	= 1	—
—	Dg.	= 1	—
—	Hg.	= 1	—

A kilogram is equal to about $2\frac{1}{2}$ lbs.

About what decimal of a pound does a Hg. equal?
A Dg.? A gram?

ORAL EXERCISES

24. Solve:

1. How many grams in 4 Dg.? In 6 Dg.? In 3 Hg.?
In 5 Hg.? In 7 Hg.? In 7 Kg.? In $9\frac{1}{2}$ Kg.? In 15
Dg.? In 12 Hg. 4 Dg.? In 16 Kg. 8 Hg. 5 Dg.?

2. What decimal of a gram is 6 dg.? 8 dg.? 5 cg.? 8 cg.? 3 mg.? 9 mg.?

3. Express in grams: 5 g. 5 dg.; 8 g. 6 cg.; 12 g. 14 mg.; 25 g. 2 dg.; 3 cg. 5 mg.; 4 dg.; 5 g. 3 dg. 7 cg. 5 mg.

4. About how many pounds do 4 Kg. equal? 6 Kg.? 10 Kg.? 20 Kg.? 5 Hg.? 11 Hg.? 12 Hg.? 15 Hg.? 25 Hg.? 5 Dg.? 12 Dg.? 15 Dg.? 10 g.? 100 g.? 1,000 g.? 4,000 g.? 5,000 g.?

WRITTEN EXERCISES

25. Solve:

1. What is the approximate value of 25 kilograms of coffee at 32 cents a pound? Of 36.5 kilograms at 28 cents a pound? Of 18.75 kilograms of tea at 48 cents a pound?

2. Mr. Anthony bought 46.75 kilograms of cheese at $24\frac{1}{2}$ cents a kilogram, and sold it at 15 cents per pound. How much did he make?

3. About how many kilograms in 22,000 lb.? In 55,000 lb.? In 401,500 lb.?

4. The Ansonia Company bought 132,000 lb. of copper at 13 cents a pound and sold it in Paris at 33 cents a kilogram. What was the gain?

5. What is the value of 5,250 g. of butter, at 56 cents a kilogram?

6. What is the value of 520 g. of cheese at \$.52 a kilogram?

7. What is the cost of 6,500 Kg. of coal at \$6.50 a ton? At \$5.75 a ton?

8. What will 50 Kg. of tea cost at \$.60 a pound? 40 Kg. at \$.50 a pound?

A TABLE OF EQUIVALENTS

26. A liter equals about .9 qt. dry measure.

A liter equals about 1.05 qt. liquid measure.

A meter equals 39.37 inches linear measure.

A kilometer equals about .625, or $\frac{5}{8}$ mile, linear measure.

An are equals about .025, or $\frac{1}{40}$ acre, square measure.

A stere equals about .25, or $\frac{1}{4}$ cord, of wood.

A gram equals about .0025, or $\frac{1}{400}$ lb., avoirdupois.

1. Louis sold 12 bushels, 3 pecks, 6 quarts of hickory nuts, at 8 cents a liter. How much did he receive for them?

2. How much are 2,736 liters of cranberries worth, at 12 cents a quart?

3. Mr. France sold 25 gallons, 3 quarts, 1 pint of vinegar, at 6 cents a liter. How much did he receive for it?

4. What is the value of 1,008 liters of molasses, at 16 cents a quart?

5. How many inches in 5 Dm. 4 m.? How many feet?

6. What will it cost to macadamize a road 24.16 kilometers long, at \$3,250 a mile?

7. What will it cost to build a Telford road 5.5 miles, at \$3,260 a kilometer?

8. What is the approximate value of 2,880 acres of land, at \$87.50 an acre?

9. What is the approximate value of 8.87 acres, at \$4.25 an acre?

10. A grocer sold 68,000 grams of sugar, at $4\frac{1}{8}$ cents a pound. About how much did he receive?

11. A package of silver weighs 3.47 grams. What is its weight in grains?

12. How many 2-gr. capsules will 5 g. of quinine fill?

CASTING OUT THE NINES

Very sure tests of the four fundamental processes are obtained by "casting out the nines"; that is, subtracting 9 as many times as possible from the sum of the digits.

Cast out the nines from 68,574.

Beginning at the left, add the digits until their sum equals or exceeds 9 :

$$6 + 8 = 14$$

Subtract 9 : $14 - 9 = 5$

Continue the addition,

beginning with the difference. $5 + 5 = 10$

Subtract 9. $10 - 9 = 1$

Continue the addition. $1 + 7 + 4 = 12$

Subtract 9. $12 - 9 = 3$

3, the last difference, is called the **excess**.

To test addition by casting out the nines. Cast out the nines from each addend and from the sum of the excesses if this sum equals or exceeds 9.

Then cast out the nines from the sum. If the excess of the addends equals that of the sum, the addition is correct.

Add and test :

68,574	Excess 3	}	$3 + 8 = 11$
43,928	Excess 8		$11 - 9 = 2$
<u>112,502</u>	Excess 2		Excess of excesses 2.

As the excess of the excesses of the addends equals the excess of the sum, the result is correct.

Add and test :

1. 15,984	2. 111,999	3. 400,004
<u>36,725</u>	<u>222,888</u>	<u>500,005</u>

To test subtraction by casting out the nines. Cast out the nines of the minuend and the subtrahend and the difference. If the excess of the minuend equals the excess of the sum of the excesses of the difference and subtrahend, the subtraction is correct.

Subtract : 75,864	Excess 3	$5 + 7 = 12$
58,912	Excess 7	Excess 3
<u>16,952</u>	<u>5</u>	

Subtract and test :

1. 19,873	2. 98,764	3. 100,000	4. 44,444
<u>14,569</u>	<u>10,801</u>	<u>98,705</u>	<u>19,999</u>

To test multiplication by casting out the nines. If the excess of the product equals the excess of the product of the excesses of multiplicand and multiplier, the multiplication is correct.

485	Excess 8	$8 \times 5 = 40$	Excess 4
<u>23</u>	Excess 5		
<u>1455</u>			
970			
<u>11155</u>	Excess 4		

Multiply and test :

1. 6783	2. 5467	3. 383	4. 68,573
<u>98</u>	<u>76</u>	<u>383</u>	<u>284</u>

To test division by casting out the nines. Multiply the excess of the quotient by that of the divisor. If the excess of the result equals that of the dividend, the division is correct.

If there is a remainder, add the excess of the product of the divisor times the quotient to the excess of the remainder and find the excess of this sum. This should equal the excess of the dividend.

Divide 6754 by 32.

211	
32 <u>6754</u>	Excess of dividend 4
64	Excess of divisor 5
<u>35</u>	Excess of quotient 4
32	Excess of remainder 2
<u>34</u>	
32	$5 \times 4 = 20$ Excess 2
<u>2</u>	$2 \times 2 = 4$ Excess of dividend.

Divide and test :

1. 33)9999; 41)8576; 57)7384; 902)876,549.

TABLES OF DENOMINATE NUMBERS

(1)

MEASURES OF LENGTH

12 inches	= 1 foot
3 feet	= 1 yard
$16\frac{1}{2}$ feet	= 1 rod
320 rods	= 1 mile

1 mi. = 1760 yd. = 5280 ft. = 63,360 in.

A *nautical mile (knot)* = 6080.27 ft., or approximately 1.15 mi.

A *furlong* = $\frac{1}{8}$ mi.; a *fathom*, used in measuring the depth of water, is 6 ft.; a *hand*, used in measuring the height of horses, is 4 in.

(2)

SQUARE MEASURES

144 square inches	= 1 square foot
9 square feet	= 1 square yard
$80\frac{1}{4}$ square yards	= 1 square rod
160 square rods	= 1 acre
1 acre	= 43,560 sq. ft.

An acre of land in the form of a square is very nearly 209 feet on a side.

A tract of land 1 mile square is often called a section.

100 sq. ft. of roofing, flooring, or slating is called a square.

(3)

CUBIC MEASURES

1728 cubic inches	= 1 cubic foot
27 cubic feet	= 1 cubic yard

A pile of wood 8 ft. long, 4 ft. wide, and 4 ft. high, or 128 cu. ft. of wood, is called a cord.

(4)

SURVEYORS' LINEAR MEASURES

100 links = 1 chain

80 chains = 1 mile

This chain, called *Gunter's chain*, is 4 rd., or 66 ft., long.
 1 link = 7.92 in.

Links are written as hundredths of a chain.

(5)

SURVEYORS' SQUARE MEASURE

10 square chains = 1 acre

640 acres = 1 square mile

(6)

LIQUID MEASURES

4 gills = 1 pint

2 pints = 1 quart

4 quarts = 1 gallon

1 gal. = 231 cu. in.; 1 cu. ft. = $7\frac{1}{2}$ gal., approximately.
 A gallon of water weighs about $8\frac{1}{8}$ lb.; a cubic foot of water weighs about $62\frac{1}{2}$ lb., or 1000 oz.

In measuring the capacity of cisterns, etc., $31\frac{1}{2}$ gal. = 1 barrel.

(7)

DRY MEASURES

2 pints = 1 quart

8 quarts = 1 peck

4 pecks = 1 bushel

1 bu. = 2150.42 cu. in., or approximately $1\frac{1}{4}$ cu. ft.

Our bushel is the *Winchester bushel*. In form it is a cylinder $18\frac{1}{2}$ in. in diameter and 8 in. deep.

(8)

AVOIRDUPOIS WEIGHT

16 ounces = 1 pound

100 pounds = 1 hundredweight

2000 pounds = 1 ton

1 *long* or *gross* ton = 2240 pounds1 av. lb. = 7000 gr.; 1 av. oz. = $437\frac{1}{2}$ gr.

(9)

TROY WEIGHT

24 grains = 1 pennyweight

20 pennyweights = 1 ounce

12 ounces = 1 pound

1 troy pound = 5760 gr. = $\frac{5760}{7000}$ av. lb.

1 troy ounce = 480 gr., or about 1.1 av. oz.

Troy weight is used in weighing gold, silver, and other precious metals and gems.

(10)

APOTHECARIES' WEIGHT

Apothecaries' measures are used to some extent by druggists in filling prescriptions. The grain, ounce, and pound are the same as those in Troy weight, but the ounce is divided differently.

20 grains (gr.) = 1 scruple . . . sc. or ℥.

3 scruples = 1 dram . . . dr. or ℥.

8 drams = 1 ounce . . . oz. or ℥.

12 ounces = 1 pound . . . lb. or ℔.

The grain is the basis of our three systems of weights. As its name implies, it was originally the weight of a single grain of wheat.

(11)

APOTHECARIES' LIQUID MEASURES

60 drops (gtt.) or minims (m.)	= 1 fluid dram	. f3.
8 fluid drams	= 1 fluid ounce	. f3.
16 fluid ounces	= 1 pint	. . . O.
8 pints	= 1 gallon	. . . Cong.

(12)

MEASURES OF TIME

60 seconds	= 1 minute
60 minutes	= 1 hour
24 hours	= 1 day
7 days	= 1 week
365 days	= 1 year
366 days	= 1 leap year
10 years	= 1 <i>decade</i> .
100 years	= 1 <i>century</i> .

Thirty days have September,
 April, June and November.
 All the rest have thirty-one,
 Save February, which alone
 Has twenty-eight, and one day more
 We add to it one year in four.

The earth revolves around the sun in 365 days, 5 hours, 48 minutes, 46 seconds. This is the *mean solar year*, and is nearly $365\frac{1}{4}$ days.

To correct the errors in the calendar, made by disregarding the fraction of a day over 365 days, centennial years divisible by 400 and other years divisible by 4 are lengthened 1 day, Feb. 29. These years are *leap years*.

The standard unit of time is the *day* ("the mean solar day," which is the average time between the noon of one day and noon of the day following).

(13)

COUNTING TABLE

2	= 1 pair
20	= 1 score
12	= 1 dozen
12 dozen	= 1 gross
12 gross	= 1 great gross

(14)

STATIONERS' MEASURES

24 sheets	= 1 quire
20 quires	= 1 ream
20 reams	= 1 bundle
5 bundles	= 1 bale

Paper is quite generally sold by the 100, 500, and 1000 sheets; also by the pound.

ANSWERS

PART TWO

Page 2.—2. 643.

Page 3.—4. 289. 5. Sums: \$45.67, \$4.85, \$7.65, \$278.36; Differences: \$7.79, \$1.19, \$2.35, \$74.26. 2. 17,205.

Page 4.—3. 17,155; \$1062.50: 74,400; 9,600,000. 1. 50, 25, 20, 10; 33, 11, 9; 49, 14; 48, 32, 24, 16, 12, 8; 19, 5; 47; 31; 46, 23; 45, 30, 18, 15, 10, 9. 3. 12. 4. 5; 10; 15; 15 $\frac{1}{4}$.

Page 6.—1. 14. 2. $\frac{1}{2}$; $\frac{1}{3}$; $\frac{1}{4}$; 24; 22; 26.

Page 7.—1. \$112.50. 2. $\frac{1}{2}$ lb.; 12 $\frac{1}{2}$ lb.; \$7.50. 3. \$37.50. 4. 100 sq. ft.; 200 sq. ft. 5. 2700 cu. ft. 6. 160 sq. rd. 8. 192 hills. 9. 24 ft. long, 18 ft. wide. 10. 36 pk.

Page 10.—1. 27; 47.1; 285.72; 1.08; 63.12; 180.06; 1.5. 2. \$1.75; \$211.25; \$951; \$3737.50. 3. \$9; \$74.34; \$195.75; \$79.83; \$1203.30; \$2331.81. 4. \$4.90; \$5.88; \$7.84. 5. \$93.75. 6. \$5.625; \$88.125. 7. 64 acres. 8. 96 acres; 50%; 160 acres. 9. \$2400. 10. \$3600. 11. \$6000. 12. \$6900.

Page 22. (28.)—2. 422; 53,244; 75,227; 369,418; 24,526; 39,417; 12,497; 99,299; 65,208.

Page 23.—3. \$10.69. 4. \$11.63. 5. \$822.14. 6. \$1091.30; \$1022.74; \$1441.55; \$1330.30; \$1672.25; \$3436.72; \$2637.72.

Pages 24 and 25. (30.)—1. \$250; \$500; \$750; \$1000; \$1150. 2. \$22.50; \$10; \$810; \$300; \$1110; \$216; \$207; \$124.80; \$45.60; \$27; \$180; \$30; \$830.40; \$279.60.

Pages 26 and 27. (35.)—1. 114,912. 2. 54,900. 3. 51,623,304. 4. \$206,172. 5. 144. 6. 1520. 7. \$9046.70. 8. \$126,150. (36.)—1. 96 skirts. 2. \$28.80. 3. \$480; \$32; \$13 $\frac{1}{2}$; \$75; \$36; \$111; \$369; \$24.60. 4. \$666.66 $\frac{2}{3}$; \$333.33 $\frac{1}{3}$ less; \$208.33 $\frac{1}{3}$. 5. 45,000 gal.; 2250 gal.; \$937.50; 8000 lb.; \$1200; \$2137.50; \$562.50. 6. 50 mi.; 85 mi. 7. \$81.50.

Page 29. (39.)—1. 71 $\frac{1}{2}$. 2. 76 $\frac{1}{2}$. 3. 138. 4. 110 $\frac{1}{2}$. 5. 113. 6. 102 $\frac{1}{2}$. 7. 29. 8. 53 $\frac{1}{2}$. 9. 115 $\frac{1}{2}$. 10. 77 $\frac{1}{2}$.

Page 32. (46.) — 1. 177 $\frac{1}{2}$. 2. 237 $\frac{1}{2}$. 3. 200 $\frac{1}{4}$. 4. 133 $\frac{1}{2}$.
5. 59 $\frac{1}{2}$. 6. 79 $\frac{1}{2}$. 7. 276 $\frac{1}{2}$. 8. 215 $\frac{1}{2}$. 9. 18 $\frac{1}{2}$. 10. 54 $\frac{1}{2}$.
11. 179 $\frac{1}{2}$. 12. 151 $\frac{1}{2}$.

Page 33. (47.) — 5. 14,375,000. (49.) — 1. 36 $\frac{5}{8}$, 24 $\frac{5}{8}$, 31 $\frac{5}{8}$,
11 $\frac{5}{8}$, 14 $\frac{5}{8}$, 12 $\frac{5}{8}$; 43 $\frac{5}{8}$, 28 $\frac{5}{8}$, 4 $\frac{5}{8}$, 2 $\frac{5}{8}$, 17 $\frac{5}{8}$, 14 $\frac{5}{8}$; 62 $\frac{5}{8}$, 41 $\frac{5}{8}$,
6 $\frac{5}{8}$, 3 $\frac{5}{8}$, 24 $\frac{5}{8}$, 20 $\frac{5}{8}$; 341 $\frac{5}{8}$, 227 $\frac{5}{8}$, 34 $\frac{5}{8}$, 17 $\frac{5}{8}$, 136 $\frac{5}{8}$, 113 $\frac{5}{8}$;
66 $\frac{5}{8}$, 44 $\frac{5}{8}$, 61 $\frac{5}{8}$, 31 $\frac{5}{8}$, 26 $\frac{5}{8}$, 22 $\frac{5}{8}$. 2. \$21. 3. \$30. 4. \$12.80.

Page 34. (51.) — 1. 9 $\frac{1}{2}$; 16 $\frac{1}{2}$; 16 $\frac{1}{2}$; 23 $\frac{1}{2}$; 25 $\frac{1}{2}$; 27 $\frac{1}{2}$; 25 $\frac{1}{2}$;
18 $\frac{1}{2}$; 20 $\frac{1}{2}$; 12 $\frac{1}{2}$; 14 $\frac{1}{2}$; 8 $\frac{1}{2}$. 2. 96,031; 150. 3. 30 $\frac{1}{2}$.

Page 36. (54.) — 1. 1584. 2. 3281.25. 3. \$9923. 4. 7920
times. 5. \$8360.55. 6. 18,900 roses; \$1968.75. 7. 151,200;
\$5,103.

Page 37. (55.) — 1. \$.10. 2. \$.77, \$1.10, \$.99, \$1.32, \$1.65,
\$.220; \$.803. 3. \$.70. 4. \$.60. 5. \$.60; \$1.90. 6. \$1.

Page 38. — 7. \$.80. 8. 36 cows. 9. 325 sheep. 10. \$3.25 gain.
11. \$1116.14. 12. 205 qt. 13. \$4.10. 14. \$7.50. 15. \$.96. 16. $\frac{1}{2}$;
 $\frac{1}{2}$; $\frac{1}{2}$. 17. 48 in.; 1 $\frac{1}{2}$ yd. 18. \$400.

Page 39. — 19. 2720; 8160; \$652.80; \$10,200; \$.94; \$7674.16.
20. 267,120 yd.; \$21,369.60; \$8013.60. 21. \$205. 22. 9 mi.
23. 216 acres. 24. \$18.75.

Page 40. — 25. \$.65. 26. \$2.15. 27. \$6173.40; \$28.71 +.
28. \$91.60. 29. 482; 482; \$1996.

Page 41. — 30. \$100, \$97.50, \$468, \$477.75, \$100; \$1243.25;
\$752.75. (56.) — 1. 756 $\frac{1}{2}$. 2. 889 $\frac{1}{2}$. 3. 954 $\frac{1}{2}$. 4. 625. 5. 7242 $\frac{5}{8}$.
6. 6,409 $\frac{1}{2}$. 7. 753 $\frac{1}{2}$. 8. 680 $\frac{1}{2}$. 9. 415 $\frac{1}{2}$. 10. 790. 11. 796 $\frac{1}{2}$.
12. 1466. 13. 2407 $\frac{1}{2}$. 14. 10,039 $\frac{1}{2}$. 15. 5297 $\frac{1}{2}$. 16. 238 $\frac{1}{2}$.
17. 584 $\frac{1}{2}$. 18. 723 $\frac{1}{2}$. 20. 40. 21. 48. 22. 9 cu. ft.

Page 44. (62.) — 1. 30 qt., 34 qt., 32 qt.; 96 qt.; 12 pk.; 3 bu.
2. \$2.40, \$2.72, \$2.56; \$7.68. 3. \$2.07. 4. 186 lb.

Page 45. — 5. \$93; 114 lb. 6. 53 gal. 7. \$7.95; \$37.60. 8. 23
yd. 9. \$.69; \$1.26. 10. 32. 11. \$.48. 12. \$1.00. 13. \$44.
14. 426 rd. 15. 2434 lb.

Page 46. (63.) — 1. 409 oz. 2. 1228 oz. 3. 623 oz. 4. 750 lb.
5. 1975 lb. 6. 224 yd. 7. 137 yd. 8. 387 pk. 9. 314 pk. 10. 684 hr.
11. 1359 hr. 12. 150 mo. 13. 84 wk. 14. 187 mo. 15. 1060 wk.
16. 33 ft. 4 in. 17. 30 yd. 2 ft. 18. 8 lb. 4 oz. 19. 8 cwt. 25 lb.
20. 2 cwt. 63 lb. 21. 6 da. 4 hr. 22. 12 da. 8 hr. 23. 12 sq. ft.
24. 20 lb. 4 oz.

Page 51. (77.) — 1. 168. 2. 420. 3. 288. 4. 525. 5. 80.
6. 144. 7. 540. 8. 2640. 9. 78. 10. 378. 11. 225. 12. 375. 13. 756.
14. 2160. 15. 6237. 16. 360. 17. 15,840. 18. 1320.

Page 54. (82.) — 1. $11\frac{1}{2}$. 2. $11\frac{1}{2}$. 3. $12\frac{1}{2}$. 4. $11\frac{1}{4}$. 5. $14\frac{3}{4}$.
6. $10\frac{3}{8}$. 7. $16\frac{3}{8}$. 8. $13\frac{3}{4}$. 9. 15. 10. 21. 11. 10. 12. $12\frac{1}{4}$.

Page 55. (88.) — 1. $1\frac{1}{2}$. 2. $2\frac{3}{4}$. 3. $2\frac{1}{4}$. 4. $2\frac{3}{4}$. 5. $2\frac{3}{4}$.
6. $2\frac{1}{2}$. 7. $2\frac{1}{2}$. 8. $2\frac{3}{4}$. 9. $2\frac{3}{4}$. 10. $2\frac{3}{4}$. 11. $2\frac{3}{4}$. 12. $2\frac{3}{4}$.

Page 60. (103.) — 1. $1\frac{1}{2}$. 2. $2\frac{3}{4}$. 3. $1\frac{1}{2}$. 4. $1\frac{1}{2}$. 5. $1\frac{1}{2}$.
6. $2\frac{3}{4}$. 7. $1\frac{1}{2}$. 8. $2\frac{3}{4}$. 9. $2\frac{3}{4}$. 10. $2\frac{3}{4}$. 11. $1\frac{1}{2}$. 12. $2\frac{3}{4}$.
13. $1\frac{1}{2}$. 14. $2\frac{3}{4}$. (104.) — 1. $\frac{1}{2}$. 2. $\frac{3}{4}$. 3. $\frac{3}{4}$. 4. $\frac{1}{2}$. 5. $\frac{3}{4}$.

Page 61. (106.) — 1. 291. 2. 1010. 3. 624. 4. 726.
5. 8236. 6. 836. 7. 1005. 8. 3620. 9. 366. 10. 779.
11. 397. 12. 666. 13. 15. 14. 21. 15. 27. 16. 44.
17. 45. 18. 47. 19. 32. 20. 34.

Page 62. (107.) — 1. 250. 2. 791. 3. 202. 4. 274;
\$1209. 5. 139. 6. 159. 7. 334. 8. 35 in. 9. 77. 10. 46
in.

Page 64. (111.) — 1. $\frac{1}{2}$. 2. $\frac{7}{8}$. 3. $\frac{5}{8}$. 4. $\frac{1}{2}$. 5. $\frac{2}{3}$. 6. $\frac{2}{3}$.
7. $\frac{7}{8}$. 8. $\frac{7}{8}$. 9. $\frac{1}{2}$. 10. $\frac{1}{2}$. 11. $\frac{1}{2}$. 12. $\frac{1}{2}$. (112.) — 1. $\frac{1}{2}$;
 $\frac{1}{2}$; $\frac{1}{2}$. 2. $1\frac{1}{2}$; $\frac{1}{2}$; $2\frac{1}{2}$. 3. $\frac{1}{2}$; $\frac{1}{2}$; $\frac{1}{2}$. 4. $\frac{1}{2}$.

Page 65. (115.) — 1. $5\frac{1}{2}$. 2. $5\frac{1}{2}$. 3. $14\frac{1}{2}$. 4. $5\frac{1}{2}$. 5. $12\frac{1}{2}$. 6. $22\frac{1}{2}$.
7. $19\frac{1}{2}$. 8. $11\frac{1}{2}$. 9. $24\frac{1}{2}$. 10. $8\frac{1}{2}$. 11. $159\frac{1}{2}$. 12. $49\frac{1}{2}$. 13. $24\frac{1}{2}$.
14. $29\frac{1}{2}$. 15. $39\frac{1}{2}$.

Page 66. (118.) — 1. $17\frac{1}{2}$. 2. $26\frac{1}{2}$. 3. $14\frac{1}{2}$. 4. $15\frac{1}{2}$. 5. $8\frac{1}{2}$.
6. $15\frac{1}{2}$. 7. $6\frac{1}{2}$. 8. $27\frac{1}{2}$. 9. $100\frac{1}{2}$. 10. $9\frac{1}{2}$. 11. $38\frac{1}{2}$. 12. $21\frac{1}{2}$.

Page 67. — 13. $7\frac{1}{2}$. 14. $14\frac{1}{2}$. 15. $11\frac{1}{2}$. 16. $15\frac{1}{2}$. 17. $37\frac{1}{2}$.
18. $15\frac{1}{2}$. 19. $20\frac{1}{2}$. 20. $65\frac{1}{2}$. 21. $13\frac{1}{2}$. 22. $14\frac{1}{2}$. 23. $30\frac{1}{2}$. 24. $21\frac{1}{2}$.
25. $12\frac{1}{2}$. 26. $9\frac{1}{2}$. 27. $8\frac{1}{2}$. 28. $37\frac{1}{2}$. 29. $51\frac{1}{2}$. 30. $32\frac{1}{2}$. (119.) —
1. $37\frac{1}{2}$ yd. 2. $13\frac{1}{2}$ yr. 3. $15\frac{1}{2}$ mi. 4. $31\frac{1}{2}$ A. 5. $19\frac{1}{2}$ lb. 6. $48\frac{1}{2}$
in. 7. $17\frac{1}{2}$ mi. 8. $18\frac{1}{2}$ min. 9. $49\frac{1}{2}$ rd.

Page 69. (124.) — 1. $\frac{3}{4}$. 2. $\frac{5}{8}$. 3. $\frac{5}{8}$. 4. $\frac{7}{8}$. 5. $\frac{3}{4}$. 6. $\frac{1}{2}$.
7. $\frac{1}{2}$. 8. $\frac{1}{2}$. (125.) — 1. $\frac{3}{8}$; $\frac{3}{8}$; $\frac{3}{8}$; $\frac{1}{2}$. 2. $\frac{3}{8}$; $\frac{3}{8}$; $\frac{3}{8}$.

Page 71. (129.) — 1. $\frac{1}{2}$. 2. $\frac{1}{2}$. 3. $\frac{1}{2}$. 4. $\frac{1}{2}$. 5. $5\frac{1}{2}$. 6. $1\frac{1}{2}$.
7. $\frac{1}{2}$. 8. $\frac{1}{2}$. 9. $\frac{1}{2}$. 10. $\frac{1}{2}$. (131.) — 1. 125. 2. 294. 3. 584.
4. 324. 5. 200. (133.) — 1. $\$81\frac{1}{2}$. 2. $\$53\frac{1}{2}$. 3. $\$177\frac{1}{2}$. 4. $\$23\frac{1}{2}$.
(134.) — 1. $2\frac{1}{2}$. 2. $1\frac{1}{2}$. 3. $4\frac{1}{2}$. 4. $1\frac{1}{2}$. 5. $20\frac{1}{2}$. 6. $21\frac{1}{2}$. 7. 21. 8. 33.
9. 35. 10. $1\frac{1}{2}$. 11. $\frac{1}{2}$. 12. $\frac{1}{2}$. 13. $\frac{1}{2}$. 14. $\frac{1}{2}$. 15. 1. 16. $\frac{1}{2}$. 17. $\frac{1}{2}$.
18. $1\frac{1}{2}$. 19. $3\frac{1}{2}$. 20. $3\frac{1}{2}$. 21. 10. 22. $3\frac{1}{2}$. 23. $5\frac{1}{2}$. 24. $22\frac{1}{2}$. 25. $7\frac{1}{2}$.

26. 30. 27. $4\frac{1}{2}$. 28. 27. 29. $8\frac{1}{2}$. 30. 28. (135.) — 1. \$69 $\frac{1}{2}$.
2. \$3.35 $\frac{1}{2}$. 3. \$4.18 $\frac{1}{16}$. 4. 125. 5. $57\frac{1}{2}$ cu. ft. 6. 129 $\frac{1}{2}$ bu. 7. 11 $\frac{1}{2}$
rows. 8. 110 $\frac{1}{2}$ knots. 9. \$2.09 $\frac{1}{2}$; \$2.45 $\frac{1}{2}$. 10. $8\frac{1}{2}$ mi.

Page 74. (139.) — 1. 4. 2. 2. 3. $\frac{1}{2}$. 4. 6. 5. 7. 6. 10. 7. 2.
8. 6. 9. 1. 10. 4. 11. 8. 12. 26. 13. $\frac{1}{2}$. 14. $\frac{1}{2}$. 15. $1\frac{1}{2}$. 16. 24.
17. 10. 18. 14. (140.) — 1. 11 families; \$4.12 $\frac{1}{2}$. 2. 11; \$2.64.
3. 11 days; \$149.50; \$82.50. 4. 12 coats; \$39.

Page 76. (145.) — 1. $\frac{1}{2}$. 2. $\frac{1}{2}$. 3. $\frac{1}{2}$. 4. $2\frac{1}{16}$. 5. $1\frac{1}{16}$. 6. $1\frac{1}{2}$.
7. $1\frac{1}{16}$. 8. $1\frac{1}{16}$. 9. $\frac{1}{2}$. 10. $\frac{1}{2}$. 11. $1\frac{1}{2}$. 12. $\frac{1}{16}$. (148.) — 1. $9\frac{1}{2}$.
2. 12 $\frac{1}{2}$. 3. $\frac{1}{16}$. 4. $\frac{1}{2}$. 5. 6. 6. 14 $\frac{1}{2}$. 7. $\frac{1}{16}$. 8. $\frac{1}{2}$. 9. $5\frac{1}{2}$. 10. $\frac{1}{16}$.
11. $\frac{1}{16}$. 12. $5\frac{1}{2}$. 13. $\frac{1}{2}$. 14. $\frac{1}{2}$. 15. $\frac{1}{16}$. 16. $2\frac{1}{2}$.

Page 77. (149.) — 1. 8 barrels; \$30. 2. 48 crates; \$204.
3. 8 coats; $\frac{1}{2}$ yd. 4. 16 fields. 5. 8 bows. 6. 21 licenses. 7. $8\frac{1}{2}$ days.
8. 10 bags. 9. \$2.25. 10. $3\frac{1}{16}$.

Page 78. (151.) — 1. 1890. 2. 2520. 3. 1680. 4. 4368. 5. 288.
6. 720. (153.) — 1. 5. 2. $6\frac{1}{2}$. 3. 9. 4. $14\frac{1}{16}$. 5. $15\frac{1}{2}$. 6. 12. 7. $21\frac{1}{2}$.
8. 32. 9. $7\frac{1}{16}$. 10. $12\frac{1}{2}$. 11. $20\frac{1}{2}$. 12. 8. 13. $14\frac{1}{2}$. 14. 20. 15. $22\frac{1}{2}$.
16. $10\frac{1}{2}$. 17. $18\frac{1}{2}$. 18. 14. 19. $4\frac{1}{2}$. 20. $10\frac{1}{16}$.

Page 79. (155.) — 1. $54\frac{1}{2}$. 2. $70\frac{1}{2}$. 3. $118\frac{1}{16}$. 4. $86\frac{1}{16}$. 5. $151\frac{1}{2}$.
6. $80\frac{1}{2}$. (156.) — 1. $4\frac{1}{2}$. 2. $24\frac{1}{2}$. 3. $10\frac{1}{16}$. 4. $13\frac{1}{2}$. 5. $56\frac{1}{16}$. 6. $\frac{1}{2}$.
(158.) — 1. 592. 2. \$23 $\frac{1}{2}$. 3. 1640. 4. $49\frac{1}{2}$. 5. \$72.72. 6. 5274.
7. \$73.35. 8. $48\frac{1}{16}$. 9. 299.39. 10. 4940. 11. \$24.56. 12. $68\frac{1}{2}$.
(159.) — 1. $1\frac{1}{2}$. 2. $\frac{1}{2}$. 3. $1\frac{1}{2}$. 4. $1\frac{1}{2}$. 5. $\frac{1}{2}$. 6. $1\frac{1}{16}$. 7. $\frac{1}{2}$. 8. $\frac{1}{16}$.
9. $\frac{1}{16}$. 10. $1\frac{1}{16}$. 11. $1\frac{1}{2}$. 12. 28. 13. $3\frac{1}{2}$. 14. $8\frac{1}{2}$. 15. 57. 16. $\frac{1}{16}$.
17. 84. 18. 45. 19. $\frac{1}{2}$. 20. $1\frac{1}{2}$. 21. $\frac{1}{2}$. 22. $27\frac{1}{16}$. 23. $15\frac{1}{2}$.
24. $38\frac{1}{2}$.

Page 80. (160.) — 1. $284\frac{1}{2}$ acres; $53\frac{1}{2}$ acres; $36\frac{1}{2}$ acres.
2. $289\frac{1}{2}$ lb.; $25\frac{1}{2}$ lb.; $11\frac{1}{2}$ lb.; \$1.65 $\frac{1}{2}$; \$3.28 $\frac{1}{2}$. 3. 8 bags. 4. \$65.
5. $60\frac{1}{2}$ doz.; 781 brooms; \$47.25, \$55.50, \$50, \$29.50; \$182.25.

Page 81. — 6. 126 lb.; \$6.30; \$31 $\frac{1}{2}$. 7. \$1.50. 8. $23\frac{1}{2}$ yd.;
\$56.10; \$70.12 $\frac{1}{2}$. 9. 160 lb.; \$4. 10. 16 hats. 11. $10\frac{1}{2}$ yd. 12. 30
aprons. 13. $109\frac{1}{2}$ yd. 14. $54\frac{1}{2}$ yd.; $36\frac{1}{2}$ yd.; $18\frac{1}{2}$ yd.

Page 82. (161.) — 1. 20 sq. in. 2. 25 ft.; 20 ft.; 90 ft.
3. 2000 sq. ft. 4. 8000 sq. ft. 5. 1728 sq. ft. 6. 6912 sq. ft.
7. 100 ft.; 25 ft.; 2500 sq. ft.; 250 ft. 8. A, $18\frac{1}{2}$ ft. by 25 ft.; B,
 $18\frac{1}{2}$ ft. by $31\frac{1}{2}$ ft.; C, $18\frac{1}{2}$ ft. by 25 ft.

Page 83. (163.) — 1. $2\frac{1}{16}$. 2. 2. 3. $1\frac{1}{2}$. 4. $57\frac{1}{16}$. 5. $67\frac{1}{2}$.
6. $141\frac{1}{2}$. 7. $68\frac{1}{2}$. 8. $60\frac{1}{2}$. 9. $130\frac{1}{2}$. (164.) — 1. $\frac{1}{2}$. 2. $\frac{1}{16}$. 3. $4\frac{1}{2}$.
4. $3\frac{1}{2}$. 5. $21\frac{1}{16}$. 6. $11\frac{1}{2}$.

Page 84. (165.) — 1. $\frac{7}{12}$. 2. $\frac{3}{10}$. 3. $2\frac{1}{2}$. 4. $9\frac{1}{11}$. 5. $14\frac{7}{11}$.
6. $5\frac{3}{11}$. (166.) — 1. 5 acres. 2. $20\frac{1}{2}$ acres. 3. \$7687.50. 4. 4120 min.
 5. 756 hr. 6. \$672. 7. 13,532 oz. 8. 8974 oz. 9. 19,784 oz.
 10. \$76.50. 11. \$3.60. 12. \$19,200. 13. \$204.75; \$236.25; \$288.75.
 14. \$20.46. 15. \$.42.

Page 85. — 16. 90 in.; 350 ft.; 576 rd.; 135 in.; $199\frac{1}{2}$ ft.;
 180 rd. 18. 1536 sq. ft. 19. 18,824 sq. ft. 20. $54\frac{1}{2}$ bu. 21. $88\frac{1}{2}$.
 22. $2\frac{7}{10}$. 23. $44\frac{1}{2}$ acres. 24. $773\frac{7}{10}$ yd. 25. $13\frac{1}{2}$ yr. 26. $23\frac{1}{2}$ yr.
 27. $6\frac{1}{2}$ yr.

Page 86. — 28. 2 hr. 10 min. 29. $25\frac{1}{2}$. 30. $70\frac{1}{4}$ mi. 31. $93\frac{1}{4}$.
 32. $\$16\frac{1}{2}$; $\$5\frac{1}{2}$. 33. $228\frac{1}{2}$ mi. 34. $14\frac{1}{2}$ hr. 35. \$1.83+. [36. $21\frac{1}{2}$
 yd. 37. $2\frac{1}{2}$ yd. 38. $\$8\frac{1}{2}$, $\$6\frac{3}{4}$, $\$12\frac{3}{8}$; $\$27\frac{1}{2}$.

Page 90. (174.) — 1. $33\frac{1}{2}$. 2. 15. 3. 100. 4. $28\frac{1}{2}$. 5. 35.
 6. 1150. 7. 300. 8. 5400. 9. 21.

Page 98. (191.) — 1. 2,250,711. 2. 56,292. 3. 2,010,839.
 4. 16,730,852.

Page 99. (192.) — 1. 45.55. 2. 124.032. 3. 80.075. 4. 169.725.
 5. 211.251. 6. 139.268. 7. 205.999. 8. 651.005. 9. 2923.4539.
 10. 11,395.889038. (193.) — 1. 85.5 ft. 2. 55.5 in. 3. 49.95 ft.
 4. 175 ft.; 37.5 ft. 5. 156.30 mi.

Page 100. (195.) — 1. 306.64. 2. 137.9734. 3. 417.95.
 4. 139.052. 5. 11.05. 6. 9.375. 7. 88.375. 8. 43.78. 9. 586.055.
 10. 57.8136. 11. 174.25. 12. 3.045. 13. .9. 14. 19.085. 15. 15.1225.
 16. .325575. 17. 1.484. 18. 141.895. 19. 2.3999985.

Page 101. (196.) — 1. \$.80, \$.25, $\$1.12\frac{1}{2}$, \$.75, \$2.25; $\$5.17\frac{1}{2}$.
 2. 284.75 ft. 3. 238.34 cu. ft. 4. 14.75 yd. 5. 114.965 rd. 6. .55 ft.;
 .95 ft.; 1.5 ft. 7. 108.125 acres. 8. 107.525 acres. 9. \$143.

Page 103. (198.) — 1. \$71.20. 2. \$16.90.

Page 106. (203.) — 1. 109.2. 2. 5.76. 3. 1.242. 4. 70.47.
 5. 58.5. 6. 336. 7. 1670.2. 8. 1038.15. 9. 13.068. 10. 3271.2.
 11. 3171. 12. 1002.186. 13. 1502. 14. 641.44. 15. 270.3. (204.) —
 1. 960 acres; 640 acres; 320 acres; 800 acres; 480 acres; 320 acres;
 30,720 bu.; 23,040 bu.; \$30,105.60; \$14,976. 2. \$22.50. [3. \$39.
 4. \$710.

Page 107. (205.) — 1. \$157.50; \$46.50; \$111; \$108.50;
 \$49. 2. \$208.75; \$192; \$172; \$155.25; \$140.25; \$111.75; \$90.25;
 \$75; 5 mo.; \$15. 3. 12 mo.

Page 110. (211.) — 1. .00016; .00531; 3.3009. 2. .00541; 5.4146+; 45.9071. 3. 2.1312; .011708+; .65431. 4. 40.6014; 7.32911+; 484628. 5. 1.46. 6. .15+. 7. .09. 8. .55. 9. 3.887+. 10. 3.44. 11. 13.04. 12. 23.5. 13. 5.45.

Page 111. (216.) — 1. 91.19. 2. 6.706. 3. 3500. 4. 180.958. 5. 17.142. 6. 4.311. 7. 3.993. 8. 69.044. 9. 3.442. 10. 2.903. 11. 2.256. 12. 3442.857. 13. .00834. 14. 9.662. 15. .018. 16. .468. 17. 69.92. 18. 2.577.

Page 112. (217.) — 1. 20,026.7857. 2. .005. 3. .6093+. 4. 281.4074. 5. 125. 6. 30,520. 7. 91.5. 8. 3.424. 9. .0139. 10. 24.6031. 11. .0035. 12. 1.1155. 13. 2.01. 14. 3004. 15. 60.0109. 16. 20.04. 17. 12,245.1948. 18. 12.5736. 19. .0389. 20. 236.8386. 21. 606571.0353+. 22. 378.3245+. 23. .0314. 24. 3025.3984. (218.) — 1. 200 bags. 2. 392 lb. 3. 363.9 lb. 4. 76 lb.

Page 113. — 5. 28.5 tons. 6. 8.93 tons. 7. \$.061+. 8. .140 spools. 9. 35.33½ bu. 10. \$.0825; \$.092; \$.113; \$.104. 11. \$.60.17½. 12. \$.8.55½. 13. \$.73.04. 14. \$.57.42. 15. \$.62.75. 16. \$.202.50. 17. \$.18.06½. 18. \$.67½. 19. \$400. 20. 600.

Page 115. (225.) — 1. .33½. 2. .66½. 3. .75. 4. .857½. 5. .444½. 6. .714½. 7. .833½. 8. .181½. 9. .375. 10. .357½. 11. .416½. 12. .437½. 13. .916½. 14. .176½. 15. .409½.

Page 116. (226.) — 1. 8.41285. 2. .25. 3. 1.59375. 4. 18.432. 5. .143½. 6. .07356. 7. .005625. 8. 14.4. 9. 14.4. 10. 1. 11. 5000. 12. 17.125. 13. 60.708½. 14. .0861. 15. 1. (227.) — 1. \$.25½. 2. 3½ reams. 3. 37½ doz. 4. 12 reams.

Page 119. (233.) — 1. 200. 2. 700. 3. 600. 4. 133½. 5. 250. 6. 50. 7. 300. 8. 66½. 9. 500. 10. 450. 11. 266½. 12. 320. 13. 350. 14. 150. 15. 550. 16. 600.

Page 121. (236.) — 1. 4. 2. 1050. 3. 455. 4. \$200; \$800, \$400, \$200. 5. \$2437.46. 6. \$72.03. 7. \$1801.62½. 8. \$308.85. 9. \$900. 10. \$190.86.

Page 122. — 11. \$1152.75. 12. 87½%, \$1400, \$3000; 25%, \$400, \$2000; 62½%, \$1000, \$2600; 12½%, \$200, \$1400; 87½%, \$1400, \$200; 62½%, \$1000, \$600; 50%, \$800, \$2400; 12½%, \$200, \$1800; 37½%, \$600, \$2200; 75%, \$1200, \$400; 25%, \$400, \$1200; 37½%, \$600, \$1000. 13. \$3300. 14. \$1500. 15. \$2500; \$1500. 16. \$1.50; \$.50. 17. \$250. 18. \$4000; \$20,000. 19. \$4000; \$12,000. 20. \$1760. 21. \$.01.

Page 124. (239.) — 1. \$182. 2. \$321.86. 3. \$905.62½. 4. \$769.13. 5. \$173.25. 6. \$3001.475. 7. \$3579.30. 8. \$6014.32. 9. \$8520. 10. \$3168.28. 11. \$4483.05. 12. \$6181.92.

Page 125. (241.) — 1. \$7.20. 2. \$21.60. 3. \$77.20. 4. \$135. 5. \$205.92. 6. \$172.80. 7. \$101.565. 8. \$23.055. 9. \$13.62. 10. \$68.67. 11. \$258.79. (242.) — 1. \$504. 2. \$280. 3. \$744. 4. \$1236. 5. \$681. 6. \$345. 7. \$315. 8. \$435.88. 9. \$585.325.

Page 126. (245.) — 1. \$75.60. 2. \$21. 3. \$122.40. 4. \$49.665. 5. \$54.056. 6. \$76. 7. \$41.70. 8. \$95.095. 9. \$27.647. 10. \$47.14+.

Page 127. (247.) — 1. \$129.60; \$43.20; \$86.40; \$172.80. 2. \$13.50; \$9. 3. \$12.40; \$25. 4. Madison \$1200, Morristown \$2400. 5. \$2125. 6. 40 cows. 7. 12 horses.

Page 128. — 8. \$.38. 9. 23½ yd. 10. \$299. 11. 40 ft.; 1600 sq. ft. 12. 320 ft.; 6400 sq. ft. 13. Gain, 66⅓%; ⅓; \$320; \$800. 14. \$640. 15. \$125 loss. 16. ⅓. 17. 5; ⅓; 4½; 6½. 18. \$164; \$1148; \$3608; \$3444. 19. \$1.45. 20. 3450 lb.

Page 129. — 21. 120 rd. 22. 200 rd. 23. 280 rd. 24. 3. 25. 5½. 26. ⅓. 27. 2. 28. 2025 sq. ft. 29. \$450. 30. \$389. 31. 1,161,930 lb. 32. 1,510,509 lb. 33. 188,813½ lb.; 62,937½ goats. 34. \$133.955. 35. 111½ days. 36. \$.00¼ per pound.

Page 130. — 37. \$.35. 38. 8766 hr.; 525,960 min.; 31,557,600 sec. 39. 127 bu. 40. 364.5 ft. 41. 35½ lb. 42. \$21.55. 43. 1007.325 sq. ft. 44. 23½ gal. 45. 80 yd. 46. 6 yd.

Page 136. (260.) — 1. 8 rd; 10 rd.; 5 rd. 2. \$750; \$1500; \$400. 3. \$32,000; \$30,000; \$72,000; \$85,333½; \$192,000.

Page 143. (275.) — 1. 1183 gi. 2. 408 gi. 3. 824 gi. 4. 191 qt.

Page 144. — 5. 789 qt. 6. 2405 qt. 7. 1240 qt. 8. 1504 qt. 9. 240 acres. 10. 50 sq. rd. 11. 35 sq. rd. 12. 22½ sq. yd.; 29,403 sq. in. 13. 16,320 acres. 14. 6200 sq. rd. 15. 1452 sq. yd. 16. 128.25 sq. ft. 17. 73,3½ sq. yd. 18. 120,000 oz. 19. 332 oz. 20. 1,152,000 oz. 21. 1,325,730 sec. 22. 70,420,220 sec. 23. 64,680 ft. 24. 64,240 yd. 25. 135,080 yd.

Page 145. (277.) — 1. \$10.20. 2. \$41.84. 3. \$70.86. 4. \$16.75; \$12.50; \$75.25. 5. 272½ sq. ft.; 6534 sq. ft.; 9801 sq. ft. 6. \$575.12½. 7. 1080 min.; 64,800 sec. 8. 30½ min. 9. 60,000. 10. 23,040 cu. ft. 11. 10 pens. 12. \$22.60. 13. \$450.

Page 146. (279.) — 1. 133 gal. 3 qt. 1 pt. 2 gi. 2. 89 gal. 3 qt. 0 pt. 3 gi. 3. 97 gal. 2 qt. 1 pt. 3 gi. 4. 165 gal. 0 qt. 1 pt. 5. 117 bu. 2 pk. 4 qt. 6. 238 bu. 3 pk. 7. 589 bu. 0 pk. 6 qt. 8. 430 bu. 0 pk. 4 qt. 9. 8 mi. 73 rd. 4 yd. 0 ft. 10 in. 10. 6 mi. 55 rd. 1 yd. 2 ft. 6 in. 11. 24 mi. 221 rd. 1 yd. 2 ft. 6 in. (280.) — 1. \$3.10. 2. \$3.80. 3. \$14.66. 4. 3; 320; 1760.

Page 147. — 5. 36 sq. rd. 16 sq. yd. 6. 76 sq. rd. 12 sq. yd.; 153 sq. rd. 15½ sq. yd.; 324 sq. rd. 27 sq. yd. 7. 4 A. 36 sq. rd. 15 sq. yd.; 12 A. 109 sq. rd. 12½ sq. yd.; 6 A. 54 sq. rd. 22½ sq. yd.; 3 A. 24 sq. rd. 2 sq. yd. 8. \$2975. 9. \$17,500. 10. \$108. 11. 4 yr. 33 da. 8 hr. 12. 86,400 sec. 13. \$525,960. 14. 27 acres. 15. 3240 bu. 16. 1160 bags. 17. \$.636. 18. \$2030. 19. \$1826.

Page 148. (282.) — 1. 2.34375 gal. 2. 8.28125 gal. 3. \$2.23 +. 4. \$26.53 +. 5. \$33.516. 6. .472 mi. 7. 70.005 sq. mi. 8. .408 T. 9. .778 da. +. 10. \$44.63. 11. \$33.03 +. 12. \$10.968. 13. \$9150.

Page 150. (285.) — 1. 140 rd. 2. 2970 ft. 3. 4224 ft. 4. 457.6 yd. 5. 22,176 in. 6. 20 sq. yd. 5 sq. ft. 116.1 sq. in.; 135 sq. rd. 8 sq. yd. 4 sq. ft. 32.12 sq. in.; 272 A. 153 sq. rd. 18 sq. yd. 1 sq. ft. 50.4 sq. in. 7. 6 cwt. 15 lb. 12 oz.; 16 cwt 12 lb. 8 oz.; 8 cwt. 14 lb. 4 oz.; 15 cwt. 85 lb. 8. 8 hr. 45 min. 36 sec.; 17 hr. 31 min. 12 sec.; 8 hr. 47 min. 2.4 sec. 9. 11 hr. 25 min. 26 sec.; 5.16 oz.; 7 sq. yd. 7 sq. ft. 81.4 sq. in.; 17 min. 45.6 sec. 10. 1382.4 cu. in.; 293 rd. 2 yd. 2 ft.; 106 sq. rd. 20 sq. yd. 1 sq. ft. 72 sq. in.

Page 151. (287.) — 1. 40 mi. 80 rd. 9 in. 2. 67 gal. 2 qt. 1 pt. 3. 17 T. 3 cwt. 38 lb. 3 oz.

Page 152. (289.) — 1. 192 mi. 97 rd. 2 ft. 6 in. 2. 56 sq. rd. 64 sq. in. 3. 53 cu. yd. 23 cu. ft. 722 cu. in. 4. 44 T. 19 cwt. 47 lb. 2 oz. 5. 205 da. 21 hr. 21 min. 55 sec. 6. 529½ lb. 7. 115 yr. 4 mo. 15 da. 8. 37 ft. 1 in. 9. 2 bu. 3 pk. 3 qt.

Page 153. — 10. \$1.65; \$1.40; \$.80; \$1.40; \$.90. 11. 15 qt. 12. \$.38, \$.30, \$.23, \$.45, \$.35; \$1.71. 13. 9 cwt. 26 lb. 5 oz. 14. 6 bu. 2 pk. 3 qt. 15. 33 gal. 1 qt. 1 pt. 16. 18 yd. 0 ft. 8 in. 17. 26 yd. 2 ft. 9 in. 18. 986 mi. 25 rd. 3 yd. 0 ft. 7 in.

Page 154. (291.) — 1. 11 bu. 1 pk. 0 qt. 1 pt. 2. 4 bu. 3 pk. 7 qt. 1 pt.

Page 155. (293.) — 1. 14.875 gal. 2. 44.625 gal. 3. \$18. 4. 667 bu. 2 pk. 1 qt. 5. 899,000 sq. ft. 6. \$117.19. 7. \$216.75. 8. \$.3.22.

Page 156. — 9. .73. 10. \$69.37½. 11. \$7.32½. 12. 330. 13. 40 sq. rd.; ¼ A. 14. \$3.96, 50 lb. 15. \$1.17. 16. 30 min.

Page 157. (295.) — 1. 169 da. 2. 172 da. 3. 215 da. 4. 304 da. 5. 46 da. (297.) — 1. 2 yr. 7 mo. 21 da. 3. 2 mo. 1 wk. 5 da. 4. 4 yr. 1 mo. 21 da.

Page 158. (299.) — 1. \$65.06½; \$15.01½. 2. \$3.80. (300.) — 1. \$5.22.

Page 159. — 2. 306.4 gal. 3. 1275.22 gal. 4. Corn, 385.6 bu.; Wheat, 301.33 bu.; Barley, 202.05 bu. 5. 65 yd. 0 ft. 2 in. 6. 122 yd. 2 ft. 3 in. 7. 23 yd. 2 ft. 8 in. 8. 10 yd. 0 ft. 4 in. 9. 3600 gal.; 7200 gal.; 4590 gal.; 4083.95 gal.; 5850 gal.

Page 160. — 10. 52 yd. 1 ft. 6 in.; 37 yd. 11. \$303.75. 12. 32 cu. yd. 13. 21½ cu. yd. 14. 7296 cu. ft.; 5568 cu. ft.; 7980 cu. ft. 15. 2062.25 cu. ft. 16. 6502.5 gal. 17. 6417.18 gal. 18. 216 cu. ft.; 8 cu. ft.; 27. 19. 281.25 gal. 20. 2 ft. 21. 1 ft. 8 in.

Page 161. (303.) — 1. 48.375 gal. 2. 40½. 3. 27 ⅔ da.

Page 162. — 4. 25½ da. 6. 40 persons. 7. 9 cu. ft. 8. 6666½ cakes. 9. 14½. 10. 48 books. 11. 144 cu. ft. 12. 30 cu. ft. 13. 120 quart bottles; 240 pint bottles; \$8.40. 14. 40 ft. 15. 264 ft.

Page 163. — 16. 148 ft. 6 in. 17. 24 panes. 18. 53½ sq. ft. 19. 131.25 cu. ft. 20. 2 ft. 6 in. 21. 5½ in. by 4½ in. by 2 ft. 6 in. 22. 60 lb.

Page 165. (306.) — 1. 8.1 qt. 2. 2 T. 4 cwt. 34 lb. 3. \$14. 4. 720 board feet. 5. 960 tiles. 6. 40. 7. \$40.62½. 8. \$27.54. 9. 132 steps.

Page 166. — 10. 124 mi. 190 rd. 11. 190 suits. 12. ⅔. 13. 10½%. 14. 134 T. 8 cwt.; 168 T.; 140 T. 11 cwt. 20 lb. 15. \$228. 16. \$270. 17. \$192. 18. 4 T. 7 cwt. 50 lb. 19. 2 T. 16 cwt. 25 lb. 20. 2 T.; 4 T.; 8 T.; 4 T. 6 cwt. 21. \$59.06½. 22. \$58.78.

Page 167. — 23. \$105. 24. 30 yd. 25. 16.66+ yd. 26. 42 yd. 1 ft. 3 in. 27. 360 ft. 28. 540 ft.; 20%. 29. 4200 ft. 30. 11½%.

Page 168. — 31. 83½%. 32. 77 ft. 3 in.; 204 ft. 6 in. 33. 8100 sq. ft.; 900 sq. yd.; 29½ sq. rd. 34. 13 rolls; \$4.55. 35. \$9.45; \$9.60; \$27.05; \$11.70; \$9.80. 36. \$26.25; \$20.25; \$33; \$62.50; \$24.75.

Page 169. — 37. 41.58 sq. rd. 38. 11,320.155 sq. ft. 39. 2640 cu. ft. 40. \$140.62½. 41. 6 cords. 42. \$43.50; \$51.60. 43. 3; 4; 3; 12. 44. \$1.98. 45. \$926.50; \$4506. 46. 1.55+ A.; 5000 sq. ft.;

7500 sq. ft.; 12,500 sq. ft.; 20,000 sq. ft.; 15,000 sq. ft.; 17,500 sq. ft. 47. 2; 3; 5; 8; 6; 7. 49. \$3750; \$7500; \$5250.

Page 170. — 50. 60 ft. 51. 144 sq. yd.; 96 sq. yd.; 216 sq. yd. 52. \$37.12. 53. \$31.55. 54. 11,951 laths; \$35.853; 4.268 da.; \$8.399; \$229.96; \$39.44; \$20.38 loss. 55. \$54; \$162.

Page 171. — 56. 11,250 sq. ft.; 1125 sq. ft.; 12,375 sq. ft. 57. 88 ft.; 2 ft.; 72 ft.; 1920 cu. ft. 58. 4 times. 59. 416.842 times. 60. 207 cars.

Page 172. — 61. 144 rails. 62. \$237.60. 63. \$230.40. 64. 450 ft.; 350 ft. 65. 18; 6. 66. \$156.25; \$2000. 67. 1632 ft. 68. 6464 sq. ft.

Page 173. — 69. $8\frac{1}{8}$ ft.; \$.53 +; $1\frac{1}{4}$ sq. ft.

Page 175. (310.) — 1. 458.4. 2. 176.89. 3. 35.3925. 4. 914.88. 5. \$126.86875. 6. \$2.403. (311.) — 1. \$3800. 2. \$2125; \$2443.75.

Page 176. — 3. $41\frac{3}{4}\%$. 5. 52.8 ft.; 79.2 ft.; 6.6 ft.; 24.75 ft.; 79.2 ft.; 118.8 ft. 6. $33\frac{1}{2}\%$. 7. 10%; 12%. 8. 25%; $16\frac{2}{3}\%$; 25%. 9. 1056 ft. 10. $9\frac{1}{8}\%$.

Page 178. (314.) — 1. 8%. 2. $14\frac{3}{4}\%$. 3. 20%. 4. $11\frac{1}{2}\%$. 5. $30\frac{1}{8}\%$. 6. $87\frac{1}{2}\%$. (316.) — 1. 900.

Page 179. — 2. \$1635. 3. 63,360 in. 4. 338.28 A. 5. 2000 qt. 6. 25 mi. 7. \$1250. 8. 112,000. 9. \$4500. 10. 350. 11. \$60; \$144. 12. 1280; 450; $2213\frac{3}{4}$.

Page 181. (319.) — 1. 18,480 bu. 2. \$13,150; \$12,656.25. 3. 1500 ft. 4. \$47.84. 5. \$172.90. 6. \$1320. 7. \$660. 8. \$400. 9. \$5.10. 10. \$3.60; 25%.

Page 182. — 11. $71\frac{1}{2}\%$. 12. Loss $11\frac{1}{2}\%$. 13. 25%. 14. $15\frac{1}{2}\%$. 15. $45\frac{5}{11}\%$. 16. $38\frac{1}{2}\%$; $61\frac{1}{2}\%$. 17. $95\frac{5}{11}\%$. 18. 4%. 19. $31\frac{1}{2}\%$. 20. 30%.

Page 184. (322.) — 1. \$35. 2. \$136.98. 3. \$824.685. 4. \$247.30. 5. \$104.40. 6. \$40.29. (323.) — 1. \$630; \$3630. 2. \$11.25; \$386.25. 3. 1 yr. 4 mo.; \$67.92. 4. \$2516.67; \$1168.75; \$1600.

Page 185. (324.) — 1. \$404.79.

Page 186. — 2. \$851.70. (325.) — 1. \$2. 2. \$102. 3. \$2.04. 4. \$104.04. 6. \$154.40.

Page 192. (330.) — 1. 90 in.; 61.275 in.; 151.125 in. 2. 855 sq. ft. 3. \$6.90. 4. \$562.50.

Page 193. — 5. 240 sq. in.; $1\frac{1}{2}$ sq. in.; 35 sq. in.; \$.02. 6. $2\frac{1}{2}$ in.; $1\frac{1}{2}$ in. 7. 24 boys.

Page 194. — 9. $6\frac{1}{2}$ sq. ft. 10. 1296 sq. in. 11. 1 mi.; 6400 sq. rd.

Page 195. (332.) — 1. 40 in. 2. 140 in. 3. 18.5 in.

Page 196. (335.) — 1. 96 sq. in.; 135 sq. in.; 198 sq. in.; 240 sq. in.; 500 sq. in. 2. 30 A.; $42\frac{1}{16}$ A.; $60\frac{1}{16}$ A.; 75 A.; $156\frac{1}{2}$ A. 3. $1\frac{1}{8}$ A. 4. 5 A. 5. $4\frac{1}{8}$ A. 6. \$80.36. 7. $2\frac{1}{2}$ A.; $1\frac{1}{2}$ A.; $1\frac{1}{2}$ A. 8. 1521 trees.

Page 197. — 9. 1 cow. 10. \$16.875. 11. $\frac{1}{16}$ T. 12. 768 sq. ft., 64 sq. ft. in each bed except Peas 24 sq. ft., Radishes 24 sq. ft. Lettuce 48 sq. ft., Parsley 48 sq. ft.; 256 qts.; \$2.56.

Page 198. — 13. $75\frac{1}{16}$ A. 16. 23,760 ft. 17. \$4752. 18. 18 sq. ft. 19. 122 sq. ft.

Page 199. (338.) — 1. 44 in. 2. 110 in. 3. 44 in. 4. 88 in. 5. 14 in. 6. 7 in. 7. 14 in. 8. 102 in. 9. 55 in. 10. $10\frac{1}{2}$ in. 11. $10\frac{1}{11}$ rd. 12. $157\frac{1}{2}$ ft. 13. $53\frac{1}{2}$ in. 14. 840 ft.

Page 200. (340.) — 1. 16.6874 in. 2. 17.822 in. 3. 65.9736 in. 4. 43.9824 in. 5. 7.0024 in. 6. 14.0056 ft. 7. 188.496 sq. ft.; 383.2752 sq. ft. 8. 25.1328 ft. 9. 21.9912 in. 10. \$27.49. 11. 56.5488 ft.

Page 201. (341.) — 1. 198 min.; 18 min. past three. 2. 56 min. past 12 A.M. 3. 3000 bu.; \$1875. 4. $52\frac{1}{4}\%$. 5. $21\frac{1}{8}\%$. 6. 14,814.81 lb. 7. 20 sq. rd.; 40 sq. rd. 8. 160 rd.; 200 rd. 9. 8 ft. 10. $\frac{1}{2}$.

Page 202. — 11. 50%. 12. $6\frac{1}{2}\%$. 13. \$8. 14. \$12.60. 15. \$5.91. 16. $\frac{1}{2}$; $\frac{1}{2}$. 17. \$7.20. 18. \$21.70. 19. 50 ft. 20. \$125. 21. 80 ft. 22. 240 ft. 23. \$3. 24. 2.

Page 203. — 25. 90; 200. 26. 5. 27. \$70; \$17.50. 28. 10%. 29. $16\frac{1}{2}\%$. 30. \$8; 20 ft.; 9 yd. 31. \$11.25; \$4.50; \$25. 32. 4; 8; $\frac{1}{2}$. 33. \$40; \$36. 34. 140 lb.; 250 lb.; 750 lb. 35. \$65. 36. \$99. 37. \$36; \$24.12; \$12.08.

Page 204. — 38. $\frac{1}{2}$; 3. 39. 1760 yd. 40. 288 sq. in.; 1440 sq. in. 41. \$45. 42. 15 da. 43. 90 da. 44. \$50. 45. 242 yd. 46. 1.44 sq. yd.; 12.96 sq. ft. 47. 40 sq. ft. 48. 56 ft. 49. 185 sq. ft. 50. $15\frac{1}{2}$ ft. 51. 74 mi. 52. 320 cu. ft.

Page 205. — 53. 4800 bu. 54. 1875 gal. 55. 2250 gal. 56. 8800 bricks. 57. 1800 bricks. 58. 3600 bricks. 59. 781 sq. in.

60. 153.9384 in. 61. 87.9648 in. 62. 140.056 in. 63. 43.9824 in.;
65.9736 in.; 175.9296 in.; 87.9648 in. 64. 14.037+ in.; 7.0025 ft.;
25.074+ ft.; 35.014 in.

Page 206. — 65. 87.9648 yd.; 263.8944 ft. 66. 14.0056 rd.
67. 45 in. 68. 11.9%. 69. 8.1%. 70. 40.2%. 71. 7200 T. 72. 5760
T. 73. 806.4 T. 74. \$56,448; \$36,520. 75. 17.314 A.; \$34,406,350.
76. \$67,505,143. 77. \$34,127,142. 78. 50.55%. 79. \$9140. 80. 29.6
+ ft. . 81. 35.65 ft. 82. \$294.19+. 83. \$6.93. 84. \$65.31+.
85. \$84.005; \$67.204. 86. \$26.064; \$60.81+. 87. \$2.23+.

Page 208. — 88. 8235 bricks. 89. 276,000. 90. 2.2 T., 3.7 T.,
1.86 T., 3.66 T., 2.32 T.; 68 $\frac{1}{11}$ %, 98 $\frac{1}{11}$ %, 1 $\frac{1}{11}$ %, 59 $\frac{1}{11}$ %, 57 $\frac{1}{11}$ %.
91. .82 T., 1 T., 3.4 T., 2.27 T., 15.7 T.; 314 $\frac{1}{11}$ %, 240%, 49 $\frac{1}{11}$ %,
116 $\frac{1}{11}$ %, 176 $\frac{1}{11}$ %.

Page 209. — 92. \$4800. 93. 2400 ft.; 4800 ft.; \$60. 94. \$66;
\$7.66 $\frac{1}{2}$. 95. 14 $\frac{1}{2}$ ft.; 2 $\frac{1}{2}$ ft. 96. \$.57 $\frac{1}{2}$; \$.10 $\frac{1}{2}$. 98. 96 books.

Page 210. — 99. 10 $\frac{1}{2}$ mi. 100. 232.2 grains; 25.8 grains.
101. \$32. 102. 525 $\frac{1}{2}$ sq. ft. 103. 11 $\frac{1}{11}$ %. 104. 20,146.9.
105. 37 $\frac{1}{10}$. 106. 11 $\frac{1}{11}$. 107. 4 da. 4 hr. 48 min.; 21 da. 10 hr. 17 $\frac{1}{2}$
sec. 108. \$6.43. 109. \$5.70.

Page 211. — 111. $\frac{1}{2}$; 90°. 112. $\frac{1}{10}$; 36°. 114. 90°; 90°. 115. 7.95
yd. 116. 19.09+ ft. 117. \$.45. 118. \$1.056; \$1.10. 119. \$375.
120. \$18.75; \$25; \$31.25. 121. \$375.56.

Page 212. — 122. \$6275. 123. \$295 loss.

ANSWERS

PART THREE

Page 7. (12.)—1. 15,454. 2. 51,577. 3. 17,759. 4. 156,687.
5. 354,544. 6. 25,886.

Page 8. (17.)—1. 50; 30. 2. 1700; 1500. 3. Luther, 65;
Richard, 55. 4. John, $4\frac{1}{2}$ mi.; Henry, $2\frac{1}{2}$ mi. 5. Charles, 17;
Alfred, 11. 6. Mary, 36 qt.; Lucy, 30 qt. 7. John, 50; James, 30.

Page 9. (18.)—1. 4,966; 14,444. 2. [31,978; 34,527. 3. 66,505.
4. Chicago. 5. Northwestern, 2290; Michigan, 228; California, 1403;
Illinois, 985; Minnesota, 1136; Wisconsin, 1242; Nebraska, 2085.
6. Harvard, 574; Chicago, 645; Northwestern, 2935; Michigan, 873;
California, 2048; Illinois, 1630; Minnesota, 1781; Cornell, 1104; Wis-
consin, 1837; Yale, 2856; Pennsylvania, 1275; Syracuse, 2848; Nebraska,
2730; New York, 2289. 7. Western, 2549. 8. \$3663.15; \$4926.90.

Page 10.—9. \$1253.21; \$3673.69. 10. 1908-9, 20,659,557; 1907-8,
17,772,460; 1906-7, 25,997,376; 5,337,819 T. larger than 1908-9;
8,224,916 T. larger than 1907-8; 2,887,097 T.

Page 13. (24.)—1. 1,168,825; 5,844,125. 2. 54,606,750. 3. 1825;
18,250; 182,500; 45,625. 4. \$6250; \$8125; \$8812.50. 5. \$1.85;
\$22.50; \$19.25; \$18; \$56.25; \$375; \$9; \$35; \$63; \$30.

Page 14. (26.)—1. \$1.44. 2. \$38.88. 3. \$163.35. 4. \$369.63.
5. \$42.57. (28.)—1. 800, 1600, 2000, 500, 875, 1650, 650, 925, 950,
1050, 1400, 1250, 3000, 3100, 6000, 4000, 6500, 4200; 400, 800, 1000, 250,
437 $\frac{1}{2}$, 825, 325, 462 $\frac{1}{2}$, 475, 525, 700, 625, 1500, 1550, 3000, 2000, 3250,
2100. 2. \$2000. 3. \$2125; \$1400.

Page 15. (30.)—1. 400; 1000; 800; 2000; 1066 $\frac{2}{3}$; 1066 $\frac{2}{3}$; 533 $\frac{1}{3}$;
10,666 $\frac{2}{3}$. 2. \$1733.33 $\frac{1}{3}$; \$866.66 $\frac{2}{3}$. 3. Oats, 1066 $\frac{2}{3}$ bu.; Corn, 1025 bu.;
Potatoes, 3000 bu. 4. \$600; \$866.66 $\frac{2}{3}$.

Page 20. (36.)—1. 134 $\frac{1}{11}$; 27 $\frac{1}{11}$; 61 $\frac{1}{11}$; 9836 $\frac{1}{11}$; 185 $\frac{1}{11}$;
46 $\frac{1}{11}$. 2. 5 $\frac{1}{11}$ mi.; 5 $\frac{1}{11}$ mi.

Page 21.—3. \$41.83. 4. 34+. 5. Gasoline, \$.01485; Kerosene,
\$.027. 6. \$7.45+; \$10.07+; \$.89+; \$.596+; \$1.66+; \$.05+;
\$.4.23+; \$.8.34+; \$.9.35+; \$.28.68+; \$.18.67+; \$.38.69+;
663.63 lb.

Page 24. (39.)—1. 18 ft. 2. 30 ft. 3. 18 ft. 4. 45 ft.

Page 25. (41.)—1. 25, 24, 14, 15, 21, 16, 12, 20. 2. 20. 3. \$9; 21 sheep.

Page 27. (45.)—1. 24. 2. 80. 3. 54. 4. 24. 5. 120. 6. 180. (46.)—1. 6; 72. 2. 7; 210. 3. 8; 1680. 4. 3; 270. 5. 4; 96. 6. 2; 180. 7. 5; 1200. 8. 4; 192.

Page 29. (48.)—4. 164,526. 5. 558,402,642,675. 6. 234,000; 16,000. 7. 364. 8. 8300. 9. Mr. Smith owes Mr. Gray \$10.62.

Page 30.—12. 12, 24, 36. 13. 4, 8. 14. 3 h.; 6, 9, and 18 mi. 15. Son, 350 A.; Daughter, 175 A. 16. James, \$3000; Peter, \$1500; Susan, \$750; Clara, \$750.

Page 32. (53.)—1. $\frac{5}{32}$, $\frac{7}{8}$, $\frac{1}{2}$. 2. $\frac{7}{16}$, $\frac{7}{12}$, $\frac{7}{11}$. 3. $\frac{3}{4}$, $\frac{7}{11}$, $\frac{1}{12}$. 4. $\frac{3}{8}$, $\frac{1}{3}$, $\frac{5}{8}$.

Page 33. (55.)—1. 45. 2. 140. 3. 90. 4. 42. 5. 30. 6. 54. (57.)—1. $\frac{728}{100}$, $\frac{10}{728}$, $\frac{84}{728}$, $\frac{80}{728}$, $\frac{45}{728}$. 2. $\frac{1120}{4200}$, $\frac{504}{4200}$, $\frac{1735}{4200}$, $\frac{1082}{4200}$, $\frac{720}{4200}$. 3. $\frac{1188}{120}$, $\frac{1188}{120}$, $\frac{1188}{120}$, $\frac{1188}{120}$, $\frac{486}{120}$. 4. $\frac{7}{10000}$, $\frac{10000}{10000}$, $\frac{10000}{10000}$, $\frac{10000}{10000}$. 5. $\frac{1}{120}$, $\frac{1}{120}$, $\frac{1}{120}$.

Page 34. (60.)—1. $5\frac{1}{2}$, $3\frac{1}{2}$, $3\frac{1}{2}$, $23\frac{3}{4}$, $4\frac{1}{2}$, 3. 2. $3\frac{1}{2}$, $4\frac{1}{2}$, $1\frac{1}{2}$, 8, $1\frac{1}{2}$. 3. $19\frac{5}{8}$, $15\frac{5}{8}$, $27\frac{5}{8}$, $58\frac{5}{8}$, $42\frac{5}{8}$. (63.)—1. $\frac{3}{2}$, $\frac{2}{3}$, $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$. 2. $\frac{5}{2}$, $\frac{4}{3}$, $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$. 3. $\frac{1208}{2122}$, $\frac{2122}{2122}$, $\frac{2122}{2122}$, $\frac{2122}{2122}$, $\frac{2122}{2122}$.

Page 36. (70.)—1. $\frac{1}{11}$. 2. $\frac{1}{10}$. 3. $\frac{1}{11}$. 4. $\frac{1}{10}$. 5. $\frac{1}{11}$. 6. $\frac{1}{10}$. 7. $\frac{1}{11}$. 8. $\frac{1}{11}$. 9. $\frac{1}{11}$. 10. $\frac{1}{11}$.

Page 37.—11. (a) $\frac{1}{10}$ more than Mr. Alfred, (b) $\frac{1}{10}$ more than Mr. Swift, (c) $\frac{1}{10}$ more than Mr. Allen. 12. $\frac{1}{10}$. 13. $\frac{1}{10}$. 14. (a) $\frac{1}{10}$, (b) $\frac{1}{10}$; (c) $\frac{1}{10}$. 15. Berries. 16. (a) $\frac{1}{10}$ larger than hay, (b) $\frac{1}{10}$ larger than oats, (c) $\frac{1}{10}$ larger than apples. (d) $\frac{1}{10}$ larger than potatoes. 17. $3\frac{1}{10}$ mi. 18. \$3.05+. 19. $\frac{1}{10}$. 20. \$ $\frac{1}{10}$. 21. $3\frac{1}{10}$ yd., $2\frac{1}{10}$.

Page 39. (73.)—1. $\frac{1}{2}$, $\frac{1}{10}$, $\frac{1}{10}$, $\frac{1}{10}$; $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{10}$. 2. $\frac{1}{2}$, $\frac{1}{10}$, $\frac{1}{10}$, $\frac{1}{10}$; $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{10}$. 3. $\frac{1}{2}$, $\frac{1}{10}$, $\frac{1}{10}$, $\frac{1}{10}$; $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{10}$. 4. $\frac{1}{2}$, $\frac{1}{10}$, $\frac{1}{10}$, $\frac{1}{10}$; $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{10}$. 5. $\frac{1}{2}$, $\frac{1}{10}$, $\frac{1}{10}$, $\frac{1}{10}$; $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{10}$. 6. $\frac{1}{2}$, $\frac{1}{10}$, $\frac{1}{10}$, $\frac{1}{10}$; $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{10}$. 7. $\frac{1}{2}$, $\frac{1}{10}$, $\frac{1}{10}$, $\frac{1}{10}$; $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{10}$. 8. $\frac{1}{2}$, $\frac{1}{10}$, $\frac{1}{10}$, $\frac{1}{10}$; $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{10}$. 9. $\frac{1}{2}$, $\frac{1}{10}$, $\frac{1}{10}$, $\frac{1}{10}$; $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{10}$. 10. $\frac{1}{2}$, $\frac{1}{10}$, $\frac{1}{10}$, $\frac{1}{10}$; $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{10}$. 11. $\frac{1}{2}$, $\frac{1}{10}$, $\frac{1}{10}$, $\frac{1}{10}$; $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{10}$. 12. $\frac{1}{2}$, $\frac{1}{10}$, $\frac{1}{10}$, $\frac{1}{10}$; $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{10}$. 13. $\frac{1}{2}$, $\frac{1}{10}$, $\frac{1}{10}$, $\frac{1}{10}$; $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{10}$. 14. $\frac{1}{2}$, $\frac{1}{10}$, $\frac{1}{10}$, $\frac{1}{10}$; $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{10}$. 15. $\frac{1}{2}$, $\frac{1}{10}$, $\frac{1}{10}$, $\frac{1}{10}$; $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{10}$. 16. $\frac{1}{2}$, $\frac{1}{10}$, $\frac{1}{10}$, $\frac{1}{10}$; $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{10}$. 17. $\frac{1}{2}$, $\frac{1}{10}$, $\frac{1}{10}$, $\frac{1}{10}$; $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{10}$. 18. $\frac{1}{2}$, $\frac{1}{10}$, $\frac{1}{10}$, $\frac{1}{10}$; $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{10}$. 19. $\frac{1}{2}$, $\frac{1}{10}$, $\frac{1}{10}$, $\frac{1}{10}$; $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{10}$. 20. $\frac{1}{2}$, $\frac{1}{10}$, $\frac{1}{10}$, $\frac{1}{10}$; $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{10}$. 21. $\frac{1}{2}$, $\frac{1}{10}$, $\frac{1}{10}$, $\frac{1}{10}$; $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{10}$.

Page 40. (75.)—1. $\frac{1}{10}$. 2. $\frac{1}{10}$. 3. $\frac{1}{10}$. 4. $\frac{1}{10}$. 5. $\frac{1}{10}$. 6. A, $\frac{1}{10}$; B, $\frac{1}{10}$; C, $\frac{1}{10}$; D, $\frac{1}{10}$; E, $\frac{1}{10}$. 7. $\frac{1}{10}$ A., $\frac{1}{10}$ A., $\frac{1}{10}$ A. 8. $\frac{1}{10}$; $42\frac{1}{10}$ A.

Page 42. (79.)—1. $1\frac{1}{2}$. 2. $1\frac{1}{2}$. 3. $4\frac{1}{2}$. 4. $\frac{1}{2}$. 5. 6. 6. $\frac{1}{2}$.
7. $\frac{1}{2}$. 8. $2\frac{1}{2}$. 9. $1\frac{1}{2}$. 10. $\frac{1}{2}$. 11. $\frac{1}{2}$. 12. $1\frac{1}{2}$.

Page 44. (84.)—1. $\frac{1}{2}$. 2. $\frac{1}{2}$. 3. $1\frac{1}{2}$. 4. $\frac{1}{2}$. 5. $1\frac{1}{2}$. 6. $1\frac{1}{2}$.
7. 2. 8. $3\frac{1}{2}$. 9. $1\frac{1}{2}$. 10. $\frac{1}{2}$. 11. $\frac{1}{2}$. 12. $\frac{1}{2}$. 13. $18\frac{1}{2}$. 14. $12\frac{1}{2}$.
15. 8. 16. $6\frac{1}{2}$ hr. 17. $2\frac{1}{2}$ hr. 18. 18 bags. 19. 8 boys. 20. $4\frac{1}{2}$.
21. 35 children.

Page 45. (86.)—1. $24\frac{1}{2}$. 2. $34\frac{1}{2}$. 3. $98\frac{1}{2}$. 4. $52\frac{1}{2}$. 5. $66\frac{1}{2}$.
6. $111\frac{1}{2}$. 7. $298\frac{1}{2}$ lb. 8. $101\frac{1}{2}$. 9. $124\frac{1}{2}$.

Page 47. (89.)—1. $4\frac{1}{2}$. 2. $7\frac{1}{2}$. 3. $7\frac{1}{2}$. 4. $8\frac{1}{2}$. 5. $10\frac{1}{2}$.
6. $17\frac{1}{2}$. 7. $7\frac{1}{2}$. 8. $20\frac{1}{2}$. 9. $21\frac{1}{2}$. 10. $8\frac{1}{2}$. 11. $3\frac{1}{2}$.
12. $6\frac{1}{2}$. 13. $13\frac{1}{2}$. 14. $19\frac{1}{2}$. 15. $12\frac{1}{2}$. 16. $2\frac{1}{2}$, $4\frac{1}{2}$, $3\frac{1}{2}$, $1\frac{1}{2}$, $5\frac{1}{2}$.

Page 48. (92.)—1. $21\frac{1}{2}$. 2. $111\frac{1}{2}$. 3. $272\frac{1}{2}$. 4. $351\frac{1}{2}$.
5. $1951\frac{1}{2}$. 6. $578\frac{1}{2}$. 7. $90\frac{1}{2}$. 8. $416\frac{1}{2}$. 9. $317\frac{1}{2}$. 10. $37\frac{1}{2}$.
13. $43\frac{1}{2}$. 14. $1811\frac{1}{2}$.

Page 49. (95.)—1. $3\frac{1}{2}$. 2. $1\frac{1}{2}$. 3. $5\frac{1}{2}$. 4. $2\frac{1}{2}$. 5. $1\frac{1}{2}$.
6. $1\frac{1}{2}$. 7. $3\frac{1}{2}$. 8. $2\frac{1}{2}$. 9. $1\frac{1}{2}$. (96.)—1. \$16.28 +.
2. 5638. 3. $14\frac{1}{2}$. 4. 450 T.

Page 51. (100.)—1. $\frac{1}{2}$. 2. $\frac{1}{2}$. 3. $\frac{1}{2}$. 4. $\frac{1}{2}$. 5. $\frac{1}{2}$.
6. $\frac{1}{2}$. 7. $3\frac{1}{2}$. 8. $14\frac{1}{2}$. 9. $37\frac{1}{2}$. 10. $24\frac{1}{2}$.
11. $708\frac{1}{2}$. 12. $367\frac{1}{2}$. 13. $2\frac{1}{2}$. 14. $\frac{1}{2}$. 15. $\frac{1}{2}$.
16. $10\frac{1}{2}$. 17. $304\frac{1}{2}$. 18. $\frac{1}{2}$. 19. $20\frac{1}{2}$. 20. $3\frac{1}{2}$.
21. $256\frac{1}{2}$.

Page 52. (102.)—1. .20. 2. .125. 3. .857 +. 4. .75.
5. .0833 +. 6. .571 +. 7. .8. 8. .2666 +. 9. .9. 10. .833 +.
11. 1.4. 12. 17.75. 13. 18.5. 14. 20.25. 15. 36.125.
16. 5.1. 17. 7.25. (104.)—1. 75.4. 2. 4.078. 3. 1.206.

Page 53.—4. Wheat, \$.81 $\frac{1}{2}$, \$.41 $\frac{1}{2}$; Corn, \$.08, \$.10; Oats, \$.08 $\frac{1}{2}$,
\$.01 $\frac{1}{2}$; Flour, \$.35, \$.60; Lard, \$.25, \$.230; Pork, \$1, \$.450; Beef,
\$.325, \$.351; Coffee, \$.01, \$.00 $\frac{1}{2}$; Sugar, \$.35, \$.45; Steel, \$1, \$.2;
Lead, \$.32 $\frac{1}{2}$, \$.22 $\frac{1}{2}$; Tin, \$.152 $\frac{1}{2}$, \$.195.

Page 54. (107.)—1. 11.25. 2. 255.792. 3. 1.748.
4. 605.36105. 5. 3280.32. 6. .001729. 7. 225.625. 8. .0016692.
9. 4.84. 10. 4.46875. 11. 8.765625. 12. 9.375. 13. 272.65625.
14. .65625. 15. 10.880625. 16. \$.640. 17. \$.5. 18. 75.9375;
82.0125; 101.27025; 131.625.

Page 59. (115.)—1. 1405.83 +. 2. 16.68. 3. 100. 4. .01.
5. 24,000. 6. 3250.666 +. 7. 2000. 8. 21.428 +. 9. 29,255. 10. 2.
11. .2065 +. 12. 118.5578 +. 13. \$.81.05.

Page 61. (117.)—1. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$. 2. 2,111. 3. $\frac{1}{12}$. 4. \$1.84.
5. \$1.68 $\frac{1}{2}$. 6. \$1664. 7. \$121.25. 8. 11 strips. 9. 41 bottles.
10. \$19.12 $\frac{1}{2}$. 11. 264.83688 ft. 12. 2.38 A. 13. \$6030; \$4221;
\$2814.

Page 62.—14. $\frac{1}{2}$. 15. 295 $\frac{1}{2}$. 16. 25,040; 70,000. 17. 534 $\frac{1}{2}$ sq. rd.
18. 250 bu.; 400 bu. 19. \$3000. 20. 27.516 mi. 21. Two sides each
37.9875 ft., two sides 100 ft. 22. \$9.65.

Page 64. (120.)—1. 156 cents. 2. 541 gi. 3. 37081 oz.
4. 8,400,000 gr. 5. 11,035 gr. 6. 2588 yd. 7. 178 in. 8. 23,580 li.
9. 15,180,411,828 sq. in. 10. 352,057,815 sq. li. 11. 511,360 cu. in.
12. 29,660 sheets. 13. 1545 doz. 14. 70 yr. 15. 468 da.

Page 65. (122.)—1. 6 $\frac{1}{2}$ bu.; 8 $\frac{1}{2}$ cwt.; 33 $\frac{1}{2}$ T.; 1.46 + hr.; 16 cu. ft.,
217 cu. in.; 7.40 rd.; 2 $\frac{1}{2}$ mi.; 10 $\frac{1}{2}$ da.; 13 $\frac{1}{2}$ sq. yd.; 3 $\frac{1}{2}$ A. (124.)—
1. 40,088 sq. in.; 33,364 sq. in.; 24,123 sq. in. 2. \$28.50. 3. \$5.
4. \$15.75.

Page 66.—5. \$4.08; \$5; \$8. 6. \$36. 7. 40 gal. 3 qt. 1 pt.
8. 30 cd. 9. 647 lb. 10. 39 ft. 11. 75 lb.; 2 cu. ft. 12. 76,250 lb.
13. 2940 tons; 305,600 cu. ft.

Page 67. (125.)—3. 4.625 gal. 4. .5873 wk. 5. .7875 T.

Page 69. (129.)—1. \$5.20. 2. \$3.79 $\frac{1}{2}$. 3. 10 yd. 2 ft. 6 in.
4. 67 yr. 9 mo. 22 da. 8. 1 A. 48 sq. rd. 2 sq. yd. 7 sq. ft. 9. 13 T.
11 cwt. 43 lb. 10. 73 gal. 1 qt. 3 gi.

Page 71. (132.)—1. 521 rd. 4 yd. 1 ft. 11 in. 2. 5478 da. 15 hr.
11 min. 30 sec. 3. 37 $\frac{1}{2}$ sq. ft. 4. 36 $\frac{1}{2}$ cu. ft. 5. 67 gal. 1 qt. 1 pt.
6. 11 lb. 10 oz. 7. 1.43 A. 8. 2 da. 22 hr. 43 min. 45 sec.

Page 73. (134.)—1. 5 bu. 6 qt. $\frac{1}{2}$ pt. 2. 6 yd. 2 ft. 1 $\frac{1}{2}$ in.
3. 5 T. 1054 lb. 9 $\frac{1}{2}$ oz. 4. 2 dollars, 5 dimes, 3 $\frac{1}{2}$ cents. 5. 4 gal.
3 qt. 1 $\frac{1}{2}$ pt. 6. 4 rd. 1 $\frac{1}{2}$ ft. 7. 9250 sq. ft.; 2 tennis courts. Left
on sides, 1599 sq. ft.; on ends, 2035 sq. ft. 8. 770 ft.; 3080 sq. ft.
9. 39 posts. 10. \$15.40. 11. \$.66 each. 12. 15 $\frac{1}{2}$ fields. 13. 24
baskets. 14. 2 $\frac{1}{2}$ bins.

Page 76. (139.)—2. 350 ft. 3. 6250 sq. ft.; 694 $\frac{1}{2}$ sq. yd. 7. 1280
sq. in. 9. 61 $\frac{1}{2}$ A. 10. 1065 sq. in.

Page 77. (140.)—1. 40 rd.; 100 rd.; 2 $\frac{1}{2}$ A. 2. \$82.50. 3. 400 ft.;
448 ft. 5. 640 A.; 40 A.; 80 A. 6. \$23,520. 7. 90 posts;
\$108.

Page 84. (146.)—1. 66 $\frac{1}{2}$ loads. 2. 118 $\frac{1}{2}$ loads. 3. \$966.66 $\frac{1}{2}$.
4. 5; 68. 5. 48 sq. ft.; \$1.68.

Page 85.—6. 720 sq. ft.; 80 sq. ft. 7. 400 sq. ft.; \$32. 8. 3600 shingles; \$28.80. 9. \$156.94. 10. 3 mi. 291 rd. 3.9098 + mi. 11. 960 ch. 75 li. 12. Over $3\frac{1}{2}$ fathoms. 13. 2889; 249. 14. 144 3. 15. .4652 lb. 16. .04479 + lb.; 4,479 + lb. 17. 71.614 + lb.

Page 86.—19. 18 min. $56\frac{2}{5}$ sec. past 8 A.M. 20. .7373 + A. 21. Area of each, 864 sq. ft. 24. Square. 25. 80 rd.

Page 89. (151.)—1. Son, 240 A.; Daughter, 272 A. 2. 34 yd.; 65 yd. 3. \$663; \$7163. 4. \$437.50. 5. $2\frac{3}{4}$ A. 6. \$144.32.

Page 90.—7. \$726.97 $\frac{1}{2}$. 8. \$9.62 $\frac{1}{2}$; \$41.12 $\frac{1}{2}$.

Page 91. (153.)—1. \$143.04. 2. \$4.80; \$5.28, \$6, \$10.20, \$11.70, \$12; Williams, \$16.98; Gillen, \$16.20; Dalton, \$18. 3. \$14.64; \$30.14 +. 4. \$.26 $\frac{1}{2}$; \$.37 $\frac{1}{2}$; \$.52.

Page 92.—5. 3552.12. 6. 1233.27; 44.73. 7. 43 $\frac{1}{2}$ %; 13. 8. 400 goats; 1600 goats. 9. \$336. 10. \$412.80. 11. 2 gal. \$.32. 12. 490 books. 13. \$277.86.

Page 93. (155.)—1. \$180. 2. \$49.50. 3. 240. 4. 455. 5. 75 yd. 6. \$65. 7. \$19.50. 8. 3200. 9. 63 qt.; 83 $\frac{1}{2}$ %. 10. \$240; \$220. 11. \$1000.

Page 94.—13. \$882.35+. 14. 512 bu. 15. \$64,000. 16. 48,675 lb. 17. 96,400 lb.; 231,860 lb.; 192,800 lb.; 231,360 lb. 18. 22,800 homes. 19. 251,600 homes.

Page 95. (157.)—1. 4 $\frac{1}{2}$ %. 2. 25+%. 3. .5%. 4. 185 $\frac{1}{2}$ %. 5. 25%. 6. 84%.

Page 96. (158.)—1. 25%. 2. 25%. 3. 50%. 4. \$10,000. 5. \$10,000. 6. \$8800. 7. \$4500. 8. Gain, 10%. 9. Loss, \$200. 10. \$1080. 11. \$450.

Page 97.—12. 92 $\frac{1}{2}$ %; 90 $\frac{1}{2}$ %; 88 $\frac{1}{2}$ %. 13. 64. 14. 160. 15. \$133.33 $\frac{1}{3}$. 16. 3536.24+. 17. 1400.24. 18. .14+. 19. 60%. 20. 17%. 21. 7 $\frac{1}{2}$ %. 22. \$630. 23. 869; 55%. 24. 33 $\frac{1}{2}$ %.

Page 98.—25. 50%. 26. 12 $\frac{1}{2}$ %. 27. 25%; 6 $\frac{1}{2}$ %. 28. .686. 29. .763. 30. 1920 ties; \$840. 31. \$13,157.4375. 32. \$306.91+.

Page 99.—33. 37 $\frac{1}{2}$ %. 34. McGann, .996; Ritchie, .966; Arndt, .965; Tucker, .944; Gessler, 100%; Bowerman, .984; Reulbach, .967. 35. .05 $\frac{1}{2}$.

Page 100. (160.)—1. \$540. 2. \$324.72. 3. \$50.46+. 4. \$.46+.

Page 101.—5. \$2180.25. 6. \$80.25. 7. \$49.60. 8. Note, \$14. 9. \$2736. 10. \$296.40. 11. 85.5%. 12. 28%. 13. 14 $\frac{1}{2}$ %. 14. 14.95%. 75. Second better by \$27.90.

Page 102. (163.)—1. \$691.18; \$16,617.82.

Page 103.—2. \$429.70½. 3. \$3445.095. 5. \$62.72; \$649.57.
6. \$37.57½; \$210.98. 7. \$14.06; \$84.33.

Page 106. (168.)—1. \$898.75. 2. \$4740.12, \$1417.50; \$6157.62.
3. \$1993.95.

Page 107.—4. \$8946; \$223.65. 5. \$1062.76. 6. \$23.73;
\$4722.27. 7. \$153.68. 8. \$303.74. 9. \$.0215. 10. \$792.81.
11. \$3135.34. 12. \$694.92.

Page 108. (170.)—1. 2.009%. 2. 3.48%.

Page 109. (172.)—1. \$.99. 2. \$.0198. 3. \$372,602;
\$135,999,730. 4. \$.0025. 5. 54,800,000 glasses. 6. \$57,474.99.
7. 8,500,000. 8. 916,400 lb.

Page 110. (174.)—1. \$108. 2. \$4.05. 3. \$708.75. 4. \$3.36.
5. \$735. 6. \$717.1875. 7. \$141.75. 8. 37½%.

Page 111.—9. \$150. 10. \$725. 11. \$682.50. 12. \$5231.75.
13. \$3681.25. 14. \$49.65. 15. Lace \$37.835, Cloth \$11.40; \$126.085.

Page 113. (178.)—1. \$35.68½. 2. \$73.71. 3. \$163.85+.
4. \$33.815. 5. \$98.444+. 6. \$86.14. 7. \$104. 8. \$78.75. 9. \$221.

Page 114. (180.)—2. \$468.45. 3. \$135.98+; \$174.16+;
\$56.88+; \$33.10+; \$36.20; \$5.02+.

Page 115. (184.)—1. \$3.3825. 2. \$2.103. 3. \$3.341½.
4. \$2.78+. 5. \$14.3416+. 6. \$6.13+. 7. \$3.505. 8. \$3.20+.
9. \$2.377+. 10. \$2.92+. 11. \$3.82+. 12. \$8.7586+.
13. \$2.5075. 14. \$4.6875. 15. \$680.05.

Page 116. (186.)—1. \$8.75. 2. \$3.204+. 3. \$3. 4. \$.93½.
5. \$.161+. 6. \$6.24+. 7. \$.97+. 8. \$1.65. 9. \$7.84.
10. \$.0585.

(187.)—1. 80⅓ mi. 2. 47½⅔%; .0238%. 3. 1750 mi. 4. 700 mi.

Page 117.—5. 2100 mi. 6. \$.44; \$.04. 7. \$550,000; \$950,000.
8. \$700,000. 9. \$50,000; \$1,700,000. 10. 14⅓%. 11. 1⅓%.

Page 118.—12. Lost, \$1. 13. Seniors won. Juniors, 42.25 sec.;
Sophomores, 59.5 sec.; Freshmen, 1 min. 56.875 sec.; Juniors, 2.9+%;
Sophomores, 4.06+%, Freshmen, 7.6+%. 14. \$1500. 15. \$.8505.
16. 2½%.

Page 119.—17. 75%; \$125.28. 18. \$20.54+. 19. \$.80; 80%.
20. 600 stitches. 21. 93¼%; 6¼%.

Page 123. (194.)—1. \$4.83. 2. \$16.11+. 3. 233.33+.
4. \$63.11+. 5. \$3.33 $\frac{1}{2}$; \$3.83; \$15; \$973; \$20.

Page 126. (199.)—1. \$72; \$3.697. 2. Edward, \$4244.83;
Rudolph, \$4504.64; Victor, \$4875.97. 3. \$603.04; \$585.

Page 127. (201.)—1. \$963.585. 2. \$389.20+. 3. \$593.28+.
4. \$497.63+. 5. \$25.23+; \$24.97+.

Page 129. (204.)—1. \$1167.795. 2. \$1290.16. 3. \$516,250.
4. \$665,000; \$148,750. 5. \$113,250. 6. \$39,500.

Page 135. (215.)—1. \$1179.15+. 2. \$410.55+. 3. \$269.14 $\frac{1}{2}$.
4. \$178.92+. 5. \$411.42+. 6. \$267.11+. 7. \$3.40+. 8. \$2.41+.
9. \$8.62+. 11. May 16, 1906; 65 da.; \$821+; \$74.979.

Page 136.—12. June 20, 1906; 69 da.; \$3.686+; \$316.914.
13. June 9, 1906; 26 da.; \$3.3495+; \$96.45+. 14. July 10, 1906;
16 da.; \$2.14+; \$80.28+.

Page 137. (222.)—1. \$7775.28. 2. \$685.125; \$680.625; \$4.50.

Page 138.—3. \$297. 4. 30 da.; \$298.50.

Page 139. (224.)—1. \$8084.066. 2. \$12,109.50. 3. \$181.86+.
4. \$164.59.

Page 140. (226.)—1. \$7900. 2. \$1000; \$50. 3. \$691.87+.

Page 145. (246.)—1. £123 9s. 1 $\frac{1}{2}$ d.; \$252.72. 2. \$69.86 $\frac{1}{2}$.
3. 3900 francs. 5. 3368 $\frac{1}{2}$ marks. 7. 1616 $\frac{1}{2}$ guilders. 9. 2083 $\frac{1}{2}$
guilders. 10. 764 $\frac{1}{2}$ francs.

Page 148. (257.)—1. \$2375. 2. \$225. 3. \$125. 4. \$150.
5. \$2000. 6. \$107.125; \$95.125; \$98.625; \$100.125.

Page 149.—7. \$3,609,487.50; \$151,315.625; \$69,300; \$197,968.75;
\$43,050; \$25,725; \$118,350; \$5512.50; \$676,700. 9. \$300. 10. 40;
\$25. 11. \$45. 12. \$35,850. 13. \$2400; 6 $\frac{1}{3}$ %.

Page 150.—14. \$4777.50. 15. \$700; 14 $\frac{1}{3}$ %.

Page 151. (260.)—1. \$3750. 2. \$3. 3. \$2. 4. 5 $\frac{25}{21}$ %, 8 $\frac{1}{3}$ %;
4 $\frac{1}{3}$ %, 5 $\frac{1}{3}$ %; 4 $\frac{1}{3}$ %, 7 $\frac{1}{3}$ %; 4 $\frac{1}{3}$ %, 4 $\frac{1}{3}$ %; 3 $\frac{1}{3}$ %, 4 $\frac{1}{3}$ %; 1 $\frac{1}{3}$ %;
3 $\frac{1}{3}$ %; 5 $\frac{1}{3}$ %, 5 $\frac{1}{3}$ %; 2 $\frac{1}{3}$ %, 4 $\frac{1}{3}$ %; 3 $\frac{1}{3}$ %, 5 $\frac{1}{3}$ %; 5 $\frac{1}{3}$ %, 10 $\frac{1}{3}$ %;
2 $\frac{1}{3}$ %, 5 $\frac{1}{3}$ %; 3 $\frac{1}{3}$ %, 5 $\frac{1}{3}$ %, 3 $\frac{1}{3}$ %, 6 $\frac{1}{3}$ %.

Page 152.—5. \$75. 6. \$10,000; \$10,200. 7. \$5925.
8. \$2543.75. 9. 37 shares; \$6300; \$222.

Page 154. (264.)—1. 984 shares; \$2952. 2. 279 + shares.
3. \$1255.50; .0523+. 4. New York City; 37 + cents.

Page 158. (#69.)—1. Fireproof, \$28; Frame, \$48.75. 2. \$1546.25.
3. Each of three companies, \$23,750; Burglary, \$4500; Plate Glass,
\$500. 4. \$137.80. 5. \$1653.60. 6. \$3346.40.

Page 159. (#70.)—1. \$2040. 2. \$2760. 3. $6\frac{1}{2}\%$. 4. \$1162.50.
5. \$996.40. 6. \$591. 8. \$2598. 9. First by \$54.55. 10. 51
shares and \$18.625 over; .081 $\frac{1}{2}\%$.

Page 160.—11. \$1920. 12. \$28,290; \$900; \$780. 13. .0023.
14. Same rate of income. 15. \$928.80. 16. \$3925.

Page 163. (#74.)—1. \$4800. 2. \$2240. 3. \$160; 50 yd.;
59 $\frac{1}{2}$ yd.; 65 $\frac{1}{2}$ yd. 4. \$12. 5. \$1080. 6. \$3200. 7. \$20. 8. 24.
9. 96. 10. \$500. 11. 1500 bu. 12. \$.96. 13. \$3000. 14. \$7000.
15. Loss, \$1600.

Page 166. (#79.)—1. — 3. 2. — 3. 3. — 4. 4. — 4. 5. — 10.
6. — 1. 7. — 1. 8. — 345. 9. — 716.

Page 167. (#81.)—1. 5a. 2. 2b. 3. — 2b. 4. 2a. 5. 3b.
6. — c. 7. d. 8. — 2c. 9. — c. 10. 3c. 11. — 2f. 12. — a.

Page 169. (#84.)—6. — 3b — 2c + 2. 7. 7a — 6c + d.
8. — 7c — 24. 9. 3a — 19c — 5d + 35. 10. — 6x + 10y — 10z — 3.
11. — ab + 5ac — 2d — 5x. 12. 15xy — xz — 2y + 2z. 13. 4x +
3y + 5z + 3xy + 4xyz. 14. 3abc + bcd + 4cde + 3a + 5 + 4bed.
15. 52x + 33xy + 29y. 16. — 3abx — 50cd. 17. 2a + 2x — 2z.
18. ab — 8abc + ax — 6d — xy + 14xyz.

Page 170. (#86.)—1. — 42. 2. — 18. 3. — 7a. 4. — 10b.
5. — 65abc. 6. — 2. 7. 23. 8. — 12. 9. 35. 10. 50. 11. — 5.
12. 18. 13. 3. 14. 60. 15. — 28y. 16. x — 5y + 45. 17. 4a
— ab — 3b + 5cd — 16. 18. — a — 16b — 2c + 8abc. 19. — x +
9xy — 15y + 16z. 20. — 4x + 4y + 4z — 4. 21. 9ab + 6ac — 12d
+ 2. 22. — 27a — 6b — bc + 9d. 23. 10ab — 10b — 40cd — 46.

Page 172. (#90.)—1. $2a^2b$. 2. a^2b^2cd . 3. abx^3 . 4. $2abc^3$.
5. $2a^2bc$. 6. a^4b^3 . 7. x^4y^4z . 8. $18a^3b^2c^3$.

Page 174. (#98.)—1. $a^2 + 2ab + b^2 + ac + bc$. 2. $2xz - 4z^2 - 2xy$
+ $4y^2$. 3. $2m + 2n + 2p - m^2 - mn - mp$. 4. $2a^2 - 2ab - 4ac + 2acd$
— $3a + 3b + 6c - 3cd$. 5. $2a^2b^2 - 5a^2bc - 2a^2bd + 3a^2c^2 + 3a^2cd$.
6. $x^2 - 2xy + y^2 + 6x - 6y + 9$.

Page 175. (#100.)—1. a^2 . 2. 2. 3. 9. 4. 12b. 5. 5c. 6. b.
7. ab. 8. ac. 9. $2x^2$. 10. 3. 11. $3c^2$.

Page 176. (302.)—1. 2. 2. *a*. 3. $\frac{2y}{18}$. 4. $2x$. 5. $2b$. 6. $2x$.
 7. -4 . 8. $-5x$. 9. $-7a$. (304.)—2. $a-b$; $a+b$. 3. $a+b$.
 4. $x^3 + 3x^2y + 3xy^2 + y^3$. 5. $2a + 2b$.

Page 178. (309.)—1. 9. 2. 14. 3. 22. 4. 2. 5. 2. 6. $3\frac{1}{2}$.
 7. 9. 8. 2. 9. 3.

Page 180. (312.)—1. 12. 2. 23. 3. 32. 4. 16. 5. 10. 6. 25.
 7. 5. 8. 17. 9. 22. 10. 80. 11. 10. 12. 15. 13. 10. 14. 13. 15. 30.

Page 181. (314.)—1. 21. 2. 24. 3. -3 . 4. -15 . 5. 30.
 6. 10. 7. 4. 8. -2 . 9. 4. 10. 1. 11. 3. 12. $-5\frac{1}{2}$. 13. 3.
 14. 16. 15. 2. 16. 29. 17. 1. 18. $3\frac{1}{2}$. 19. 1. 20. $\frac{7}{19}$.

Page 182. (316.)—1. 73. 2. 145. 3. 225. 4. 32. 5. 109.
 6. 268 rd. 7. 16 rd. 8. 35 rd. 9. 9. 10. 300. 11. 30. 12. 32.
 13. \$17.50.

Page 183.—14. 1200. 15. 45 rd. 16. 125 sheep. 17. \$.30.
 18. \$200. 19. Rye, 200 bu.; Oats, 400 bu.; Wheat, 600 bu.
 20. Charles, \$.10; Horace, \$.25. 21. Mason, \$.5; Plumber, \$.4. 22. 8 lb.
 23. 24 da. 24. 10 da. son, 30 da. Mr. Danforth.

Page 185. (318.)—1. 60. 2. $8\frac{1}{2}$. 3. 30. 4. 15. 5. $2\frac{1}{2}$.
 6. 2. 7. 1. 8. 2. 9. 4. 10. 1. 11. 19. 12. 1. 13. 4.
 14. 3. 15. 5. 16. 1. 17. 2. 18. 4.

Page 186. (320.)—1. Horse, \$170.66 $\frac{2}{3}$; Carriage, \$106.66 $\frac{2}{3}$. 2. \$80.
 3. 48 yr. 4. Mr. Lewis, \$2000; Mr. Childs, \$4000; Mr. Hudson,
 \$8000. 5. Coal, 2500 lb.; Team, 2000 lb.; Wagon, 1000 lb. 6. Celia,
 \$.38; Harry, \$.26. 7. 35 A., 45 A.

Page 187.—8. 540 boys, 660 girls. 9. 10 5-cent stamps, 40 2-cent
 stamps, 70 post cards. 10. 16, 14. 11. 425 boys, 440 girls. 12. 1st,
 \$300; 2d, \$600; 3d, \$1500. 13. 35. 14. 16, 32, 192. 15. Frank,
 44; Edward, 28. 16. Father, 54 yr.; Son, 14 yr.

Page 188.—17. 25 5-dollar bills, 20 10-dollar bills. 18. \$.45;
 \$1.50. 19. Father, 62 yr.; Son, 36 yr.

Page 191. (323.)—1. $93\frac{1}{2}$ sq. ft. 2. 390.96 sq. ft. 3. 280 sq. ft.
 4. 9600 sq. ft. 5. 16 rd. 6. 11 sq. ft. 7. $74\frac{1}{2}$ A. 8. $17\frac{1}{2}$ sq. ft.
 9. 6 sq. ft. 11. $3\frac{1}{2}$ in.; $5\frac{1}{2}$ in.

Page 193. (326.)—1. 15 ft. 2. $12\frac{1}{2}$ in. 3. 75.3984 in.
 4. 50.2656 ft. 5. 65.9736 in.

Page 194.—6. 9.4248 in. 7. 19.09 + ft. 8. 1680.67 + ft.;
 11.4 + in. 9. 15.91 + ft. 10. 168.067 ft. 11. 141.947 in.

Page 195. (328.)—1. 50.2656 sq. in. 2. 78.54 sq. in. 3. 201.0624 sq. ft. 4. 3,738,325.815 + sq. ft. 5. 15.9 + sq. ft. 6. 1.239 + sq. ft. 7. .7854 sq. in. 8. 3.1416 sq. in. 9. (a) 7.0686 sq. in.; (b) 16; (c) 25.

Page 196. (331.)—1. $12\frac{1}{2}$ cu. ft.

Page 201. (338.)—1. 207.3456 sq. in. 2. 286.1952 sq. in. 3. 64.7955 + sq. ft. 4. 1055.5776 sq. ft. 5. 1125 sq. ft. 6. 384 sq. ft. 7. 360 sq. ft. 8. \$35.74 +. 9. 47.124 sq. in.

Page 203. (341.)—1. 201.0624 sq. ft. 2. \$2.45+. 3. 7.0686 sq. ft. 4. 113.0976 sq. in. 5. 38.4846 sq. in. (343.)—1. 72 cu. in. 2. 64 cu. in.; 125 cu. in.; 216 cu. in.; 512 cu. in. 3. 8. 4. 8.

Page 204. (345.)—1. 32 cu. ft. 2. 6,880,000 cu. ft. 3. 27 cu. ft. 4. 1759.32 cu. ft. 6. 17,280 bricks. 7. 3 hr. 21 min. 36 sec.

Page 205. (346.)—1. 150.7968 cu. in. 2. 1206.374 + cu. in.

Page 206. (347.)—1. 226.1952 cu. ft. 2. 26,998,125 cu. ft. 3. 339,2918 cu. ft. 4. 1060.29 cu. ft. 5. 81.6816 cu. ft. 6. 678.5856 cu. ft. 7. 4 ft. 8. 1507.968 gal. 9. 96 sq. ft. 10. 628.32 sq. ft.

Page 207.—11. 78.54 sq. ft. 12. 785.4 cu. ft. 13. 1262.23 + bu. 14. 1036.72 + cu. ft. 15. 19,635 cu. ft. 16. 31.416 in.; 78.54 sq. in.; 235.62 cu. in. 17. 3.927 in. 18. 45° .

Page 208. (349.)—1. 75,398.4 gal.; 70,628.5642 gal. 2. 577.266 gal.; 2209.52655 gal. 3. 17,318.07 gal. 4. 9.81 + hr.

Page 209.—5. 795.217 gal. 6. 995.4945 gal. 7. 16,964.64 gal. 8. 81.6 gal. 9. 7952.175 gal. 10. 2 bbl.; 4 bbl.; 10 bbl. 11. 256.56 bbl. 12. 1036.728 sq. ft. 13. 29,157.975 gal. 14. 28,274.4 cu. ft. 15. 107.712 bbl. 16. 96.2115 cu. ft.

Page 210. (351.)—1. 25.1328 cu. ft. 2. 30 cu. ft. 3. 38.785 + loads. 4. $853\frac{1}{2}$ cu. ft. 5. 613,593.75 cu. ft. 6. 4.7124 cu. ft.

Page 211. (353.)—1. 14.1372 cu. in.; 4.1888 cu. ft.; 113.0976 cu. in.; .5236 cu. ft. 2. 268.0832 + cu. in.; .5236 cu. ft. 3. 27. 4. 64; 8. 5. 3-in., 14.1372 cu. in.; 1-in., .5236 cu. in.; 2-in., 4.1888 cu. in.; 4-in., 33.5104 cu. in.

Page 212. (354.)—1. 14 + in. 2. 140 sq. in. 3. 240 sq. in. 4. 231 sq. in. 5. 66 in. 6. 356.37 cu. ft. 7. 2.5 ft. 8. 40 A. 9. 130.9 yd. 10. 2261.952 cu. ft.; 1817.62 + bu. 11. 96 sq. in. 7854 sq. in.; 12.5664 sq. in.; \$.026 $\frac{1}{2}$.

Page 213.—12. $5\frac{1}{4}$ sq. ft. 13. 6 linear feet. 14. 441 cu. in.; 346.3614 cu. in.; 94.6886 cu. in.; \$.45.

Page 214.—16. $\frac{1}{4}$ mi.; $\frac{1}{4}$ mi.; $\frac{1}{4}$ mi.; 165 A. 17. 15 A.
18. $9\frac{1}{11}\%$ 19. \$87.50. 20. \$5414.06 $\frac{1}{2}$. 21. \$135.35 +.
22. 7.854 A.; 25,659,018 gal. 23. 60 A.; $36\frac{1}{11}\%$ 24. 11.25 A.
25. 6.8 + %. 26. 165 A. 28. \$178.66 +.

Page 215.—29. \$110. 30. 10.59 A. 31. Oak, 150; Chestnut, 800; Pine, 180; Maple, 70. 32. \$99.13 +. 33. 10 rd. 34. \$192.115.
35. 2035.7568 cu. ft. 36. 791.6832 sq. ft.

Page 216.—37. 15 A. 38. \$112.50. 39. \$589.50.

Page 217.—41. \$2.50 +. 42. 7 hr. 43. \$.246. 44. $45\frac{1}{2}\%$
45. $744\frac{1}{2}\%$ 46. Corn, 3360 lb.; Oats, 1920 lb. 47. \$225. 48. \$3.375.

Page 218.—49. 5 hr. 49 min. 40 + sec. 50. 6 ft. 3 in. 51. 12.12 + ft.
52. \$118. 53. 11,250 cu. ft. 54. 281 $\frac{1}{2}$ cu. ft. 55. 82 $\frac{1}{2}$ lb.; \$21.54 +.
56. 100 ft. 57. \$1584. 58. \$4.50. 59. 1 mi. 60. 972 lb.

Page 219.—61. $41\frac{1}{9}$ rd. 62. 22.4 + times. 63. 139.32 cu. ft.
64. 1; 1.24 +. 65. 8; 64.

Page 223. (363.)—1. 19. 2. 27. 3. 85. 4. 77. 5. 36. 6. 41.

Page 226. (369.)—1. 82. 2. 246. 3. 612. 4. 152. 5. 68.
6. 769.31 +.

Page 227. (370.)—1. 1.6. 2. 7.5. 3. 4. 4. .353 +. 5. 6.024 +.
6. $\frac{7}{8}$ +; 6 $\frac{7}{8}$. 7. 5. 8. $\frac{1}{2}$. 9. .042. 10. 20.03. 11. $\frac{1}{2}$. 12. 6.
13. 1.812 +. 14. .707. 15. $\frac{3}{4}$. 16. $\frac{1}{4}$. 17. 40 rd. 18. 1.1.

Page 231. (373.)—1. 8 yd. 2. 50.59 + rd. 3. 25 rd. 4. 20 rd.; 80 rd.
5. 6 ft.; 31,104 sq. in.; 72 in. 6. 9 ft. $\times$ 9 ft. $\times$ 9 ft.
7. 486 sq. ft. 8. 10 ft.; 100 ft. 9. 15 ft. $\times$ 15 ft.; 60 ft. 20 yd.
10. \$20.

Page 233. (376.)—1. 4; 5.29; 13; 5.19. 2. 15 ft. 3. 8 ft.
4. 13.23 + ft.

Page 234.—5. 63.2 + ft. 6. 40 ft. (378.)—1. 13 ft. 2. 30 ft.
3. \$42.50.

Page 235.—4. $16\frac{1}{2}$ ft. 5. \$48.75. 6. 6 ft. 7. 10 ft. 8. $20\frac{3}{4}$ ft.
9. 8 ft. 10. 12 ft. 11. 15 ft.

Page 236. (379.)—1. 62 $\frac{1}{2}$ A. 2. 21; 10, 8, 11. 3. 8 ft., 17.5 ft., boards on back placed horizontally; 20.12 ft., including top and bottom; $9\frac{1}{2}$ sq. ft. 4. \$2.51; 1 ft. wide. 5. 12 in.; \$.21 +. 6. 24 books on each. 7. 3962.789 + mi.; 7925.579 + mi.

Page 237.—8. 12,449½ mi.; 7925.579+ mi. 9. 987.22+ rd.; 2970 A. 10. 39.27 sq. ft.; 9.8175 sq. ft. 12. 873.88+ sq. ft. 13. 70.686 A. 14. 5.6 ft.; 23.75+ ft. 15. 5 hr.

Page 238.—16. 54 min. 20 sec. 17. 8 min. past 4 p.m. 18. 52 min. past 12 p.m. 19. 120 sq. ft. 20. 28.28+ ft. 21. 200 ft. long, 80 ft. wide. 22. 50 ft.; 40 ft.; 100 ft.; 4600 sq. ft. 23. \$56. 24. \$77.77+. 25. \$162.

Page 239.—26. \$72.22+. 27. \$43.125. 28. 3375 cu. ft.; 81 thousand bricks. 29. \$5469. 30. 67+%; 24+%; 6+%; 2+%. 31. 36+%; 24+%. 32. \$15.18+; \$115.3125. 33. \$14.96; 34. \$87.21.

Page 240.—35. \$51.75; \$69. 36. \$112.50. 37. \$919.65+. 38. \$923.50. 39. \$923.50. 40. \$935.04+. 41. \$5737.71. 42. \$498.12. 43. 2827.44 sq. in. 44. \$5.40.

Page 241.—45. 53.66 rd. 47. 65 A. 48. 25 A. 49. 40 A. 50. 153.9384 ft.; 962.115 sq. ft. 51. 840.33+ ft. 52. \$1450.60+. 53. \$496.69. 54. 9½ yd. 55. \$3. 56. \$1260.

Page 242.—57. 15 gal. 1 qt. 58. 400 bu.; \$304.50. 59. \$8700. 60. 43,560 sq. ft. 61. \$46.875. 62. 50% gain. 63. \$88. 64. \$247,800. 65. \$1,820,000. 66. Lost 4%. 67. Tuttle, \$116,000; Rice, \$66,500; Ludlow, \$24,600; Total, \$207,100.

Page 243.—68. \$2080; \$2560. 69. Retail dealer, \$1.60. 70. 33¼%. 71. 51 yr. 1 mo. 10 da. 72. \$576; 10%. 73. 547.2 ft. 74. 3283.2 min. 75. 415 bu. 76. \$320. 77. \$389.25. 78. \$31.68.

Page 247. (6.)—1. (1) 15,985 m.; (2) 432 mm.; (3) 8456 Dm. 2. 46 Km. 8 Hm. 7 Dm. 5 m. 4 dm. 3 cm. 5 mm.; 125 Km. 2 Hm. 4 Dm. 8 m. 6 cm.; 33 Km. 2 Hm. 5 m. 5 dm. 4 mm. 3. (1) 12 Km. 1 Hm. 6 Dm. 2 m. 5 dm.; 1 Km. 2 Hm. 1 Dm. 6 m. 2 dm. 5 cm.; 1 Hm. 2 Dm. 1 m. 6 dm. 2 cm. 5 mm.; (2) 6 Km. 3 Hm. 3 Dm. 7 m. 6 dm.; 6 Km. 5 Hm. 2 Dm. 4 m.; 4 Km. 9 Hm. 3 Dm. 9 m. 6 dm.; (3) 9 Km. 3 Hm. 8 Dm. 7 m. 5 dm.; 9 Hm. 3 Dm. 8 m. 7 dm. 5 cm.; 9 Dm. 3 m. 8 dm. 7 cm. 5 mm.; (4) 9.6875 mil.; 22.53½ mil.; 55.46875 mil. 5. 25.5 Km.; 51 Km.; 42.5 Km. 6. 14 mi. 207 rd.; 21 mi. 226 rd. 7. 1 mi. 180 rd.

Page 250. (10.)—1. 51,800 sq. m. 2. 26,500.09 m. 3. 9.60 sq. m. 4. 90,000 sq. m. 5. 175,000 sq. m.

Page 251.—3. 1½ A.; 2½ A.; 2½ A.; 1½ A.; 1½ A.; 1½ A.; 1½ A. 4. \$.06½; \$338.21½; \$976.95. 5. \$2430; \$5520; \$7008; \$4166.25; \$2430. 7. \$7350; \$5022.50; \$7800; \$19,250; \$26,250; \$15,750; \$12,600; \$30,000; \$10,800; \$248,625; \$918.75; \$32,500.

Page 254. (16.)—1. \$750. 2. \$250; \$312.50; \$687.50; \$1031.25
3. \$100; \$345; \$450. 4. 32 cu. ft.; 248 cu. ft.; 392 cu. ft.; 4,960 cu.
ft.; 336 cu. ft.; 267.2 cu. ft. 5. \$199; \$134.

Page 256. (19.)—1. 4526.378 l. 2. 6 Kl. 2 Hl. 7 Dl. 5 l. 3 dl.
2 cl. 5 ml. 3. 17 Kl. 4 Hl. 8 dl. 4 cl. 9 ml. 4. 3117.35 l.
5. 115.05 l.

Page 257. (22.)—3. \$54.60. 4. \$2.58 +. 5. \$25.
6. \$36.96. 7. \$50.40. 9. 1018.3 + l. 10. 599.44 + l.

Page 258.—11. 1546.11 + l. 13. 326.428 + l. 14. 247.023 + l.
15. 57.73 + l. 16. \$5.95 +. 17. \$9.84 +.

Page 260. (25.)—1. \$17.60; \$22.484; \$19.80. 2. \$3.97.
3. 10,000 Kg.; 25,000 Kg.; 182,500 Kg. 4. \$2640. 5. \$2.94.
6. \$.27 $\frac{1}{8}$. 7. \$46.47 $\frac{1}{2}$; \$41.11 $\frac{1}{4}$. 8. \$66; \$44.

Page 261. (26.)—1. \$36.80. 2. \$295.488. 3. \$5.91 +. 4. \$169.34.
5. 2125.98 in.; 177.16 + ft. 6. \$49,075. 7. \$28,688. 8. \$252,000.
9. \$37.69 +. 10. \$7.01 $\frac{1}{4}$. 11. 50.315 grains. 12. 36 +.

